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Modeling of the Temperature Field on the Cathode's Surface during Electrophysical Impact

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Abstract

The work contains result of development of the method of calculation of the temperature field in the cathode's surface during electrospark alloying.

Keywords: electrospark alloying, an anode, a cathode, a working surface, temperature field, the heat transfer

1. Introduction

Among different methods of the surface hardening of metals and alloys that use concentrated energy currents electrospark alloying (ESA) is one of the methods that is used widely. This method is known for low energy consumption, simple technology, opportunity of the local coating in a wide range of physical and chemical characteristics.

To date, extensive experimental material of the studying of the process of ESA, electrode erosion and creating of the electrode materials, is accumulated [1]. Further development of the ESA method and its practical usage (thermophysical properties and option charge optimizing, getting the surface levels with specified properties) is hampered by lack of numerical methods for calculation of temperature fields on the working surface. Absence of the full mathematical model of

ESA process is explained by the difficulty of the electrical, physical and chemist processes that happen during the realization of the method [1-4].

This work is the continuation of the works that developed the ESA mathematical model.

2. Mathematical model for determination of the temperature field of the cathode's surface

Interaction of the cathode (processed detail) and anode (alloying instrument) happens during the ESA. As a result, a hole, filled with the material, received on the reaction of cathode, anode and electrode medium, appears on the cathode. Particle transfer of the material erosion of anode on cathode happens in liquid and solid phases. One of the options is transferring of the hot particle (its temperature is close to melting point) on the cold surface (its temperature is close to the environmental temperature. During this process binding of the particle on the surface without formation of a zone mutual crystallization is possible. For this case mathematical model for determination of the temperature field in the cathode's surface during ESA process will be investigated further.

Here is the problem of the heating of cathode-parallelepiped Q_0 with the working surface S_0 в in a Cartesian coordinate system (Figure 1)

$$Q_0 = \left\{x \in R^3 : |x_i| < a_i,\ i = 1, 2,\ 0 < x_3 < a_3\right\},$$
$$S_0 = \left\{x \in R^3 : |x_i| < a_i,\ i = 1, 2,\ x_3 = 0\right\}.$$

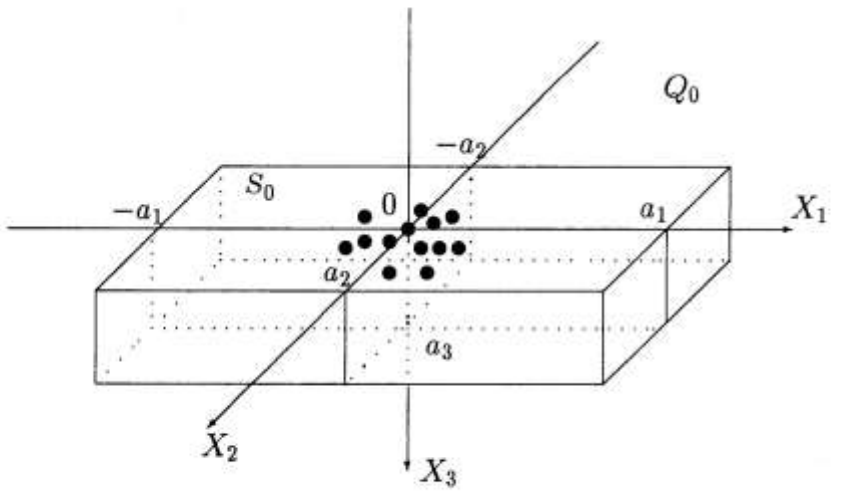

Figure 1. Cathode with the drops on its surface

A source of heating is surface heat current $q = \{x_1, x_2, t\} = \{\vec{x}, t\}$, that was created as a result of electrospark discharge between electrode and cathode-parallelepiped, and warmed-over liquid metal, that as drops Q_i gets from anode to the working surface S_0. Here $t > 0$ – time, $i = \overline{1, N}$, N – number of drops.

It is assumed that during one electrospark discharge drops are formed simultaneously and evenly fill some part of the surface (Figure 1). After falling on it, drops becomes solid immediately, giving the cathode heat energy. By moving of the cathode periodically full covering of the surface is achieved.

