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Application of the Denavit - Hartenberg Method to Estimate the Positioning Errors of an Automated XYZ Cartesian Table

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Abstract

This document describes the fundamental criteria in the design of an automated Cartesian table with three XYZ axes with an interchangeable head for plasma and oxyfuel cutting, as well as the selection of the processes implemented for its construction. For this purpose, the table is divided into four fundamental parts: X axis, Y axis, Z axis and control system. As it is a table for a high precision process such as plasma, a study of machine errors is carried out using the Denavit - Hartenberg method and the errors associated with deformation using simulation software. To conclude, it is determined that with the design detailed in this document, it is possible to build a CNC table with high precision and low cost.

Keywords: CNC table, design, CAD, error calculation

1. Introduction

The machine tools are a type of precision machine that because of their particular performance are expected to be high-performance and high reliability machines. The automated cutting table design is a cross-disciplinary project that includes computational, electronical and electrical-mechanical concepts, among other disciplines of interest. The study of errors can be carried out by means of homogeneous transformation matrices, since this method allows for one part of the machine to be taken into consideration at a time, transforming coordinate systems from one reference system to another, transforming errors having into account the different parts of the machine and it take them to a reference system for the cutting tool and the working material [1].

In an effort to improve accuracy, authors as Guoyong Zhao [2] have conducted studies to quantitatively determine geometric and kinematic errors, concluding that the methods presented in their research significantly increase accuracy and it's maintained for a long time. Similarly Bermejo et al. [3] In their degree project presented at the Universidad del Atlántico, they were dedicated to finding and quantifying the geometric errors presented in a double column milling machine, developing a methodology that converges in a calculator that allows quantifying the positioning and speed errors for any position of the milling machine studied.

On the other hand, research conducted by LI-BAO AN [4] and Mohd Fadzil et al., were focused on studying the operating parameters and determining the optimal values to carry out this process in order to minimize operating costs for particular case studies.

This work focuses on the calculation of errors of a CNC machine with interchangeable head and good precision that allows different processes to be performed to obtain a wide field of research. Based on the methodology of Denavit and Hartenberg, a study of kinematic and geometric errors is carried out for their minimization and high precision, achieving with this a construction with low budget elements.

2. Methodology

Precision is of vital importance in CNC machines, that is why this project seeks to quantify the geometrical errors presented in the CNC Cartesian table that has been designed. To achieve this, an error prediction methodology is used, using models based on direct and inverse kinematics. These models allow the geometrical errors of the machine tool axes to be determined independently of the process carried out. These models are intended to quantify the position (orientation) and velocity (linear and angular) errors at the tip of the tool as a function of the errors in the kinematic pairs. These errors are the result of the lack of parallelism and perpendicularity resulting from a defective assembly [5].

To determine the relationship between tool speeds and joint movement speeds, the differential model expressed in a Jacobian matrix is used. The Denavit-Hartenberg method is also used to determine the spatial relationship between the machine elements and a fixed reference system.

2.1 Denavit & Hartenberg model

Denavit and Hartenberg [6] established an organized method to describe and represent the spatial geometry of the elements of a kinematic chain over a fixed reference system as shown in Figure 1. The method uses a homogeneous transformation matrix to describe the spatial relationship between adjacent elements, converting the problem to the resolution of a 4 x 4 homogeneous transformation matrix that relates the position of the tool to the stationary reference system.

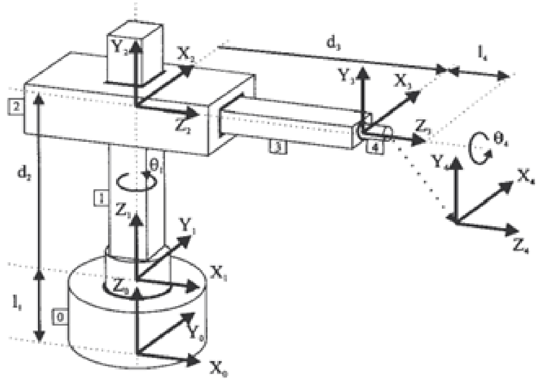

Figure 1. Robot with reference systems of the Denavit - Hartenberg method.

