Contemporary Engineering Sciences, Vol. 9, 2016, no. 6, 289 - 302
HIKARI Ltd, www.m-hikari.com
http://dx.doi.org/10.12988/ces.2016.512325

A New Approach to the Problem of Rotor-Bearing Stability

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Abstract

Rotors instability is one of the most serious problems of modern engineering. There are many approaches for solution of the problem which is especially important for high-speed machinery. A known model of piece-wise linear system of oscillator supported by bearings having clearances is chosen for analytical consideration of a rotor motion and its stability. Since the procedure of prediction of the dynamical stability of investigated motions near harmonic oscillations is based on analysis of the first derivative of restoring force it is convenient to represent the derivative as expansion of Fourier series. Two cases of symmetry and asymmetry of acting external force are studied and compared. Approximate equations for computing and plotting maps of instability zones are found. The existence of a difference in principle between both considered cases is resulted from different types of expansions of Fourier series. Hence, it follows that if in a symmetric case the main (first) instability zone of the motion, supposed close to harmonic, coincides with the region of the primary resonance, asymmetry of restoring force shifts the first instability zone to the right direction.

Keywords: Piece-wise Linear Instability Zones, Fourier Series

1. Introduction

Recently, there has been a tendency to increase the power and efficiency of rotating machinery. Consequently, various rotor dynamic effect, which in some cases may be due to existing nonlinearities such as bearing clearances and asymmetry of acting external force become increasingly important in the design and operation of such machinery.

In the past, some aspects of the associated phenomena had been considered for a simple piece-wise linear system. Hence, Maezawa [1] was, possibly, the first, who studied analytically and observed practically the superharmonic resonance in piece-wise linear system and took into consideration the role of asymmetry of elastic elements. The influence of asymmetric spring on the real motion of a rotor system and its character of bifurcation analysis of oscillator was researched in [2]. Periodic motions were analyzed by Dr. Albert C.J. Luo [3] in three wise linear system under periodic excitations. Investigating rotor dynamics, M. Karlsberg [4] took into account special kinds of rotating machinery, supported by bearings with clearance which are further clamped in a supporting structure. Due to gravity the shaft often vibrates close to a static equilibrium position leading to the possibility to linearize equation of motion. Although several studies on bearing with clearance exist, there is a question how such clearance affects the stiffness coefficient close to a static equilibrium position. The author [4] found that natural frequencies decrease significantly with clearance.

Up to date, the attention of researchers has been focused on taking advantage of nonlinear phenomena for the benefit of a system. Devices of such kind may be created by using effect of sub- and superharmonic responses in strongly nonlinear system for transformation input industrial frequency into another one necessary for machinery. The knowledge in this topic allows us to do this now, because recent investigations in rotor dynamics brought new results and new ideas. Significant achievements on this direction were obtained by known researchers as M. Wiercigroch [5], Ali H. Nayfeh and B. Balachandran [6].

The problem of stable operation of rotating machinery elements (such as rotors and shafts) is of crucial importance, and no successful solution was found up to date.

In this study we have made an attempt to show that one of possible instability reasons of horizontal rotor may be joined influence of gravity and radial clearance during shaft journals' take-off from bottom supports. Such effect leads to the fact that common piece-wise linear system with clearance becomes strongly non-linear, with formed extensive instability zone in afterresonance area, which causes generation of powerful ½-order subharmonic component which is dangerous for rotor operation. Our investigation will be focused entirely on this horizontal rotor model. A region lying on the right direction from frequency-response curves of the primary resonance is the main object of our study and it will be called further as afterresonance area.

2. Derivation of the simplified oscillator

Hereafter, we suppose that bearings consist of two halves: top and bottom, that is, they are sliding type of bearings. Besides that, to simplify the task, we shall not consider the shaft rotation, and we review it as a beam type oscillator, with a mass in the middle, laid on two supports. The model includes periodical common type

external vertical force, contrary to the real force acting on the rotor due to residual unbalance. To compare the instability charts (shown below) of the forced regime of oscillations under inquiry, which is close to harmonic oscillations, we would choose two of the most frequent rotor installation types: vertical and horizontal.
With these assumptions, a motion of unit mass oscillator may be described with differential equation:

$$\ddot{x} + \alpha\dot{x} + R(x) = p_0 + p\sin\nu t\,, \tag{1}$$

where *x* – coordinate, *α* - viscous damping factor, supposed to be positive and small, *R(x)* – static characteristic of elastic restoring force containing clearance 2b (Fig. 1a)