The following boundary value problem characterizes heat balance in the cathode and drops:

$$\lambda_1 \Delta T_0 - c_1 \rho_1 \frac{\partial T_0}{\partial t} = 0, \qquad \vec{x} \in Q_0, \qquad t_0 < t < t^*,$$

$$\lambda_2 \Delta T_i - c_2 \rho_2 \frac{\partial T_i}{\partial t} = 0, \qquad \vec{x} \in Q_0, \qquad t_0 < t < t^*,$$

$$T_0\big|_{t=0} = \Phi_0(\vec{x}), \quad \vec{x} \in Q_0, \quad T_i\big|_{t=0} = \Phi_i(\vec{x}), \quad \vec{x} \in Q_i, \quad i = \overline{1, N},$$

$$\frac{\partial T_0}{\partial x_1}\Big|_{x_1 = \pm a_1} = 0, \quad \frac{\partial T_0}{\partial x_2}\Big|_{x_2 = \pm a_2} = 0, \quad \frac{\partial T_0}{\partial x_3}\Big|_{x_3 = a_3} = 0,$$

$$\lambda_1 \frac{\partial T_0}{\partial x_3} + \alpha_1 (T_0 - T_s) = 0, \quad x_3 = 0, \quad (x_1, x_2) \notin \bigcup\nolimits_{i=1}^{N} \Omega_i,$$

$$T_0 = T_i, \quad \lambda_1 \frac{\partial T_0}{\partial x_3} = \lambda_2 \frac{\partial T_i}{\partial x_3}, \quad x_3 = 0, \quad (x_1, x_2) \in \bigcup\nolimits_{i=1}^{N} \overline{\Omega}_i,$$

$$\lambda_2 \frac{\partial T_i}{\partial \vec{n}} + \alpha_2 (T_i - T_s) = -q(T_i), \quad \vec{x} \in \partial Q_i, \quad x_3 < 0.$$

Here $\Delta = \sum_{i=1}^{N} \partial^2 / \partial x_i^2$ – Laplace operator, T_0 – cathode temperature; T_i – temperature of the *i*-numbered drop; T_s – environmental temperature; λ_1, λ_2, c_1, c_2, ρ_1, ρ_2, α_1, α_2 – thermal conduction coefficients, heat capacity, specific gravity and heat transfer of the cathode (index 1) and drops (index 2), $\vec{n}$ – outer-pointing normal to $\partial\Omega_i$, $\kappa = \lambda/(c\rho)$ – thermal diffusivity coefficient, $\Omega_i = Q_i \bigcap \{a_3 = 0\}$, $\overline{\Omega}_i = \Omega_i \bigcup \partial\Omega_i$, $Q_i = \{a_{1i}, a_{2i}\} \times \{b_{1i}, b_{2i}\} \times \{0, d\}$ – size of a drop, $q(T_i) = \kappa\sigma T_i^4$ – Stefan-Boltzmann law non-liner emission.

That problem is non-liner and is determinated in the difficult three-dimensional space $Q_0 \bigcup \left(\bigcup_{i=1}^{N} Q_i \right)$ with the changing of the covering after every electrospark discharge.

Par of the difficulties can be avoided with the help of extra hypothesis of the task. Let us consider that size of the drops is small. Then, obviously, temperature within the drop is almost constant and changes significantly only during the time. That gives an opportunity to average problem with the volume Q_i, using only volume for the area Q_0.

As a result of using new variables $U = T - T_s$, $U_i = T_i - T_s$, $a_0^2 = \lambda_1 / (c_1 \rho_1)$ and averaging he task with the volume Q_i, the task for Q_0 takes the following view:

$$\begin{aligned}
&\Delta U - \frac{\partial U}{\partial t} = 0, \quad \vec{x} \in Q_0, \quad \tau_0 < \tau < \tau^*, \\
&U\Big|_{t=\tau_0} = \varphi(\vec{x}), \quad \vec{x} \in Q_0 \bigcup \left(\bigcup_{i=1}^{N} Q_i \right), \\
&\frac{\partial U}{\partial x_1}\Big|_{x_1 = \pm a_1} = 0, \quad \frac{\partial U}{\partial x_2}\Big|_{x_2 = \pm a_2} = 0, \quad \frac{\partial U}{\partial x_3}\Big|_{x_3 = a_3} = 0, \\
&\frac{\partial U}{\partial x_3} + h_1 U = 0, \quad x_3 = 0, \quad (x_1, x_2) \notin \bigcup_{i=1}^{N} \Omega_i, \\
&\frac{\partial U}{\partial x_3} + h_1 U = \psi(t), \quad x_3 = 0, \quad (x_1, x_2) \in \bigcup_{i=1}^{N} \overline{\Omega}_i.
\end{aligned} \tag{1}$$