To apply the Denavit-Hartenberg method the following algorithm must be followed [7]. Step 1: Number the links starting with 1 (first link of the chain) and ending with n (last moving link). It shall be listed as link 0 to the fixed base (reference system); Step 2: Number each joint starting with 1 and ending with n; Step 3: Locate the axis of each joint. If it is rotary, the axis will be its own rotary axis. If it is prismatic, it will be the axis along which the displacement takes place; Step 4: For i from 0 to n-1, place the zi axis on the axis of the i+1 joint; Step 5: Place the origin of the system of the base (S0) at any point of the z0 axis. The x0 and y0 axes will be positioned so that they form a dextrogyro system with z0; Step 6: For i from 1 to n-1, place the S0 system (in solidarity with link i) at the intersection of the z0 axis with the normal line common to zi-1 and zi. If both axes were to be cut, it would be placed (Yes) at the cutting point. If they were parallel (Si) it would be in the i+1 joint; Step 7: Place xi on the normal line common to zi-1 and zi; Step 8: Place yi so that it forms a dextrorototation system with xi and zi; Step 9: Place the Sn system at the end of the tool so that zn coincides with the direction of zn-1 and xn let it be normal to zn-1 and z0. Step 10: Get Ɵi as the angle to be rotated around zi-1 so that xi-1 and xi be parallel; Step 11: Obtain di as the average distance along zi-1 that would have to be displaced by Si-1 so that xi and xi-1 would be aligned. Step 12: Obtain ai as the average distance along xi, the new Si-1 would have to be displaced to match its origin with Si; Step 13: Obtain αi as the angle that would have to be rotated around xi to make the new Si-1 fully coincide with Si; Step 14: Obtain the i-1Ai; transformation matrices defined in equation 2; Step 15: Obtain the transformation matrix that relates the system of the base to the one of the end of the tool (equation 1); Step 16: The T matrix (equation 1) defines the orientation and position of the end referred to the base as a function of the n joint coordinates, where θ_i is the angle formed by the x_{i-1} and x_i axes measured in a plane perpendicular to the z_{i-1} axis, using the right-hand ruler. It is a variable parameter in rotating joints; d_i is the distance along the z_{i-1} axis from the origin of the coordinate system (i-1)-th to the intersection of the z_{i-1} axis with the x_i axis. It is a variable parameter in

prismatic joints; a_i is the distance along the x_i axis from the intersection of the z_{i-1} axis with the x_i axis to the origin of the i-th system, in the case of rotating joints. In the case of prismatic joints it is calculated as the shortest distance between the z_{i-1} and z_i axes and α_1 is the separation angle of the z_{i-1} and z_i axis, measured in a plane perpendicular to the x_i axis, using the ruler of the right hand.
The data were considered yielding the homogeneous transformation matrix as follow is Equation1.

$$ {}^{i-1}A_i = \begin{pmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1} $$

The $^{j\text{-}1}\boldsymbol{A_i}$ matrices are then introduced in Equation 1 to finally obtain the T matrix in Equation 2.

$$ T = {}^{0}A_1\,{}^{1}A_2\,{}^{2}A_3\,{}^{3}A_4 \tag{2} $$

Based on the D-H algorithm, we will apply it to our Cartesian table. Beginning with the diagrams in Figures 2a and 2b that allow us to observe the links, joints and reference systems of the Cartesian table.

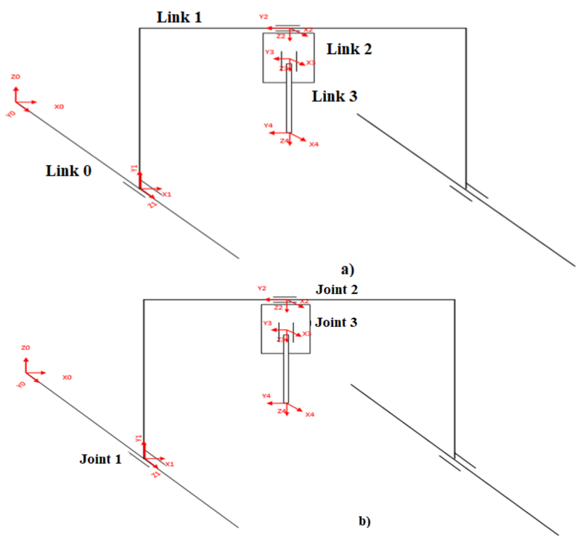

Figure 2. Cartesian table diagram with numbered, a) links, b) joints.