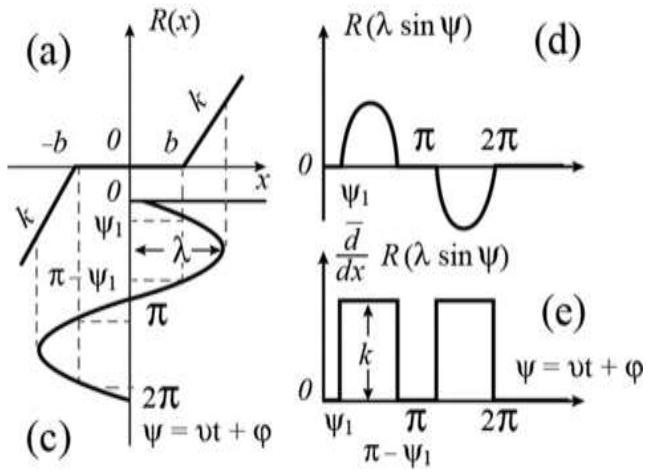

Figure 1. A symmetrical variant of external force application to a clearance system: (a) – general view of elastic system, (c) – input signal curve, (d) – output function of nonlinear characteristic, (e) – periodical function of changing stiffness

$$R(x) = \begin{cases} 0 & at|x| \le b, \\ k(|x| - b)\sin x & at|x| \ge b; \end{cases}$$

p_0 – constant or slowly varying in comparison with frequency ν component, that shifts the operating point on the *R(x)* characteristic at amount of $\Delta = b + p_0/k$; where *p* is an amplitude of external force's harmonic component, *k* is a stiffness constant of elastic support. Now and further we will apply the method of E. P. Popov and I. P. Paltov [7] of periodical signal transition through piece-wise linear element.
In general, depending on relation between frequency *v* and natural frequency of vibrator, forced solution of equation (1) may significantly differ from harmonic one. However, for frequency *v* intervals near the primary resonance, and in afterresonance area, stationary motion of the system (1) may approximately be described by the following law,

$$x_0 = \lambda \sin(\nu t + \varphi) + z\,, \tag{2}$$

where λ, ν, φ are the amplitude, frequency and phase angle; z is a permanent component (vibration centre shift).

To investigate the dynamic stability (2) under small perturbations, we will set up a variational equation. After substitution of $x = x_0 + \xi$ into (1), where ξ is small deviation from stationary state and from transition to the new variable η by alternation $\zeta = \eta \exp(-\delta t)$, where $\delta = 1/2\alpha$, we come to the first approximate equation regarding η

$$\ddot{\eta} + \left[(dR/dx)_0 - \delta^2\right]\eta = 0\,. \tag{3}$$

Here, the designation $(dR/dx)_o = (dR/dx)_{x\text{-}xo}$ is implemented.

Further investigation procedure is associated with stability analysis of zero solutions equation (3).

Considering the approximate character of solution (2), and that obviously, approximate time points of discontinuous function $(dR/dx)_0$ within time period of $T = 2\pi/\nu$ may be compliant with this solution, this would be reasonable to be restricted with calculating boundaries of periodic solutions of the equation (3) which separate stability and instability zones. In order to use approximate approach to the equation (3), it is convenient to represent the derivative $(dR/dx)_0$ as appropriate Fourier series, and accordingly come to linear Hill equation.

3. The discussion of symmetric case

Let us review symmetrical case (Fig. 1), when $\rho_0 = 0$, and accordingly, $z = 0$. By using known ideas [7] of periodical signals transition through the piecewise linear elements, we expand function $(dR/dx)_0$ (Fig.1, e) as appropriate Fourier series

$$(dR/dx)_0 = \theta_0 + \sum_{n=1}^{\infty} \theta_n \cos n\psi\,,\ \psi = \nu t + \varphi\,,\ \text{where}$$

$$\theta_0 = \pi^{-1} \int_{\psi_1}^{\pi-\psi_1} k\,d\psi = k(1 - 2\pi^{-1} \arcsin b/\lambda)\,,$$

$$\theta_n = 2\pi^{-1} \int_{\psi_1}^{\pi-\psi_1} k \cos n\psi\, d\psi = \begin{cases} 0 & \textit{where}\ n\ \textit{is odd} \\ -4(\pi n)^{-1} k \sin(n\psi_1) & \textit{where}\ n\ \textit{is even} \end{cases}.$$