Here

$$\varphi(\vec{x}) = \begin{cases} \varphi_0(\vec{x}), \ \vec{x} \in Q_0, \\ \varphi_i(\vec{x}), \ z = 0, \ \vec{x} \in \bigcup\limits_{i=1}^{N} \Omega_i, \end{cases}$$

$$\varphi_0(\vec{x}) = \Phi(\vec{x}) - T_s, \ \varphi_i(\vec{x}) = \Phi_i(\vec{x}) - T_s,$$

$$\psi(t) = q - \left[c_0 \frac{\partial U}{\partial t} + c_1 U + c_2 f(U) \right],$$

$$h_1 = \frac{\alpha_1}{\lambda_1}, \ \tau_0 = a_0^2 t_0, \ \tau^* = a_0^2 t^*, \ c_0 = \frac{\lambda_1}{\lambda_2} \frac{c_2 \rho_2}{c_1 \rho_1} d,$$

$$c_1 = \frac{h_2}{\gamma_1} \left(1 + 2d \frac{\Delta a_i + \Delta b_i}{\Delta a_i \Delta b_i} \right), \ c_2 = \frac{1}{\gamma_1} \left(1 + 2d \frac{\Delta a_i + \Delta b_i}{\Delta a_i \Delta b_i} \right), \ \gamma_1 = \frac{\lambda_1}{\lambda_2},$$

$$h_2 = \frac{\alpha_2}{\lambda_2}, \ f(U) = \frac{\kappa T}{\lambda_2} (U + T_s)^4, \ \Delta a_i = a_{2i} - a_{1i}, \ \Delta b_i = b_{2i} - b_{1i}.$$

That problem is determinated only for Q_0 area, because boundary condition for the temperature U replaces all the equations of the heat balance in drops. However, another difficulty here is nonlinear heat exchange between drops and parallelepiped. Because of that we will divide the task into two parts.

In the first part heat current $\psi(t)$ from one average drop is calculated approximately. The second part studies a lot of identical small drops, that are put on the covering with the given and known heat current $\psi(t)$.

For solving the problem Green's function $G(\vec{x},\xi,t-\tau)$ that satisfies boundary value problem will be used.

Size of the parallelepiped is much bigger than the size of the drop and because of that for making task easier it can be considered as a halfspace $R^3_+ = \{x \in R^3 : x_3 > 0\}$, putting $a_1 = a_2 = a_3 = +\infty$.

Let us put coordinate system the way that its beginning will coincide with the center of the drop *Q*, for which the problem is solved, and axes Ox_1, Ox_2, Ox_3 were axes of symmetry of the problem.

The decision should be found in the octant $Q_0^1 = \{x_1 > 0,\ x_2 > 0,\ x_3 > 0\}$ with extra conditions on its borders $\partial U / \partial x_j = 0$, $j = 1, 2$.

If $\alpha_1 = 0$ (and hence *h* = 0), then Green's function G is written through the fundamental solution $g_3(\vec{x},t) = g_1(x_1,t)\cdot g_1(x_2,t)\cdot g_1(x_3,t)$ of the Cauchy problem for parabolic equation:

$$G(x,\xi,t) = \prod_{r=1}^{3}\left(\sum_{s=1}^{2} g_1(x_r + (-1)^s \xi_r, t\right),$$

where $g_1(x_i,t) = 1/2\kappa\sqrt{\pi t}\cdot \exp(-x_i^2/4\kappa^2 t)$.

Considering that, it is possible to change the problem to its equivalent integral relations:

$$U(x,t) = V(x,t,t') - \kappa^2 \int_{t'}^{t} d\tau \int_0^a\int_0^b G(x,\xi',0,t-\tau)\cdot \Psi(\xi',0,\tau)d\xi',\ x \in Q_0^1 \bigcup S_1,\ t > t',$$

$$V(x,t,t') = \int_0^\infty\int_0^\infty\int_0^\infty G(x,\xi,t-t')U(\xi,t')d\xi + \kappa^2 \int_{t'}^{t} d\tau \int_0^a\int_0^b G(x,\xi',0,t-\tau)\cdot q(\xi',0,\tau)d\xi',$$

$$\Psi(\xi',0,\tau) = c_0 \partial U(\xi',0,\tau)/\partial\tau + c_1 U(\xi',0,\tau) + c_2 f(U(\xi',0,\tau),$$

$$U(x,0) = \varphi(x),\quad \vec{x} \in Q_0^1 \bigcup S_1,$$

where $S_1 = \{x : 0 < x_1 < r,\ 0 < x_2 < r,\ x_3 = 0\}$, *r* – radius of the drop.