2.2 Axis movement errors

As mentioned above, displacement errors are classified into two groups, parallelism errors and perpendicularity errors both from a defective assembly. The perpendicular error as shown in Figure 3 is caused by the lack of ideal perpendicularity between two axes.

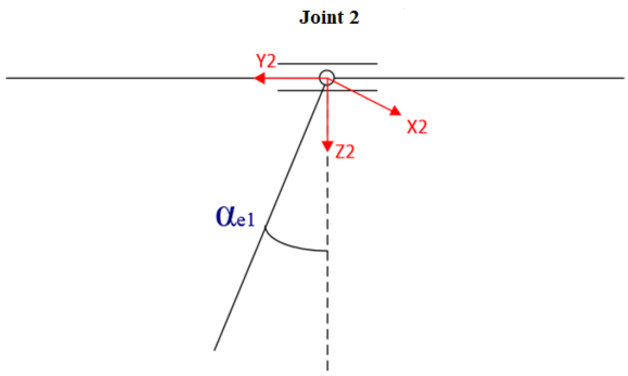

Figure 3. Perpendicularity error.

In the model on which this document is based, two simultaneous perpendicularity errors are proposed in both joints. For these types of errors, ISO standards are followed, which define the conditions that must be followed for a correct perpendicularity error test. In the selected tests, the perpendicularity errors occur in the 2nd and 3rd articulation and based on the ISO 10791-2:2001 [8] standard, adjusting the tolerances dictated by that standard, the transformation matrix T is recalculated and subtracting this matrix with the matrix without errors gives the geometric error of the machine for any position.
The first error represented in the Z axis perpendicularity can be quantified using Equation 3 [8].

$$\alpha_{2real} = \alpha_{2ideal} - \alpha_{e1} \,. \tag{3}$$

Using the Equation 4, the Error resulting from the deviation can be quantified.

$$\alpha_{e1} = arctan\frac{\delta_{z2}}{l} \,, \tag{4}$$

where $\delta_{z2}$is the deformation of the Y axis about the X axis; l is the critical length for Y axis deformation.

Based on the normalized error tolerances, the ISO perpendicularity error has a value of 0.002 [8]. Therefore we have to $\alpha_{e1} = 0.00013544°$, later replacing this value in Equation 4, $\alpha_{2real} = -90.00013544°$.

Related to the the second error in the 3th articulation, as shown in Figure 4 the effects derived from the deviation of perpendicularity in the Z axis. The consequences are θ_e2 which would be a deflection of the tool in clockwise and α_e2, which would be a deflection of the tool counterclockwise. By regulation this deflection has a value of 0,016 [5], [8].

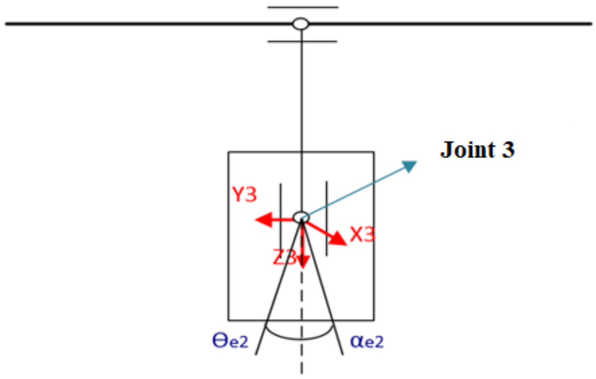

Figure 4. Perpendicularity error in articulation

This second error changes the value of α_3, and introduces a new matrix to the T matrix product, because an error on the Z axis also moves the axis. Therefore using the equation 3 the result obtained wase $\pm 0.0010835°$ for the counterclockwise movement [3], [5].

2.3 Errors with Solidworks® simulation

Looking for the way to approximate as closely as possible the result of errors obtained by means of the study to the real value of errors that the machine will present when is working, the method proposed in this section is proposed. The method consists of using the values given by the Solidworks® software simulation tool.

