Contemporary Engineering Sciences, Vol. 11, 2018, no. 69, 3421 - 3427
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/ces.2018.87335

Production Scheduling for a Job Shop Using a Mathematical Model

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Abstract

Today, all organizations are continually seeking to produce high quality, reliable, and timely goods and services that meet the needs of their customers. Therefore, production scheduling is one of the most complex activities in production management in organizations. In the present work, the production projection is developed in a Job Shop system using a mathematical model of linear integer programming whose objective is to minimize the total lateness. The developed model is validated by means of an application case with three jobs and four machines, obtaining the optimum programming according to the requirements. The results obtained have been compared with a scheduling software to evaluate the performance of the proposed model.

Keywords:: Job Shop, production scheduling, linear programming, operations research

1. Introduction

Production scheduling is a decision-making process that is frequently used in manufacturing and service companies for allocating resources to specific tasks or jobs in specific time periods in order to optimize one or more objectives. [1]
Factors such as globalization set an important standard in the competitiveness of companies and the development of decision making tools for production scheduling is vital for companies to meet their commitments to customers. The applications of this type of tool make production processes more efficient.

Production scheduling is one of the most relevant and complex activities in production management and is an operational response that seeks to optimize the fixed delivery of goods or service [2]. Therefore, for the programming to be successful, it is vitally important that whoever develops it has a thorough knowledge of its productive configuration [3]. Depending on the flow pattern of work through the plant, many types of production configurations can be distinguished, of which one of the most studied in the literature is the Job Shop. The configuration in job shop corresponds to the group of production systems that present a flexible flow strategy, in this system, the machines are organized by activity, that is, grouping those of the same type in order to maximize their use [4]. The concept of Job Shop Scheduling, although its origin is not very clear, can be attributed to Muth and Thompson for their Industrial Scheduling work [5]. According to [6] production scheduling in this type of environment has a considerable impact on the efficiency of the system and this represents a competitive advantage for the company that influences even its customers. The problem of programming a Job Shop work system is a part of production planning that involve a set of more difficult problems in combinatorial optimization [7], which is why this article develops a mathematical model of linear optimization that responds to the solution of this type of problem as a production management tool.
For this reason, this paper proposes a production scheduling model for a Job Shop using a linear optimization model whose objective is to minimize the total delay or decrease the delivery date. The problems of programming of Job Shop (JSSP) are the most difficult to solve within the combinatorial problems [8], which is why many researchers have devoted their interest, as well as its many applications in the manufacturing industry where this type of problem can solve multi-target situations in the real world. Several recent works have been developed [9], [10], [11], [12], [13].

2. Methodology

This is the case when several jobs must be programmed in different machines. Each of the works has a sequence and processing times, as well as a delivery date. The objective is to determine the start dates of work in each of the machines to minimize the delivery time of each of the tasks, given the configurations and specifications of the manufacturing system. It is important to keep in mind that the following restrictions are met in the developed model:
a. The sequence of each job on the machines is known
b. Each job must be processed without interruption.
c. The machines are available at zero time and the jobs are released at zero time.
d. Transport and setup times are negligible between machines and operations

2.1 Mathematical model

The following mathematical model is proposed to provide a solution to the problem of production scheduling in a Job Shop environment that minimizes the delivery time of all jobs. Is a model Mixed Integer Linear Programming (MILP). The advantage of a MILP approach is that it give a general framework for to model a wide variety of problems, and its greatest difficulty lies in the computational expenditure that must be invested in large-scale problem solving, since these problems are of computational complexity NP-hard. [14], [15].

Variables:
t_{ij} = time for starting work i on the machine j.
T= time of delivery of all the works.
Yk= binary variable
Parameters:
P_{ij}= processing time of job i on machine j.
di = delivery time of the work i
m= number of jobs
ni= sequence of work i

The model is stated as:

$$\text{Min } Z = T \quad (1)$$

s.t:

$$t_{ij(n+1)} - t_{ij(n)} \geq p_{ij(n)} \; ; \; \forall_i \quad (2)$$

$$t_{ij} + p_{ij} \leq d_i ; \; \forall_i \quad (3)$$

$$t_{ij} + p_{ij} \leq T ; \; \forall_i \quad (4)$$

$$t_{i(m)j} - t_{i(m+1)j} + p_{i(m)j} \leq M y_k \quad (5)$$

$$t_{i(m+1)j} - t_{i(m)j} + p_{i(m+1)j} \leq M(1 - y_k) \; ; \; \forall_j \quad (6)$$

The objective function of model is stated in (1). Constraint (2) ensures the sequence of each of the jobs. Constrains (3) and (4) ensures ensures delivery time of the work i. The constraints (4) and (5) ensure that no two jobs can be scheduled on the same machine at the same time.