θ_0 value determines average steepness (differential stiffness) of the elastic characteristic within the period T = 2π/ν. By giving that $\psi_1 = \arcsin y$, where $y = b/\lambda$, the function of $\sin(n\psi_1)$, which is included into θ_n, and with $\lambda > b$ this function may be represented as an identity function,

$\sin(n \arcsin y) = C_n^1 y(1-y^2)^{(n-1)/2} - C_n^3 y^3 (1-y^2)^{(n-3)/2} + \ldots$, where C_n^1, C_n^3 – are binomial coefficients.
By setting *n = 2j*, the equation (3) is arranged as

$$\ddot{\eta} + \left[\theta_0 - \delta^2 - 2\sum_{j=1}^{\infty} \theta_j \cos 2j(vt+\varphi)\right]\eta = 0, \quad (4)$$

where $\theta_j = (j\pi)^{-1} k \sin(2j \arcsin b/\lambda)$.

Considering that Hill equation (4) includes only even components with frequencies of *2jv*, and main period of *π/v*, then according to [8], sufficient in the first approximation solution of stability condition may be represented as

$$\left[\theta_0 - (jv)^2\right]^2 + 2\left[\theta_0 - (jv)^2\right]\delta^2 + \delta^4 > \theta_j^2, \qquad j = 1, 2, 3... \quad (5)$$

where j is instability zones order in case the condition (5) is not in compliance.
The criterion (5) may be calculated from substitution of approximated specific solution into equation (4) as per Whittaker method

$$\eta = \exp(\mu t)\sin(jvt - \sigma), \quad (6)$$

where μ is a characteristic exponent; σ is a phase parameter having values between 0 and –π/2 in instability zones.
Then, according to the solution (6) representation, it is clear that the implementation of criterion (5), with any j value, excludes occurrence of conditions for generation of frequency v vibration components (provided j = 1) and multiple of v (provided j > 1), i.e. generation of superharmonic vibrations.
If we substitute θ_0 and θ_j into (5) for j =1, and neglect damping (δ = 0) for sakes of simplicity, we will come to relation for calculating boundaries of the first (main) instability zone

$$v^2 = k - 2\pi^{-1} k(\arcsin b/\lambda \pm b/\lambda \sqrt{1-(b/\lambda)^2}), \quad (7)$$

which accurately matches within a single radical sign of formula [7]:
$q(b/\lambda) = k - 2\pi^{-1} k(\arcsin b/\lambda + b/\lambda\sqrt{1-(b/\lambda)^2})$ for coefficient of harmonic linearization describing curve of the system's autonomous oscillations at δ = 0. Primitive computations coming from amplitude frequency response [2]

$$\lambda^2\left[(q - v^2)^2 + \alpha^2 v^2\right] = p^2 \quad (8)$$

show that relation (7) with minus sign before the root, will form the right zone's boundary for $j = 1$ that complies with provision of $d\lambda/dv = \infty$ (existence of vertical tangent lines for λ(v) series of curves).

Similarly from (5), with j = 2 and δ = 0, we get the following relation

$$v^2 = \frac{1}{4} k \left\{1 - 2\pi^{-1}\left[\arcsin b/\lambda \pm b/\lambda(1-2b^2/\lambda^2)\sqrt{1-(b/\lambda)^2}\right]\right\}$$

for the boundaries of the area where we could expect a motion component generation with frequency 2*v*, in accordance with solution (6).

Figure 2, a shows the results of calculation of the first three (j =1, 2, 3) instability zones for solution (2) at z = 0, superimposed on amplitude frequency curves (8) for display purposes. The curves are accompanied with some results (marked in circles) of analog computer process modeling. Unstable conditions of curves are shown in dotted lines. System's analog computer modeling (1) proved appropriateness of the situation shown in Fig. 2(a).

Note that the discussed symmetrical option task may act as the approximate model of vertical rotor, and essentially, it brings a few new data, as the influence of only one clearance on motion state is fairly to be noted. Such conclusion is verified with comparison of Fig. 2(a) with results achieved by W. Szemplinska-Stupnicka [9] for Duffing equation, which is commonly known to contain polynomial cubic term, and it may also be considered as symmetrical case. Whereas the author used a harmonic balance method, the results are close to those shown in Fig. 2(a).