3. Results and Discussion

3.1 Parameter table and application fo the method D-H

Taking into account the values obtained in section 2.2, where the perependicularity error of each axis was found, these values were entered into a new D-H matrix with an intermediate matrix called M as shown in Equation 5.

$$M = \begin{bmatrix} C\theta_{e2} & 0 & S\theta_{e2} & 0 \\ 0 & 1 & 0 & 0 \\ -S\theta_{e2} & 0 & C\theta_{e2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (5)$$

where θ_{e2} as well as $\alpha_{e2} = 0.0010835°$

The Denavit-Hartenberg algorithm including the errors is then performed amd obtained the result presented in Table 1 and equation 6 to 13

Table 1. Actual values of the D-H parameters of each matrix

Articulation	**Θ**	**D_i**	**a_i**	**α_i**
1	0	D_1	A_1	90
2	90	D_2	A_2	α_{2e}
3	0	D_3	A_3	α_{3e}
3-4	0	0	0	[M]
4	0	D_4	A_4	0

$$^{0}A_1 = \begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6),$$

$$^{1}A_2 = \begin{bmatrix} 0 & -C\alpha_{2e} & S\alpha_{2e} & 0 \\ 1 & 0 & 0 & a_2 \\ 0 & S\alpha_{2e} & C\alpha_{2e} & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

$$^{2}A_3 = \begin{bmatrix} 1 & 0 & 0 & a_3 \\ 0 & C\alpha_{3e} & -S\alpha_{3e} & 0 \\ 0 & S\alpha_{3e} & C\alpha_{3e} & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8),$$

$$^{3}A_4 = \begin{bmatrix} 1 & 0 & 0 & a_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (9)$$

$$M = \begin{bmatrix} C\theta_{e2} & 0 & S\theta_{e2} & 0 \\ 0 & 1 & 0 & 0 \\ -S\theta_{e2} & 0 & C\theta_{e2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

While the Error Matrix is now defined as shown in equation (11)

$$T_e = {}^0A_1\ {}^1A_2\ {}^2A_3 M\ {}^3A_4 =$$

$$\begin{bmatrix} -S\theta_{e2}C\alpha_{2e}S\alpha_{3e} - S\theta_{e2}S\alpha_{2e}C\alpha_{3e} & S\alpha_{2e}S\alpha_{3e} - C\alpha_{2e}C\alpha_{3e} & C\theta_{e2}C\alpha_{3e}S\alpha_{2e} + C\theta_{e2}S\alpha_{3e}C\alpha_{2e} & a_1 - a_4S\theta_{e2}S\alpha_{3e}C\alpha_{2e} - a_4S\theta_{e2}C\alpha_{3e}S\alpha_{2e} + d_4S\alpha_{2e}C\alpha_{3e}C\theta_{2e} + d_4C\alpha_{2e}C\theta_{e2}S\alpha_{3e} + d_3S\alpha_{2e} \\ S\theta_{e2}C\alpha_{2e}C\alpha_{3e} - S\theta_{e2}S\alpha_{2e}S\alpha_{3e} & -S\alpha_{2e}C\alpha_{3e} - C\alpha_{2e}S\alpha_{3e} & C\theta_{e2}S\alpha_{2e}S\alpha_{3e} - C\theta_{e2}C\alpha_{2e}C\alpha_{3e} & a_4C\alpha_{2e}S\theta_{e2}C\alpha_{3e} - a_4S\alpha_{2e}S\theta_{e2}S\alpha_{3e} + d_4S\alpha_{2e}C\theta_{e2}S\alpha_{3e} - d_4C\alpha_{2e}C\alpha_{3e}C\theta_{e2} - d_2 - d_3C\alpha_{2e} \\ C\theta_{e2} & 0 & S\theta_{e2} & a_4C\theta_{e2} + d_4S\theta_{e2} + a_2 + a_3 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (11)$$

As many of the terms have known numerical values, this value is replaced to obtain a simpler matrix:

$$\begin{bmatrix} 1{,}891064 \times 10^{-5} & -1{,}6546768 \times 10^{-5} & -0{,}999999999684296 & a_1 + 0{,}00001891a_4 - 0{,}999999999684d_4 - 0{,}99999999972d_3 \\ 3{,}129100 \times 10^{-10} & 0{,}9999999998631 & -1{,}65467685 \times 10^{-5} & 3{,}1291 \times 10^{-10}a_4 - 1{,}65468 \times 10^{-5}d_4 + 0{,}00000236387d_3 - d_2 \\ 0{,}99999999982 & 0 & 0{,}00001891064 & a_2 + a_3 + 0{,}99999999982119a_4 + 0{,}000018910d_4 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (12)$$

