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Generation of Fractional Resonance in Nonlinear Physical Systems due to Their Internal Forces Motion’s Instability

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Abstract

The work discusses the results of the author’s discovery: the existence of a fundamental association between the phenomenon of self-excited fractional resonance in nonlinear physical systems and the instability of forced oscillations, accepted as harmonic ones, which is acceptable in the first approximation. In previous works, the author showed a wide distinction between the behavior of symmetric and asymmetric systems with respect to the restoring force of the system. The behaviors of both systems' types deviate significantly from each other. The asymmetric behavior is of much more interest both for practical and theoretical studies. The main goal of the article is to combine various types of physical systems with energy-intensive parameters (like rigidity) with, for example, a general approach and to show that their behavior in a critical state of instability reveals one of two outcomes – a generation of fractional resonances or an amplitude jump phenomenon, in a case when a critical state coincides with the main resonance. It should be noted that the amplitude jump occurs in the first zone of unstable symmetric systems. The main resonance and the jump effect in asymmetric systems occur in the second instability zone.

Keywords: fractional resonance, state of motion instability

1 Introduction

The feature of nonlinear oscillatory systems is known to be the ability to generate harmonics with the highest frequency. Thus, it is obvious that the parameters of the systems change accordingly, both at free and forced oscillations, especially those that are energy-intensive (rigidity, capacitance, inductance).

Forced oscillations are of practical interest. Besides modulating the nonlinear energy-intensive parameter, they also provide a constant flow of energy into the system.

It is also known that the periodic modulation of this parameter under certain combinations of the external influence frequency and the natural frequency of the vibrator (oscillator), can cause a parametric resonance, developing in accordance with the Mathieu-Hill equation exponentially. To date, the parametric resonance phenomena as a result of the instability of rest (instabilities “in the small”) related to linear systems, have been studied in detail. The non-linearities play an auxiliary role of the amplitude limiter of the arising resonance.

A great contribution to the theory and practice of parametric systems was made by V.V. Bolotin [1]. It is important to note that in such systems a periodic external force acts directly on the energy-intensive parameter, causing its modulation and, as a result, leading to the resonance of the oscillator.

2 Behavior characteristics of fractional resonance in nonlinear systems

Before moving to nonlinear systems, it is important to note that the issue of the modulation of the energy-intensive parameter by forced oscillations (which is the only possible option in the absence of additional conditions for this purpose), has not been developed for a long time, although no one has challenged it.

Such ideas began to contradict the known experimental observations. In particular, there were reports of so-called fractional resonances. The new topic arose interest of L.I. Mandelstam and N.D. Papalexi [2]. Working on the theoretical part of the problem, they suggested that, due to the modulation of parameters by forced oscillations in nonlinear systems, the resonances of the ½, 1/3, ¼ order, etc. may occur. That is the frequency division resonances at the input to the oscillator by 2, 3, 4, times (subharmonic responses of the system located on the frequency axis to the right of the main resonance). However, when experimentally observing a potentially self-oscillating system (which is not excited to a state of spontaneous generation), namely, a non-oscillating system, the researchers found that only one oscillator response, namely, a subharmonic resonance of ½ order, possess the property of self-excitation of fractional resonances. This contradiction between the developed theory and the results of the experiment seemed irresolvable for a long time.

The author experimentally discovered and theoretically substantiated a previously unknown property of systems with a characteristic (asymmetric with respect to the restoring force) to form the only zone of the instability of forced oscillations in the resonance zone. Namely, it alone is possible to excite the subharmonic resonance of ½ order, which was observed in [2], and the system itself contained a quadratic term, without which a resonance of ½ order would be impossible. The property of asymmetric systems to form the first (main) instability zone was checked by the author for the systems described by the Duffing equation [3, 5], potential self-oscillatory systems, as well as piecewise-linear systems [7]. The desired first zone for a somewhat simplified potentially self-oscillating system turned out to be close to an oval in shape.

