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Weak Novikov Identity in the Join of (-1,1) Rings

K. Subhashini and P. Vijasekhara Reddy

Department of Basic Scinces , G.Pulla Reddy Engineering College (Autonomous)
Nandyala Road, Kurnool-518007, Andhra Pradesh
subhashini.gprec@gmail.com

Abstract

If R is a (-1,1) ring of characteristic $\neq$ 2,3 with Weak Novikov identity (w,x,yz) = y(w,x,z) then Strong Novikov identity x(yz) = y(xz) holds good in a prime ring R. With this it is proved that R is assoviative.

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1. INTRODUCTION

The identity

(w,x,yz) = y(w,x,z) ………………..(1)

is known as Weak Novikov identity. Where as the identity

x(yz) = y(xz) ………………..(2)

is referred as Strong Novikov identity. If a ring is Strong Novikov then it is Weakly Novikov. Moreover Weakly Novikov rings are a sub class of associative rings where as Strong Novikov rings are not. In [1] E.Klenfield proved that a prime non-associative Weakly Novikov ring with (x,y,z) = (x,z,y) must be Strong Novikov. Again Kleinfeld [3] proved that a semi prime ring of characteristic $\neq$ 2 satisfying the Variations of the novikov identities (xy)z = (xz)y and (x,y,z) = - (x,z,y) is associative. In the another

paper of Kleinfeld [2], it is proved that a prime right alternative ring with minimum condition on right ideals which satisfies the identity (1) must be associative.
A (-1,1) ring R is a non-associative ring in which the following identities hold:

$$(x,y,z) + (x,z,y) = 0 \quad \ldots\ldots\ldots.(3)$$
$$(x,y,z) + (y,z,x) + (z,x,y) = 0 \quad \ldots\ldots\ldots (4)$$

for all x,y,z in R. The associator (x,y,z) is defined by (x,y,z) = xy.z – x.yz and the commutator [x,y] = xy-yx. If there exists a positive integer n such that na = 0 for every element a of the ring R, the smallest such positive integer is called the characteristic of R. Throughout this paper R denotes a (-1,1) ring of characteristic $\neq 2,3$ satisfying Weak Novikov identity given by equation (1).

2. PRELIMINARIES

In R we have the following identity (identity (4) in [4]):

$$(y,(x,y,x)) = 0. \quad \ldots\ldots\ldots\ldots\ldots..(5)$$

From (3) this implies that

$$(y,(x,x,y)) = 0 \quad \ldots\ldots\ldots\ldots\ldots (6)$$

By linearizing the identies (5) and (6) , we have

$$(y,(x,y,z)) = - (y,(z,y,x)), \quad \ldots\ldots\ldots\ldots\ldots.(7)$$

$$(y,(x,z,y)) = - (y,(z,x,y)) \quad \ldots\ldots\ldots\ldots\ldots.(8)$$

From equations (3),(6),(7) and again using (3)

$$(y,(y,z,x)) = - (y,(z,x,y)) = - (y,(x,z,y)) = (y,(x,y,z)) \quad \ldots\ldots\ldots(9)$$

Commuting (4) with y, we have (y,((x,y,z) + (y,z,x) + (z,x,y))) = 0.
From (8) this equation becomes 3(y,(x,y,z)) = 0. Since R is of char $\neq 3$

$$(y,(x,y,z)) = 0. \quad \ldots\ldots\ldots\ldots\ldots(10)$$

The following identity $\overline{L}$ in [4] holds in R: (x,(y,y,z))- 3(y,(x,z,y)) = 0.
From (8) and (9) this equation becomes (x,(y,y,z)) = 0. Thus

(R,(y,y,z)) = 0. …………… (11)

Linearize equation (10), we obtain

(w,(x,y,z)) = - (w,(y,x,z) ……………(12)

Applying equations (3) and (11) repeatedly, we get

(w,(x,y,z)) = - (w,(y,x,z) = (w,(y,z,x)) = - (w,(z,y,x) = (w,(z,x,y)) ……(13)

Commute equation (4) with w and apply (12).
Then 3(w,(x,y,z)) = 0. Since char $\neq 3$,

(w,(x,y,z)) = 0 . ……………(14)

3. MAIN SECTION

The nucleus N of any ring is defined as N = { n € R / (n,R,R) = (R,R,n) = (R,n,R) = 0 }.