To find the errors, was subtracted the T_e matrix which has been corrected with the T matrix obtained initially, the fourth column will throw the equations to obtain the geometrical deviations or errors on the X-axis, Y-axis, and Z-axis as shown in equation (13)

$$T_e - T =$$

$$\begin{bmatrix} 1{,}891064 \times 10^{-5} & -1{,}6546768 \times 10^{-5} & -0{,}999999999684296 & 0{,}00001891a_4 + 4 \times 10^{-10}d_4 + 3 \times 10^{-10}d_3 \\ 3{,}129100 \times 10^{-10} & -2 \times 10^{-10} & -1{,}65467685 \times 10^{-5} & 3{,}1291 \times 10^{-10}a_4 - 1{,}65468 \times 10^{-5}d_4 + 0{,}00000236387d_3 \\ -2 \times 10^{-10} & 0 & 0{,}00001891064 & -2 \times 10^{-10}a_4 + 0{,}000018910d_4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (13)$$

From the previous matrix we can observe that the errors depend on the values of d_4 y d_3. Based on this we created a calculator presented in Figure 5, which allows us to determine the geometrical errors of the Cartesian table by entering the numerical values of d_4 y d_3.

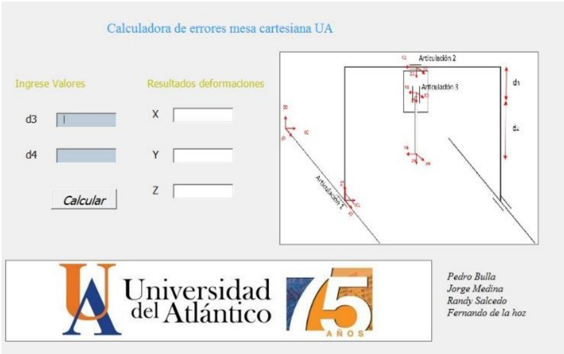

Figure 5. Geometric error calculator.

3.2 Solidworks® simulation

Simulations are performed for different head positions and the results are summarized in Figure 6a. This table relates the head position in the XY plane and the corresponding deflection in the Z axis. These results are parameterized to obtain a coordinate based error calculator, the calculator is shown in Figure 6b.

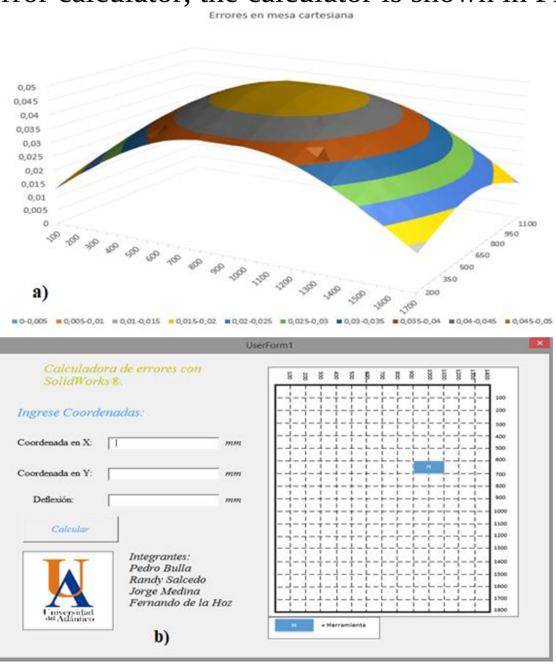

Figure 6. Errors in Cartesian table, a) using solidworks®, b) using Matlab software.

4. Conclusions

In this document we developed the error calculation of an automated Cartesian XYZ table with interchangeable head for plasma cutting and oxyfuel cutting, ensuring accuracy and a zone where the table could fail. In addition, using the Denavit-Hartenberg method and the Transformation Matrix, a model was developed that allowed the geometrical error caused by the different positions and deflections of the equipment joints to be estimated using a direct kinematic problem. Additionally, the main deflections caused by the weight of the components were estimated using Solidworks®, the entire process concluded in two calculators that allow the errors to be successfully calculated for each position of the tool head. On the other hand, as a result of this process organized in this document is the sequence of the design process of a machine that meets all the technical and technological specifications of the Colombian market, compact, rigid, safe and precise, with a study of errors that will minimize the inherent errors to the components and configuration of the equipment, allowing this equipment to maintain a high level of competitiveness in the market.

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