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Mathematical Modeling of Nonlinear Dynamic System of the Truck Crane

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Abstract

The mathematical model of the mechanical system of truck crane with three degrees of freedom is considered. The truck crane includes the mechanisms of bridge and electric hoist, which are modeled by translational kinematic pairs. For the automation of handling operations are determined the position of cargo, speed and acceleration of cargo in the initial coordinates system. The device for soft start-stop of the truck crane is simulated by nonlinear polynomial with generalized coordinates. By modified method of polynomial transformations we obtain the analytical solution of nonlinear mathematical model of truck crane. For control the accuracy of analytical results we carried out calculation of mathematical model by the method Runge-Kutta fourth order. The power of devices necessary for lift and move the cargo is defined.

Keywords: truck crane, mathematical modeling, nonlinear dynamical system, the method of polynomial transformations

1 Introduction

The truck crane applied for moving heavy cargos in the warehouse, for installing large units of equipment and for the movement of goods between the technological areas. The automation of the truck crane allows increasing the performance.

The truck crane (figure 1) includes: support, beams, bridge and hoist [1, 2].

Fig.1 The truck crane

The truck crane includes the mechanism for soft start-stop of crane, the brake mechanism of truck crane, the brake mechanism for lifting, the brake mechanism for load carriage [3, 4].

The truck crane controlled automatically or remotely.

The electric hoist is fixed to the trolley that moves along the monorail path (figure 2). Along the I-beam the electric hoist is moved. The beam is connected to the two beams on which are located the running wheels.

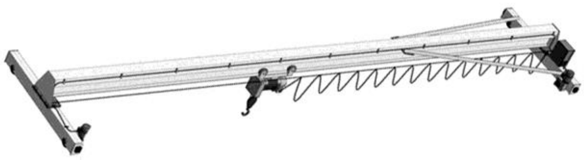

Fig.2 The electric hoist

The technical specifications the crane are determined by the following main parameters: the load, length of flight, lifting height, the speed of bridge, the speed of lift truck, the speed of lifting.

The truck crane is applied for spans from 4.5 to 28 m. and cargo capacity from 0.5 to 10 tons.

2 Mathematical modeling of the mechanical system of bridge and electric hoist

The mechanical system of bridge and electric hoist has three degrees of freedom. Figure 3 shows the kinematic scheme of truck crane.

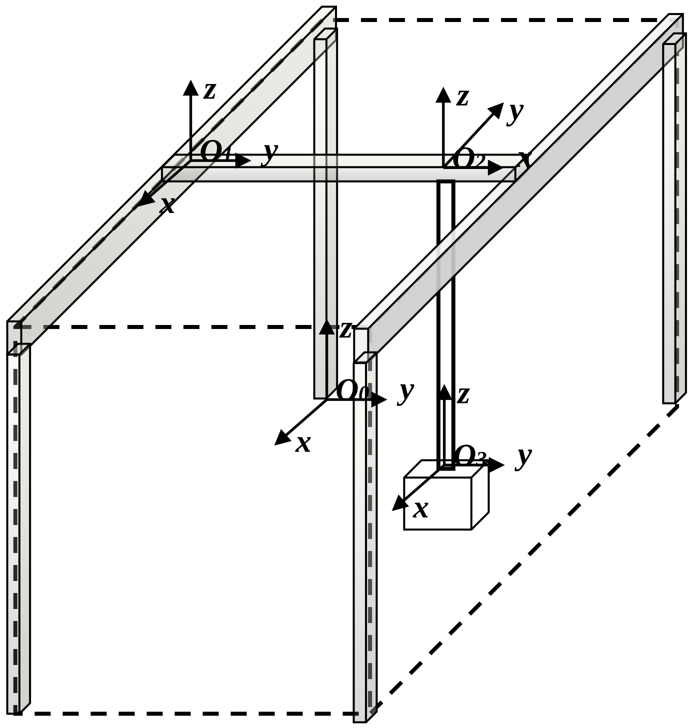

Fig.3 The kinematic scheme of truck crane

The mechanical system of the truck crane has geometric characteristics: the length and the width of the span - l_1, l_2 and height of lifting - h.

We introduce the relative coordinate system O_1, O_2, O_3 associated with bridge of crane, with the lifting truck and with the cargo.

The initial coordinate system we denote O_0 and associate with supports of crane.

The coordinates the cargo we choose for generalized coordinates q_1, q_2, q_3

The transition from coordinate system O_0 to O_1 occurs through displacement along z-axis by h and displacement along axis x by q_1.