The scientific significance of the discovery is the fact that at the design stage it is possible to predict the possibility of the appearance of certain resonances that are dangerous for the operation of machines, which usually operate in the after primary resonance frequency domain.

The practical significance is the fact that for the first time both experimentally and theoretically the dangerous role of the gap in bearings [7], which creates favorable conditions for the generation of a quite powerful (compared to other resonances) subharmonics of ½ order was revealed. The power of a subharmonic directly depends on the degree of asymmetry of the restoring force, and that, in turn, determines the depth of modulation. For example, in a piecewise linear system with a gap shown in [7], the modulation depth of a parameter can approximate to a unity, especially if the gap is theoretically capable of becoming infinitely large, as in the example given in the section “Proof of the discovery accuracy”.

3 Essence of the discovery

The essence of the discovery lies in the fact that the author has experimentally discovered and theoretically substantiated the fundamental association between the phenomenon of resonance self-excitation in nonlinear oscillatory systems and the state of their instability or, more specifically, the instability of forced oscillations in them. Since the above-mentioned association is established, there is a direct path to search for the desired resonances in the sense of determining both the order of these resonances, usually fractional and their location on the frequency axis. This path obviously leads to the study of the stability of the forced motion mode. The ultimate goal of this procedure is to find the boundaries of these regions, where one can expect the phenomenon of self-excitation of fractional resonances. The task of studying the stability of forced oscillations is considerably simplified if, in the first approximation, the main harmonic coinciding in frequency with the external periodic action is taken as their basis. This approach is quite reasonable if the study is conducted near the main resonance. These assumptions lead to the equations in Mathieu-Hill variations, for which the stability-instability criteria have already been established.

It is important to note that the harmonic composition of these equations is fundamentally different for symmetric and asymmetrical systems with respect to the restoring force [7]. Thus, in the symmetric variant, only even harmonics occur in them. Hence the first instability region for symmetric systems always coincides with the main resonance (the phenomenon of amplitude jump), all other regions are of the frequency range of superharmonics. The after resonance zone for these systems is stable everywhere [4, 7].

For asymmetric systems, the location of the instability regions is mixed: from the first (main) region, where it can be expected a self-excitation of ½ order in the resonance zone (to the right of the main resonance), to the superharmonic in the pre-resonance frequency range, starting from the third instability zone, where sub-subsuperharmonics of 3/2 order are possible. In analog modeling, this area was clearly visible [7].
The second zone of instability is revealed as an amplitude jump and coincides in frequency with the main resonance of the asymmetric system under study. A characteristic feature of asymmetric systems is their peculiarity that, at small and medium amplitudes of external influence, the skeletal curves of all resonances lean to the left ("soft" properties). Then with increasing frequency of the external force, they move to rigid properties.

4 Proof of the discovery's accuracy

The idea of the existence of a direct connection between the phenomenon of self-excitation of fractional resonances and the state of instability of the forced motion of a vibrator arose while observing the behavior of an experimental sample of an oscillatory system described by the Duffing equation (the Duffing system).
The developed sample consists of a support part (sole) and has a shape of a curved line, the curvature of which was calculated according to the known formulas depending on the coefficient of the cubic term of the Duffing equation. The support part was mounted on a standard vibration table with variable vibration parameters. A flat spring made of high-speed steel and carefully polished was rigidly fastened on top of it. A load with interchangeable mass was fixed at one end of the spring. Thus, an asymmetric model of the Duffing system with quadratic and cubic terms of the restoring force was obtained. The lateral surface under the spring had a smooth downward slope. With forced oscillations of the mass and the spring itself, it was easy to see that during the upward movement of the mass, the spring rigidity decreased due to the increase in its length from the point of junction with the support. On the contrary, the stiffness increased when moving down. That is, without any stretch, one can consider the device as a vibrator with a variable energy-intensive parameter-stiffness. Therefore, when the frequency of the vibration table approached the value of the vibrator's doubled natural frequency (measured at small oscillations), the system began to be slightly excited, passing into a fairly powerful resonance at a frequency that is half the frequency of the external source and the corresponding sharp tuning to the desired oscillation mode. Thus, all attempts to implement a subharmonic vibrator response of ½ order were successful in any combination of the mass and length values of the spring.
It was not possible to cause self-excitation of other subharmonic resonances in the same way, by analogy with the results obtained earlier in [2]. Sometimes it was possible to implement a similar-to-resonance effect with a strong external influence (impact on the mass). However, such a situation cannot be associated with instability "in the small", which implies infinitesimal initial conditions. In addition,