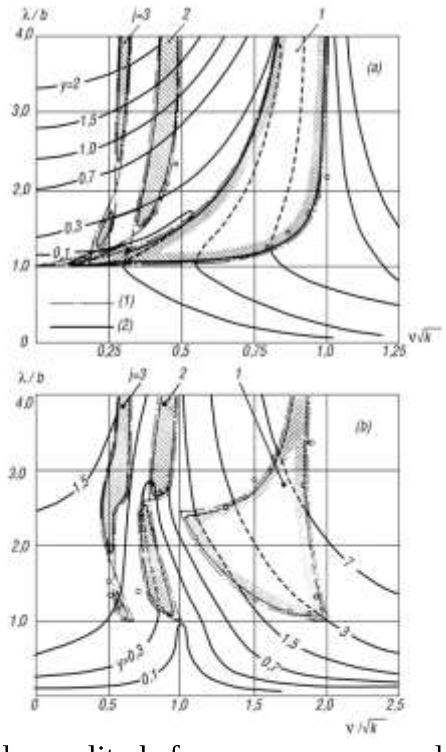

Figure 2. Forced mode amplitude frequency curves and instability charts for symmetric (a) and asymmetric (b) external influence

$(\gamma = p(bk)^{-1})$:

(1) $-\ \alpha/\sqrt{k} = 0$, (2) $-\ \alpha/\sqrt{k} = 0.02$

4. The analysis of asymmetric case

Let us review more common and more informative case of external force application (Fig. 3), when $p_0 \neq 0$, and accordingly, $z \neq 0$. The periodical function dR/dx_0, relative to input signal (2), is shown in Fig. 3(d). Whereas factors q(z, λ) calculation method is discussed in details in [7], we take without deduction these ready-made results considering the influence of shift Δ. In general case for oscillator motion that includes all parts of restoring force $R(x)$, i.e. where $\lambda > \Delta + b + z$, for $q(z, \lambda)$ we have the following relation:

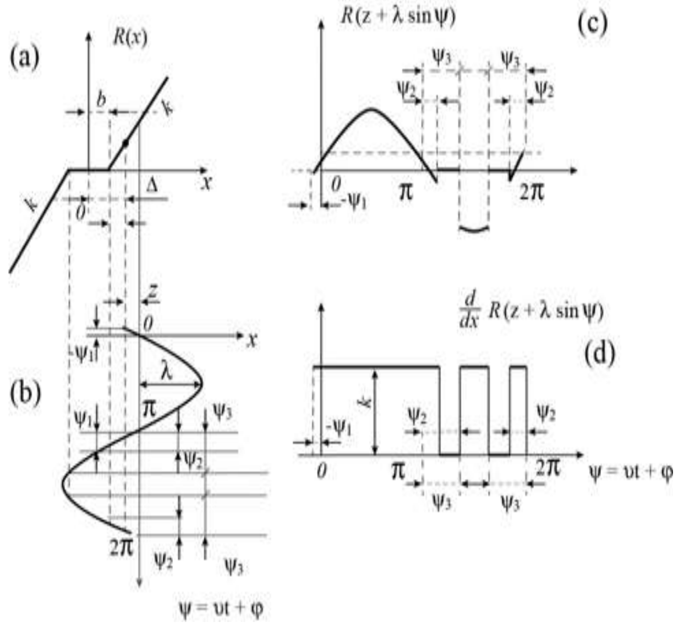

Figure 3. Asymmetric case of external force application. Reference characters are the same as in Figure 1.

$$q(z,\lambda) = (\pi\lambda)^{-1}\int_0^{2\pi} R(z+\lambda\sin\psi)\sin\psi d\psi = k(1+\pi^{-1})[\arcsin(\Delta-b+z)/\lambda - \tag{9}$$
$$-\arcsin(\Delta+b+z/\lambda+(\Delta-b+z)/\lambda\sqrt{1-(\Delta-b+z)^2/\lambda^2} -$$
$$-(\Delta+b+z)/\lambda\sqrt{1-(\Delta+b+z)^2/\lambda^2}\,].$$

The function to calculate the shift z value looks as shown below:

$$F^0(z,\lambda) = (2\pi)^{-1}\int_0^{2\pi} R(z+\lambda\sin\psi)d\psi = (b+z)k + \\ + k\pi^{-1}[(\Delta-b+z)\arcsin(\Delta-b+z)/\lambda - (\Delta+b+z)\arcsin(\Delta+ \\ +\lambda\sqrt{1-(\Delta-b+z)^2/\lambda^2} - \lambda\sqrt{1-(\Delta+b+z)^2/\lambda^2}] = 0. \qquad (10)$$