The transition from coordinate system O_1 to O_2 occurs through rotation around z-axis by $\pi/2$ and displacement along axis x by q_2.

The transition from the coordinate system O_2 to O_3 occurs through rotation around z-axis by $-\pi/2$ and displacement along axis z by $-q_3$.

We applied the matrix method and Lagrange equations in matrix form to produce the equations of motion.

For radius vector of a point in the coordinate system *i*, we define the column:

$$R_i = \begin{bmatrix} x_i & y_i & z_i & 1 \end{bmatrix}^T .$$

The communication of radius vectors in coordinate systems *i*-1 and *i* by the transition matrix A_i with formula: $R_{i-1} = A_i R_i$

The transition matrix from O_0 to O_1, the transition matrix from O_1 to O_2, the transition matrix from O_2 to O_3 are equal:

$$A_1 = \begin{pmatrix} 1 & 0 & 0 & q_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & q_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -q_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The transition matrix from O_0 to O_1, the transition matrix from O_0 to O_2, the transition matrix from O_0 to O_3 are equal:

$$A_{01} = \begin{pmatrix} 1 & 0 & 0 & q_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad A_{02} = \begin{pmatrix} 0 & -1 & 0 & q_1 \\ 1 & 0 & 0 & q_2 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad A_{03} = \begin{pmatrix} 1 & 0 & 0 & q_1 \\ 0 & 1 & 0 & q_2 \\ 0 & 0 & 1 & h - q_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

We obtain the kinetic energy for bridge of crane, lift trucks and lifted load.
For calculate the kinetic energy we apply matrix form with transition matrix: $T_i = tr(\dot{A}_{0i} H_i \dot{A}_{0i}^T)/2$,

where $\dot{A}_{0i}$ - derivative of the transition matrix, H_i - matrix of inertia.

The weights bridge of crane, lift trucks and lifted load are equal: m_1, m_2, m_3

The total kinetic energy is equal the sum of the kinetic energy of the three units:
$T = 0.5(m_1 + m_2 + m_3)\dot{q}_1^{\,2} + 0.5(m_2 + m_3)\dot{q}_2^{\,2} + 0.5 m_3 \dot{q}_3^{\,2}$

In matrix form, the potential energy equals: $P_i = -m_i G^T A_i R_i$,

where $G_i^T = \begin{bmatrix} 0 & 0 & -g & 0 \end{bmatrix}$ - the matrix-row for the acceleration of free fall

The total potential energy equal to: $P = m_3 g(h - q_3)$

We apply the Lagrange equations in matrix form for the equations of motion.
$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial P}{\partial q_i} = Q_i$, where Q_1, Q_2, Q_3 - generalized forces of units.

The nonlinear device for soft start-stop of the bridge and electric hoists is simulated by cubic polynomial.
The generalized forces for soft start-stop of the bridge, hoist and lift represented in the form: $f_1 q + f_2 q^2 + f_3 q^3 + f_0$

By substituting kinetic energy, potential energy and the generalized forces in the Lagrange equation, we obtain the system of equations with three degrees of freedom.

$$\begin{cases}(m_1+m_2+m_3)\ddot{q}_1+b_1q_1+b_2q_1^2+b_3q_1^3+b_0=0,\\(m_2+m_3)\ddot{q}_2+c_1q_2+c_2q_2^2+c_3q_2^3+c_0=0,\\m_3\ddot{q}_3+d_1q_3+d_2q_3^2+d_3q_3^3+d_0-gm_3=0\end{cases}$$

For the solution of nonlinear differential equations is applied various analytical methods [5-10]: the harmonic balance method, van der Pol method, the small parameter method, the averaging method, Krylov-Bogolyubov method, the Poincare perturbation method and the polynomial transformations method.
The exact solution of the nonlinear system of equations is obtained numerically by the method of Runge-Kutta fourth order with the following parameters:

$$m_1=500, m_2=100, m_3=1000, h=10, g=9.81, l_1=10, l_2=10$$
$$b_1=0.5, b_2=0.2, b_3=0.1, c_1=0.4, c_2=0.3, c_3=0.1, d_1=0.3, d_2=0.2, d_3=0.1$$