the nature of the vibrator movement after such an impact on it consisted probably of many harmonics (although it was periodic), that is, was very far from being harmonic.

Besides useful experimental observations of the behavior of an asymmetric variant of the Duffing system, one can easily prove theoretically by elementary calculations that the boundary of the instability region of forced oscillations (in the first harmonic approximation) clearly coincides with the subharmonic zone of ½ order. Using the method of harmonic balance, find the explicit expressions for both the amplitude A of the main harmonic solution and the amplitude of the subharmonic A½. When investigating the stability of a harmonic solution and comparing it with another, previously obtained expressions for a subharmonic at A½=0, one can see that these two expressions coincide, as was to be proved.

Of course, in more complex cases, when the characteristic of the restoring force is non-analytic and, possibly, with a gap (piecewise-linear systems), it is quite problematic to find analytical expressions for the amplitudes of the fundamental oscillations and fractional resonances. The first part of the problem, which is related to finding instability regions, is solvable, albeit with some difficulties. For this purpose, it is recommended to use the already proven approach [4] for an approximate consideration of the issue widely used in nonlinear automatic control systems (ACS). This approach was successfully applied in the task of the stability of an elastic system with a gap [7, 8]. The results of the analytical calculation turned out to be quite close to the boundaries of the instability regions, obtained by an analog simulation.

The proof of the discovery accuracy can be continued with examples of other oscillatory systems with an energy-intensive parameter. It was found that in symmetric systems the parameter modulation depth cannot be significant, therefore the instability zones of them have narrow and principally possible self-excited resonances (always even-order superharmonics) are weak and easily suppressed by damping in the system. In addition, the counting of zones starts with the second, since the first zone coincides with the main resonance.

In [8], the problems of stability in a piecewise-linear system with a gap are considered in detail, which is of undoubted interest in calculating machine parts and mechanisms based on sliding bearings. Accordingly, for the cases of using balls or rollers as supports, the task is complicated by several times, since these bearing elements themselves have pronounced non-linear properties. The piecewise linear system with a gap itself has unique capabilities that contribute to the deep modulation of the energy-intensive parameter, because under the detachable oscillations of the shaft or rotor from the supports, the rigidity of the system can vary in a large range from zero in the trunnion separation phase from the lower support to some constant value known at a design stage, in a phase of contact of a shaft trunnion with the lower bearing support. For experimental verification of the active role of the gap in the formation of conditions for self-excitation of a subharmonic of ½ order, a simple experiment was conducted. Instead of two supports for the shaft installation, a steel ring with a diameter of 140 mm, a height of 50 mm and a wall thickness of 3.5 mm was manufactured. The lower part of the ring was securely fixed on the surface of the vibration table with adjustable vibration parameters.