For smaller amplitudes *($\lambda \le b + \Delta + z$)* relations (9) and (10) are more simplified:

$$q(z,\lambda) = \tfrac{1}{2}k + k(\pi\lambda)^{-1}[\lambda\arcsin(\Delta-b+z)/\lambda + (\Delta-b+z)\sqrt{1-(\Delta-b+z)^2/\lambda^2}],$$

$$F^0(z,\lambda) = \tfrac{1}{2}k(b+z-\Delta) + \pi^{-1}k[\lambda\sqrt{1-(\Delta-b+z)^2/\lambda^2} + \\ +(\Delta-b+z)\arcsin(\Delta-b+z)/\lambda] = 0.$$

Fourier transformation of *$(dR/dx)_0$* slope (Fig. 3d) results in the following variational equation:

$$\ddot{\eta} + \left[\theta_0 - \delta^2 - 2\sum_{j=1}^{\infty}\theta_j \cos j(vt+\varphi)\right]\eta = 0, \qquad (11)$$

where $\theta_0 = k + k\pi^{-1}[\arcsin(\Delta-b+z)/\lambda - \arcsin(\Delta+b+z)/\lambda]$

$$\theta_j = \begin{cases} (-1^{(j-1)/2})(j\pi)^{-1}k(\cos j\psi_2 - \cos j\psi_3)\, \textit{where } j \textit{ is odd}, \\ (-1^{(j+2)/2})(j\pi)^{-1}k(\sin j\psi_2 + \sin j\psi_3)\, \textit{where } j \textit{ is even.} \end{cases}$$

Here, $\psi_2 = \arcsin(b-\Delta+z)/\lambda$, $\psi_3 = \arcsin(b+\Delta+z)/\lambda$ and function $\cos(j\arcsin y_{2,3})$, where $y_2 = (\Delta-b+z)/\lambda$, $y_3 = (\Delta+b+z)/\lambda$, has identical representation

$$\cos(j\arcsin y) = (1-y^2)^{j/2} - C_j^2 y^2(1-y^2)^{(j-2)/2} + C_j^4 y^4(1-y^2)^{(j-4)/2} - \ldots$$

Thus giving approximate particular solution for instability zones for equation (11) as

$$\eta = \exp(\mu t)\sin(\tfrac{1}{2} jvt - \sigma) \qquad (12)$$

and substituting it into (11), we get sufficient criterion [7] for the first approximation of equation (2) stability:

$$\left[\theta_0 - (\tfrac{1}{2} jv)^2\right]^2 + 2\left[\theta_0 + (\tfrac{1}{2} jv)^2\right]\delta^2 + \delta^4 > \theta_j^2, \quad (13)$$
j = 1, 2, 3... .

By introducing θ_0, θ_j into (13), at $j=1$ and $\delta=0$, we come up to the relation

$$v^2 = \tfrac{1}{4} k\{1+\pi^{-1}[\arcsin(\Delta - b + z)/\lambda - \arcsin(\Delta + b + z)/\lambda \pm$$
$$\pm(\sqrt{1-(\Delta - b + z)^2/\lambda^2} - \sqrt{1-(\Delta + b + z)^2/\lambda^2})]\},$$

that forms the main instability zone boundaries which falls into afterresonance area $(v/\sqrt{k} > 1)$. Due to (12) in this area we shall expect self-excited oscillations with frequency ½v, i.e. subharmonic component of ½-order. Accordingly, with j = 2 and δ = 0, the following formula is coming out from (13):

$$v^2 = k(1+\pi^{-1})[\arcsin(\Delta - b + z)/\lambda - \arcsin(\Delta + b + z)/\lambda \pm$$
$$\pm(\Delta - b + z)/\lambda\sqrt{1-(\Delta - b + z)^2/\lambda^2} - (\Delta + b + z)/\lambda\sqrt{1-(\Delta + b + z)^2/\lambda^2}],$$

which accurately matches within a single radical sign of equation (9) for $q(z, \lambda)$. Substitution into (13) of θ_0, θ_3, δ = 0 at j = 3 shall emphasize oscillation frequency band boundaries of $1.5v$ component (subsuperharmonic) in the $\lambda(v)$ plane:

$$v^2 = \tfrac{4}{9} k + \tfrac{4}{9} k\pi^{-1}[\arcsin(\Delta - b + z)/\lambda - \arcsin(\Delta + b + z)/\lambda] \pm$$
$$\pm \tfrac{4}{27} k\pi^{-1}\{[1-(\Delta + b + z)^2/\lambda^2\sqrt{1-(\Delta + b + z)^2/\lambda^2}] -$$
$$-[1-4(\Delta - b + z)^2/\lambda^2\sqrt{1-(\Delta - b + z)^2/\lambda^2}]\}$$