The analytical solution is obtained by the modified method of polynomial transformations [11-13].
We computed the generalized coordinates and the speed of the bridge for distance l

$$q_1=-a\sin^2\left(t\sqrt{w}/2\right)\left(\cos\left(t\sqrt{w}\right)+8\cos\left(2t\sqrt{w}\right)+57\right)/30,$$

$$\dot{q}_1=-a\sqrt{w}\left(15\sin\left(t\sqrt{w}\right)-5\sin\left(2t\sqrt{w}\right)+\sin\left(3t\sqrt{w}\right)\right)/15,$$

where $a=\left(6b_2\sqrt[3]{R}+6\sqrt[3]{2}\left(10b_1b_3-3b_2^2\right)-2^{2/3}R^{2/3}\right)/\left(30b_3\sqrt[3]{R}\right)$

$$w=\frac{6b_2^2R^{2/3}+12\sqrt[3]{2}b_2\left(3b_2^2-10b_1b_3\right)\sqrt[3]{R}+2\,2^{2/3}b_2R+18\,2^{2/3}\left(3b_2^2-10b_1b_3\right)^2+\sqrt[3]{2}R^{4/3}}{120b_3\left(m_1+m_2+m_3\right)R^{2/3}}$$

$$R=-54b_2^3+270b_1b_2b_3+84.375b_3^2l\left(l\left(5b_3l-6b_2\right)+8b_1\right)+$$
$$\sqrt{\left(84.375b_3^2l\left(l\left(5b_3l-6b_2\right)+8b_1\right)-54b_2^3+270b_1b_3b_2\right)^2+4\left(30b_1b_3-9b_2^2\right)^3}$$

We computed the generalized coordinates and the speed of lifting truck along the beam of bridge for distance l

$$q_2=a\left(\cos\left(t\sqrt{w}\right)-1\right)-a\left(\cos\left(2t\sqrt{w}\right)-1\right)/8+a\left(\cos\left(3t\sqrt{w}\right)-1\right)/15,$$

$$\dot{q}_2=-a\sqrt{w}\sin\left(t\sqrt{w}\right)+a\sqrt{w}\sin\left(2t\sqrt{w}\right)/3-a\sqrt{w}\sin\left(3t\sqrt{w}\right)/15,$$

where $a=\left(6c_2\sqrt[3]{R}-18\sqrt[3]{2}c_2^2+60\sqrt[3]{2}c_1c_3-2^{2/3}R^{2/3}\right)/\left(30c_3\sqrt[3]{R}\right)$

$$w=\left(6c_2^2\left(R^{2/3}-180\ 2^{2/3}c_1c_3\right)+\sqrt[3]{2}\left(1800\sqrt[3]{2}c_1^2c_3^2+R^{4/3}\right)\right)/\left(120c_3\left(m_2+m_3\right)R^{2/3}\right)+$$
$$+\left(36\sqrt[3]{2}c_2^3\sqrt[3]{R}+2c_2\left(2^{2/3}R-60\sqrt[3]{2}c_1c_3\sqrt[3]{R}\right)+162\ 2^{2/3}c_2^4\right)/\left(120c_3\left(m_2+m_3\right)R^{2/3}\right)$$
$$R=84.375c_3^2l\left(l\left(5c_3l-6c_2\right)+8c_1\right)-54c_2^3+270c_1c_3c_2+$$
$$\sqrt{\left(84.375c_3^2l\left(l\left(5c_3l-6c_2\right)+8c_1\right)-54c_2^3+270c_1c_3c_2\right)^2+4\left(30c_1c_3-9c_2^2\right)^3}$$

We computed the generalized coordinates and lifting speed for height h

$$q_3=-1.9a\sin^2\left(t\sqrt{w}/2\right)-a\sin^2\left(t\sqrt{w}/2\right)\cos\left(t\sqrt{w}\right)/30-4a\sin^2\left(t\sqrt{w}/2\right)\cos\left(2t\sqrt{w}\right)/15$$
$$\dot{q}_3=-a\sqrt{w}\sin\left(t\sqrt{w}\right)+a\sqrt{w}\sin\left(2t\sqrt{w}\right)/3-a\sqrt{w}\sin\left(3t\sqrt{w}\right)/15,$$

where $a=\left(6d_2\sqrt[3]{R}+6\sqrt[3]{2}\left(10d_1d_3-3d_2^2\right)-2^{2/3}R^{2/3}\right)/\left(30d_3\sqrt[3]{R}\right)$

$$w=\frac{6d_2^2R^{2/3}+12\sqrt[3]{2}d_2\left(3d_2^2-10d_1d_3\right)\sqrt[3]{R}+2\ 2^{2/3}d_2R+18\ 2^{2/3}\left(3d_2^2-10d_1d_3\right)^2+\sqrt[3]{2}R^{4/3}}{120d_3m_3R^{2/3}}$$
$$R=-54d_2^3+270d_1d_2d_3+84.375d_3^2\left(8hd_1-6h^2d_2+5h^3d_3+16gm_3\right)+$$
$$2\sqrt{\left(-9d_2^2+30d_1d_3\right)^3+729\left(d_2^3-5d_1d_2d_3-1.5625d_3^2\left(8hd_1-6h^2d_2+5h^3d_3+16gm_3\right)\right)^2}$$

The figure 4 presents generalized coordinates and the speed of lifting electric hoist for height of 10 meters obtained by analytical method of polynomial transformation (dashed line) and by the numerical method of Runge-Kutta (solid line).