A 150 mm cylindrical steel rod of 4 mm in diameter with a vibration sensor fixed on its center, the signals from which were sent to an oscilloscope, was freely placed on the top of the ring. It resulted in a simplest piecewise linear system with only one elastic element and an infinitely large gap, which is noticeable only when the detachable oscillations of the rod with the sensor from the ring began. Obviously, the detachable oscillations could be realized under the condition that the amplitude of forced oscillations exceeded the magnitude of the deflection of the rod from its weight and the weight of the vibration sensor, which, moreover, played the role of the vibrator mass. It is obvious that basically the same vibrator would have been obtained after removing the top covers from the bearings of a real installation. Under small forced vibrations, the assembled vibrator does not differ from the usual linear system. When the rod was detached from the ring with an increase in the amplitude of the external action, the system immediately turned into strictly nonlinear, due to the formation of two oscillation phases in the linear mode instead of two, which are fundamentally different from each other: one is in isolation, another is in contact with the ring. The rigidity of the vibrator changed from zero at the upper phase and returned to a constant value when contact was restored. With phase alternation, a sufficiently deep modulation of the parameter took place, and at a frequency from the vibration table twice as high as the natural frequency of the system, a bright subharmonic resonance of ½ order was observed, corresponding to a sharp tuning of the system.

It is interesting to note that the role of the gap in the piecewise-linear system was not easy to reveal, but mathematics did it. Here mathematics is ahead of the researcher. Since the name of the discovery goes beyond the framework of mechanical systems, the author will pay some attention to physical systems, for example, electronic, where the ½ order subharmonic works very widely and for a long time, which, as was shown, also occupies the main and special place among the other, usually weak self-excited system responses.

In 1962, the Bulletin of Moscow State University published an article by T.V. Ilinova and V.V. Migulin. It reported that the authors had discovered in the experiment a powerful generation of a subharmonic of ½ order, due to the inclusion of a semiconductor diode in the oscillatory circuit with a pronounced inductance asymmetry of a diode. Note that if the diode had had a characteristic close to a symmetric one, then the effect of self-excitation of a subharmonic of ½ order would have been a quite ordinary case. The phenomenon observed in8 was called a parametric resonance, with the proviso – in a "nonlinear system".

The proposed approach has a fairly large generality in the sense of covering a significant number of nonlinear systems with an energy-intensive parameter. Therefore, the active subharmonic generation in a system with one degree of freedom, described in [6], can also be included in the range of phenomena that have already been considered, starting with the Duffing equation. Thus, the ½ order subharmonic automatically falls into the first (main) zone of the instability of forced oscillations of asymmetric systems. At the same time, instability should not be understood primitively as the beginning of some random motion, because in the theory of oscillations, it is often the point of bifurcation of a certain process, called in this case in the first approximation the fundamental harmonic of forced oscillations. At the point of bifurcation, a new quality is often generated, giving important and not always predictable results.

5 Scientific and practical significance of the discovery

The proposed discovery affects the essential basics of the physics of oscillatory processes, among which the phenomena of resonances of various nature are of primary interest both to practitioners operating high-speed techniques and researchers of the resonance occurrence nature, often representing a real danger to the stability of the mechanisms. On the other hand, it is impossible to explain the fantastic progress in this field of science and technology without the use of resonances in modern electronics. A special place among other resonances (also called "responses" of oscillatory systems), is occupied by self-excited subharmonic responses of ½ order, leading to a significant progress in electronics.

It should be noted that the term "self-excited" resonance used in this article for the analysis of nonlinear oscillatory systems have not yet found sufficient distribution. Moreover, the existing scientific ideas in the field of the study of oscillatory processes do not suggest the existence of self-excited responses in the most common non-linear systems in practice. The scientific significance of the proposed discovery consists in changing the existing scientific concepts. In particular, it is shown that essentially asymmetrical systems necessarily form the main region of unstable forced oscillations in the resonant frequency range, that is, to the right of the main resonance, where ½ order self-excitation is guaranteed with moderate attenuation in the system.

The practical significance of the proposed discovery is the development of a method for finding self-excited responses of nonlinear systems with energy-intensive parameters. When analyzing piecewise-linear systems, the role of the gap has been determined for the first time. The gap leads to the effect of separation oscillations when the modulation depth of the energy-intensive parameter (rigidity) can reach maximum values contributing to the expansion of the main instability region and the generation of the subharmonic of ½ order, based on shafts and rotors. The useful application of self-excited resonances can be observed in mechanical frequency converters in vibration technology, where the current industrial frequency of electric current is too high for the vibration transport of some delicate units.