In Fig. 2(b) you can see results of computation performed using iteration method of amplitude-frequency curves and the first three zones (j = 1, 2, 3), where solution (2) is unstable in small. The diagram corresponds to particular case where $\Delta = 2b$, and it is followed by some analog computer simulation results that properly confirm the computations.

Let us review properly the special considerations of system motion at $p_0 \neq 0$. As it can be seen from Fig. 2(b) and Fig. 3, this case is defined by two properly different motion modes separated by the boundary λ/b = 1. There is a trivial linear oscillation below the specified boundary. At λ/b > 1, motion nature is strongly nonlinear and it consists of two phases, adding up in sum to time period $T = 2\pi/v$. One phase takes part of the specified period at any λ/b values, and this part is not lesser than a half. It corresponds to the system oscillating mode when its stiffness $(dR/dx)_0 = k$.

The other phase is taken by parabolic motion of the vibrator under permanent force p_0 action after the mass take-off the elastic component, at that $(dR/dx)_0 = 0$.
Hereby the system stiffness turns to be jumping within the T period from 0 to k, with its average value of θ_0. Whereas such detailed review of vibrator's motion phases does not play essential role for approximate computation which supposes that after the mass take-off the elastic component, the harmonic nature of oscillations shall continue to persist, in general, it gives the key to understanding of the mechanism for appearance instability in afterresonance area. Following the ideas described in V. A. Taft's works [10], we express the modulation depth factor as $\varepsilon = (k - \theta_0)/\theta_0$. With this determination, the ε value, as computations show, will have variation ranges from 0 to 1. From the expression above, one can see that ε may be increased with respect to θ_0 decrease. Indeed, with the mass motions started from boundary $\lambda/b = 1$ (Fig. 2b) and partially occupying the clearance, but not yet exceeding its boundaries, the duration of the system stay in the mass take-off phase shall increase with λ/b raise. Whereas this phase is corresponded with the value $(dR/dx)_0 = 0$, increase of take-off phase causes drop of θ_0 average value, and accordingly, increase of ε value. It is clear that this effect causes expansion of the main instability zone ($j = 1$, Fig. 2b). Further λ/b increase causing mass contact with the second (top) elastic component, will be accompanied by the adverse effect, - the θ_0 value is increased, that leads to sharp degeneracy of the main zone, and the system approaches close to symmetrical case.

Thus, the maximum ε value will be reached by the moment of complete filling of the clearance with the (2) oscillation while only one elastic component (the right one in Fig. 3) is in operation. The λ/b value corresponding to this moment can be calculated ($\lambda/b \approx 2,4$, Fig. 2b). Considering that herewith the period of changing of differential stiffness (dR/dx) at $p_0 \neq 0$, as you can see in Fig. 3d, is equal to the external force period, $T = 2\pi/\nu$ (it is twice less in symmetrical systems), it turns out that stiffness in afterresonance areas of asymmetrical systems may vary in relation to its average value θ_0 with double frequency as compared to natural oscillation frequency of the vibrator.

Such modulation of the parameter, as it is well known, may cause the instability of rest state or the instability of motion. Here we shall emphasize that this is not only the joint effect of clearance and permanent force that may break stability in afterresonance area. As computations and analog computer modeling results show, this property is in general fundamental for asymmetrical quasielastic systems [11, 12].

The mathematical model under consideration, in case $p_0 \neq 0$, may be simply represented as a weight p_0, fastened to the middle of weightless beam with stiffness k freely supported on two symmetrical fixed supports. The weight of the load uniquely defines static deflection of a beam and natural frequency of the system.