Lemma 1 : let n € N then (R,N) $\subseteq$ N.

Proof : Let w,x,y,z € R and n € N. Consider Teichmuller identity which holds in any Ring:

(wx,y,z) – (w,xy,z) + (w,x,yz) + (w,x,y)z …………(15)

we now take a turn letting one of four elements in (15) be in the nucleus N. Thus
(nx,y,z) = n(x,y,z)
(wn,y,z) = (w,ny,z)
(w,xn,z) = (w,x,nz) …………..(16)
(w,x,yn) = (w,x,y)n …………..(17)

By using equations (14), (3) , (17) , (3) and (16) we have
n(x,y,z) = (x,y,z)n = -(x,z,y)n = -(x,z,yn) = (x,yn,z) = (x,y,nz).
But from (8) (x,y,z)n = (x,y,zn) = (x,y,nz)
Hence (x,y,zn) –(x,y,nz) = 0. Thus

$$(x,y,(z,n)) = 0. \qquad \ldots\ldots\ldots\ldots\ldots(18)$$

From (3) and (4) equation becomes $(R,N) \subseteq N$.

Lemma 2 : The nucleus N of R is an ideal such that NA = 0. If R is prime and non-associative then N = 0.

Proof : For arbitrary elements x,y,z € R and n € N, from (1) we have (x,y,zn) = z(x,y,n) = 0. Also from (18) (x,y,nz) = (x,y,zn) = 0. Therefore N is both left and right ideal hence an ideal of R. Again using (1) and (14) (x,y,nz) = 0 = n(x,y,z) = (x,y,z)n. That is NA = AN = 0. Since R is prime and not associative, and hence N = 0.

Lemma 3 : If R is prime and not associative then R is Strongly Novikov.

Proof : Through the repeated use of (1) and (3) for any a,b € R we obtain
(a,b,x.yz) = x(a,b,yz) = -x(a,yz,b) = -(a,yz,xb) = (a,xb,yz) = y(a,xb,z) = -y(a,z,xb) = -y.x(a,z,b) = y.x(a,b,z) = y.(a,b,xz) = (a,b,y.xz).
Therefore (a,b,x,yz – a,b,y.xz) = 0, this implies that
(a,b,x.yz-y.xz) = 0. Therefore x.yz – y.xz € N. From lemma 2, N = 0, hence we have the Strong Novikov identity x.yz = y.xz holds in R.

Lemma 4 : If a,b,c,d,x,y and z be arbitrary elements in R. Then we have

(a) [a,b](xy.y) = 0,
(b) c(a,b,c) = 0 ,
(c) d(a,b,c) = -c(a,b,d) and
(d) (a,b,c)(xy.y) = 0.

Proof : Using equation (1) the following commutator can be written as
z [y,z] = z(yz) - z(zy) = z(zy) – z(zy) = 0, so that

$$z\,[y,z] = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(19).$$

Linearizing (19) gives

$$x\,[y,z] = -z\,[y,x] \qquad \ldots\ldots\ldots\ldots\ldots\ldots(20).$$

Using the equations in the order (1) , (20) , (1) and (19) gives
[a,b](xy.y) = xy.(y[a,b]) = -xy.(b[a,y].xy) = -b(x.y[a,y]) = 0. Thus equation (a) is proved. Using (1) , we have c(a,b,c) = c(a.bc) – c(ab.c) = c(ab.c) – c(c.ab) = c(ab.c) – c(ab.c) = 0. Thus equation (b) is proved. Linearizing equation (b) gives equation (c). similarly when we use the equations in the order (1), (c) , (1) , (1) and (a) we get (a,b,c)(xy.y) = xy.(y(a,b,c)) = -xy.(c(a,b,y)) = c((a,b,c).xy) = c(x.y(a.b,y)) = 0. With this (d) is proved and hence lemma 4 is proved.

4. MAIN THEOREM : If R is a prime (-1,1) ring of characteristic$\neq 2,3$ then R must be commutative and associative.

Proof : From [Lemma 9 of [5], if R is not associative, R must be alternative, but a prime alternative ring which is not associative must have an identity element. Using identity element in the equation (d) implies R is associative, a contradiction. Thus R must be associative . Once R is associative, it is easy to show that it must also be commutative .

REMARK : In view of equations (3) and (4) a commutative (-1,1) ring of characteristic $\neq 2,3$ is associative.

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