6 Conclusion

The phenomenon of self-excited resonances in nonlinear physical oscillatory systems due to the instability of forced oscillations in them is proposed as the discovery in this work. The discovery relates to the field of physics of oscillations,

forced oscillations of nonlinear physical systems, and their stability.
At the same time, the instability of the forced movement in systems with one degree of freedom and one or several energy-intensive parameters that are necessarily presented in them means the state of the systems when bifurcation points appear in them due to modulation of the parameter, resulting in a bifurcation of the stable movement existing up to this point. Now the new steady state of the oscillating system will correspond to a binary mode of motion with keeping the basic oscillation, as a first approximation of a harmonic and a new component, usually in the form of fractional resonances, of orders ½, 2/2, 3/2, etc. For the transition to a new mode, only infinitesimal initial conditions are sufficient, that is, instability "in the small". It is shown that the nature of self-excited fractional resonances is certainly parametric, but with a number of features that distinguish them from classical parametric resonances, for example, the existence of a "threshold" and "ceiling" [5, 7] for them, depending on the amplitude of the external force and its frequency.
The essence of the discovery is that for the first time since the first studies of fractional resonances of L.I. Mandelstam and N.D. Papalexi, a fundamental connection between the phenomenon of self-excitation of fractional resonance and the state of instability of forced oscillations was found8. The discovery changes the existing ideas about the behavior of nonlinear systems that do not imply the existence of self-excited resonances. However, in electronics one of the fractional resonances of ½ order (subharmonic resonance) is of the same nature as observed in the study [2]. It has been used quite widely since the mid-60s, although the theoretical side of the phenomenon has not been developed completely correctly since even in a somewhat unusual non-linear oscillating circuit using a semiconductor diode as a parameter cannot exist in any special case of classical parametric resonance. Some of the proposed by L.I. Mandelshtam variants: autoparametric (in the author's opinion, most adequately corresponding to the parameter modulation mechanism), N-kind resonance, fractional, etc. One can choose one of them.
The practical significance of the discovery lies in the fact that the proposed approach develops a technique to search for self-excited responses of different types of systems that have an energy-intensive parameter as a restoring force. In addition, the approach can be the basis for the experimental study of the instability of forced oscillations in devices with a support on sliding bearings.

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References

[1] V. Bolotin, *Dynamical Stability of Elastic System*, Gostekhizdat, Moscow, 1956.

[2] L. Mandelstam and N. Papalexi, Concerning resonances phenomena of the N-class, *Zhurnal Tekhnicheskoi Fiziki*, **II** (7-8) (1932).

[3] E. Popov and E. Paltov, *Approximate Methods for the Investigation of Non-linear Automatic Control Systems*, Fizmatgiz, Moscow, 1960.

[4] W. Szemplinska-Stupnicka, Higher harmonic oscillations in heteronomous non-linear systems with one degree of freedom, *Int. J. of Non-Linear Mech.*, **3** (1) (1968), 17-30. https://doi.org/10.1016/0020-7462(68)90022-x

[5] L. Vorotyntsev, Generation of subharmonic from instability in system with quadratic non-linearity, *Reports from Georgian Academy of Sciences*, **86** (1977).

[6] L. Vorotyntsev, On forming of dynamic instability of asymmetric systems, *Vibrotechnique*, **4** (1978), 29.

[7] L. Vorotyntsev, On stability of forced vibrations in elastic system with clearance, *Mashinovedenie*, **1** (1982).

[8] L. Vorotyntsev, A new approach to the problem of rotor-bearing stability, *Contemporary Engineering Sciences*, **9** (6) (2016), 289-302. https://doi.org/10.12988/ces.2016.512325

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