It is obvious that proposed vibrator model meets a case of indefinitely great clearance, and its oscillations within deflection limits p_0/k shall be linear. On further increase of amplitude, a vibrator shall transfer into nonlinear mode with take-off from supports. Note that in this mode, with any amplitude λ/b values, vibrator shows soft tendency of function $\lambda(v)$ to tilt left; which is understandable if we recall a character of behavior of parameter θ_0 variation. It is typically that the main instability zone of vibrator under consideration does not have upper limits. An installation of additional top supports in distance equal to finite clearance $2b$ gives vibrator properties typical of symmetrical option, and now demonstration of soft properties of the system shall be only restricted by clearance limits. It is obvious that behavior of the proposed vibrator shall comply with Fig. 2b.

5. A short experiment description of physical model of oscillator

In addition to analog computer simulation of the both discussed cases of rotor installation, we also performed experiment with a simple physical model of beam oscillator placed on vibrating foundation. A small diameter steel rod of l=170mm was freely supported on thin ring (150 mm diameter, 15 mm high) fastened on the amplitude and frequency controlled shaker. The ring was installed in dead-level.
The load was a piezoelectric sensor fastened in the middle of the rod and connected to oscillograph. After the oscillator passed primary (linear) resonance and after some amplitude increase that provided the rod take-off the ring, strongly nonlinear mode of oscillator vibration had started. At the frequency close to double natural system frequency, ½-order intensive subharmonic resonance was observed and it was recorded by oscillograph. The beginning of this process evidently matched with passage of bifurcation point of initial mode which is close to harmonic. This experiment allowed us to observe both the rod take-off the ring moment, and further contact with the ring. The experiment results proved to be quite in compliance with the computations made in Fig. 2.

6. Conclusion

6.1. The elementary analysis of the main instability zone (Fig. 2b) has lower and upper amplitude λ/b limits, where a subharmonic either just starting or terminating, allows us to draw such conclusions.
In afterresonance area the instability effect shall be most noticeable at average relative magnitude unbalance values that provide for partial rotor take-off bottom supports, as far as without take-off (heavy models) the rotor operation mode complies with linear oscillations ($\lambda/b \leq 1$). In case of complete take-off the system switches to rolling-in of journals inside the bearings, and mathematical model of the rotor approaches to symmetrical option (Fig. 2a), where forced mode in afterresonance area is always stable.
The decrease of relative clearance enables the subharmonic oscillation amplitude limitation, as far as the clearance size specifies position of upper boundary of the zone, and accordingly, its width.

Because the area of linear (non take-off) vibrator oscillations is specified by a static deflection value, it is clear that the instability effect is easier to be occurred in case of hard supports. Therefore, even slight softening of down support causes noticeable stabilization effect, as it increases static deflection. Accordingly, the main method to prevent the instability occurrence is a replacement of one of hard supports with elastic one, which also reduces load on foundation.

6.2. Each instability zone calculated for basic oscillation mode which is close to the harmonic one shown in Fig. 2, is corresponded with a resonance of specific oscillation frequency. It is obvious that the most intensive shall be ½-order resonance which is only specific for systems with asymmetric R(x) and corresponding to the main zone of motion instability (2). At that, the correlation in this case is direct: the more is asymmetry *R(x)*, the wider is the zone of this phenomenon. It is necessary to emphasize that all these resonance responses including the well known amplitude jumping, have common parametric nature. Hence, the main input into any instability zone's boundaries development is made by one or another Fourier series *dR/dx* harmonic. Particularly, the main zone of asymmetric system is formed by the first harmonic of this series of its frequency *v*, and that is clear due to ordinary reason: the main characteristic of any harmonic is frequency.

6.3. In summary, forced oscillations modulate energy capacious system parameters (stiffness, capacity, inductance) thus preparing their instability in small that appears in generation of a new harmonic component which amplitude increases exponentially. Similar phenomenon was observed by L. Mandelstam and N. Papalexi [13] under investigation of potentially self-exciting system; at that it was noted that only one ½-order subharmonic component in afterresonance zone was generated easily, and its amplitude had both threshold and ceiling limits as well as in our discussed case. The shape of subharmonic component existence zone left unstudied, as there was no task set to investigate stability of basic forced oscillation mode. Apparently, due to the same reason no direct connection could be determined between these two events: instability of one state and birth of the other one. It can be expected that it was for the first time suggested in [13] to use this phenomenon in particular oscillating circuits to halve the frequency in systems with single degree of freedom, and such phenomenon is most commonly used in modern electronics nowadays [14]. It is interesting to remark that the main instability zone of potentially self-exciting system described by Van der Pol equation has a form close to an oval, what was found by the author in added investigation.

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Received: January 7, 2016; Published: March 14, 2016