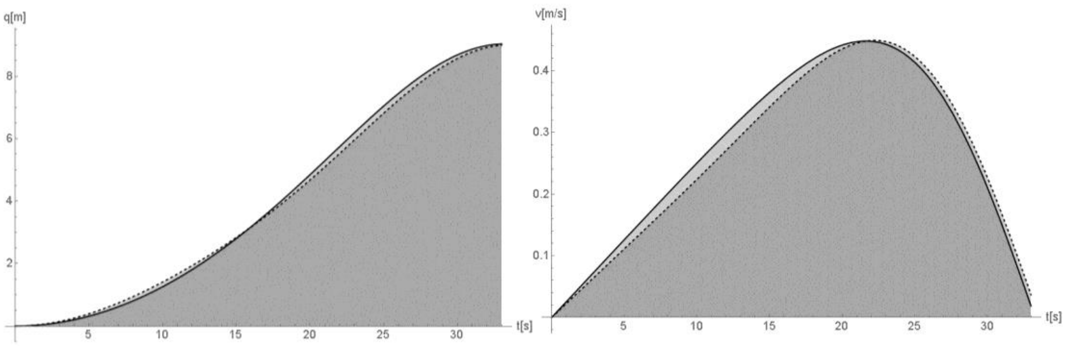

Fig.4 the generalized coordinates and cargo lifting speed

We computed the power of devices that needed for lift and move the cargo.
The maximum power required for lift on height is equal:

$$N = \frac{1}{960\sqrt{30}R^{1/3}d_3}\left(5d_3h^3 - 6d_2h^2 + 8d_1h + 16gm_3\right)\left(2^{2/3}R^{2/3} - 6R^{1/3}d_2 + 6\,2^{1/3}\left(3d_2^2 - 10d_1d_3\right)\right)\times$$
$$\sqrt{\frac{2^{1/3}R^{4/3} + 2\,2^{2/3}Rd_2 + 6R^{2/3}d_2^2 + 12\,2^{1/3}R^{1/3}d_2\left(3d_2^2 - 10d_1d_3\right) + 18\,2^{2/3}\left(3d_2^2 - 10d_1d_3\right)^2}{R^{2/3}d_3m_3}}$$

The figure 5 shows graphs of the generalized forces for move to the distance l of the beam (solid line) and for lifting truck (dashed line).

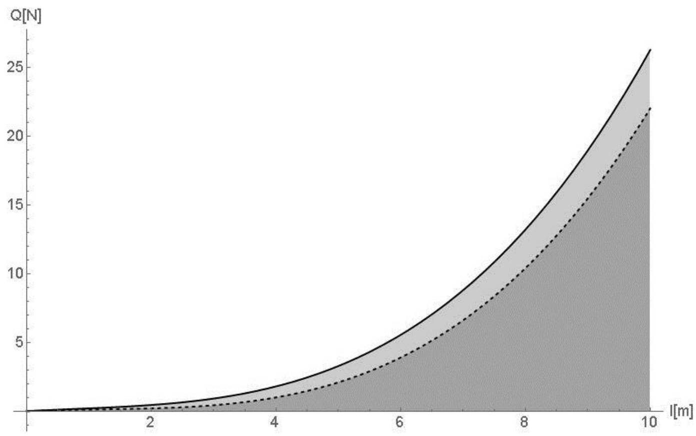

Fig.5 the generalized forces to move to distance l

Conclusion

The simulation of mechanical systems of the truck crane with a nonlinear device for soft start-stop is carried out. For solve the problem of automation for handling operations is defined the position, speed and acceleration of cargo in the initial coordinate system. By modified method of polynomial transformations we obtained the analytical solution of nonlinear mathematical model of truck crane. For control the accuracy of analytical results is carried out numerical solution of nonlinear model by method of Runge-Kutta fourth order. The power of devices necessary for lift and move cargo is defined.

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