2.2 Case study

Consider the programming of a company performs 3 different jobs associated with the development of 3 different products. The company has 4 machines that are used to perform the tasks.

The following table shows the sequence of machines that each job uses and the time in hours that the task takes up on the corresponding machine. Also shown in the last column is the maximum delivery time for each job. Each machine can only perform one task simultaneously

Table 1. Jobs and Machine sequence

Jobs	Machine sequence (machine occupancy time)	Time of delivery (h)
T1	M1(4h) →M3(3h) →M4(5h) →M2(3h)	22
T2	M1(2h) →M2(5h) →M3(1h)→M4(2h)	20
T3	M3(6h)→M1(4 h)→M4(5h)→M2(3h)	24

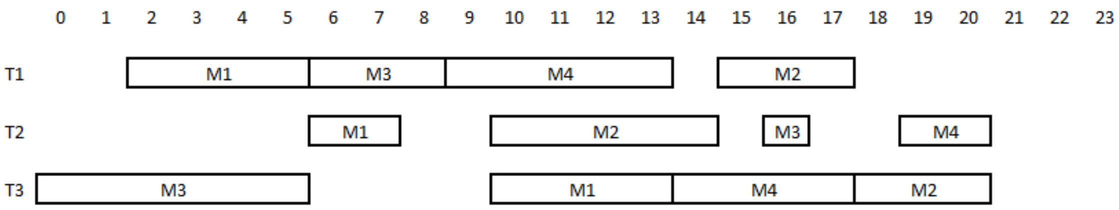
3. Results

Once the model was developed for this particular case, the Premium Solver Pro software was used to find the solution and the results are shown below. Figure 1 shows a Gant graph showing the jobs processed on each of the machines.

Table 2. Start time of jobs on each machine.

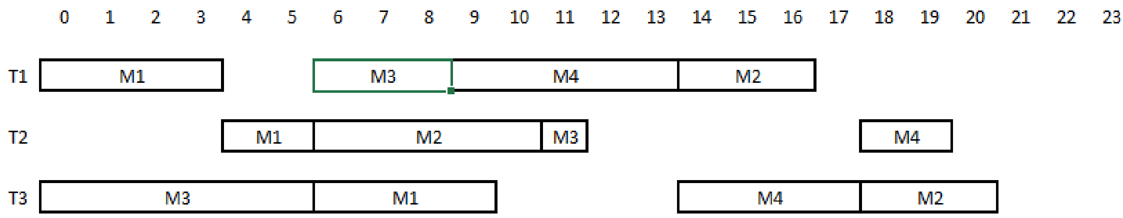
t_{11}	2	t_{21}	6	t_{31}	10
t_{12}	15	t_{22}	10	t_{32}	18
t_{13}	6	t_{23}	16	t_{33}	0
t_{14}	9	t_{24}	19	t_{34}	14

Fig.1: Model results



In order to validate the performance of the developed model, the production schedule of the proposed case was made using the Lekin software, these results are shown in figure 2.

Fig. 2: Lekin software results



When comparing the results obtained, it can be seen that the solution regarding the completion time of all tasks is the same.

Conclusions

The scheduling production is fundamental for a great number of companies, in this work a MILP model has been developed to give answer to this type of situations, a model was presented for minimizing the lateness . The paper carried out shows that the application of optimization techniques based on deterministic mathematical models supports the strategic decision making process in relation to production scheduling in job shop systems.
The mathematical model used here meets the expectations of being a decision support tool due to the high complexity of the large amount of data, decision variables and multiple interrelationships between variables that exist in production planning and can provide reliable information for managing sound business decisions.

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Received: August 3, 2018; Published: August 30, 2018