Integral relations, as a task (1), approximately describe the process of the heat transfer to the heat conducting substrate from the drop, that is put on its birder.

3. Numerical calculations

To solve the problem of material heat transfer in electrochemical effect was created a number of programs. Programs were tested with different materials relatively thermophysics constants and materials of drops and coverings. Different refractory metals were considered: such as: titanium (*Ti*), wolfram (*W*), tantalum (*Ta*). Parallelepiped's material is ferrum (*Fe*).

Since for small values of time the solution $U(\vec{x},t)$ decreases to zero while increasing x_1, x_2, x_3, then spatial grid nodes were taken close to the origin. The results are illustrated by the drawing, that show temperature *U*, that is equal to the difference between temperatures of the substrate and the environment: $U = T - T_s$.

Figures 2 – 4 shows changes of the temperature *U* in the hotspots on the axis $0x_3$: $x_1 = 0$, $x_2 = 0$ for $x_3 = x_{3k}$, in case: *Fe – Ti* (Figure 2), *Fe – Ta* (Figure 3), *Fe – W* (Figure 4) during the time *t*. (Here and further, for example *Fe – Ti*, means that parallelepiped material is ferrum, and drop material is titanium).

Curves on the Figures 2 – 4, that are near, shows the temperature change. Numbers *k*, $k = \overline{0,19}$, lines (counting from top to down) shows the temperature in the hotspots $(0, 0, x_{3k})$.

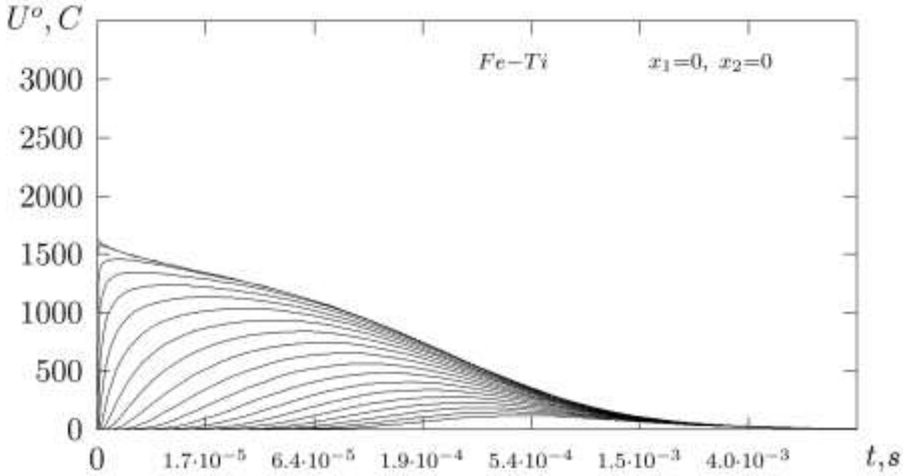

Figure 2. Changing of the temperature of the points on the axis $0x_3$ with the time in case *Fe – Ti*

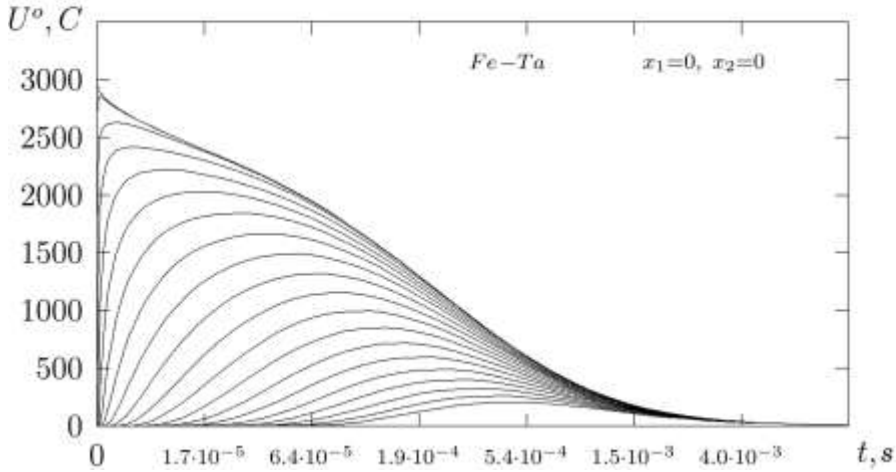

Figure 3. Changing of the temperature of the points on the axis $0x_3$ with the time in case *Fe – Ta*

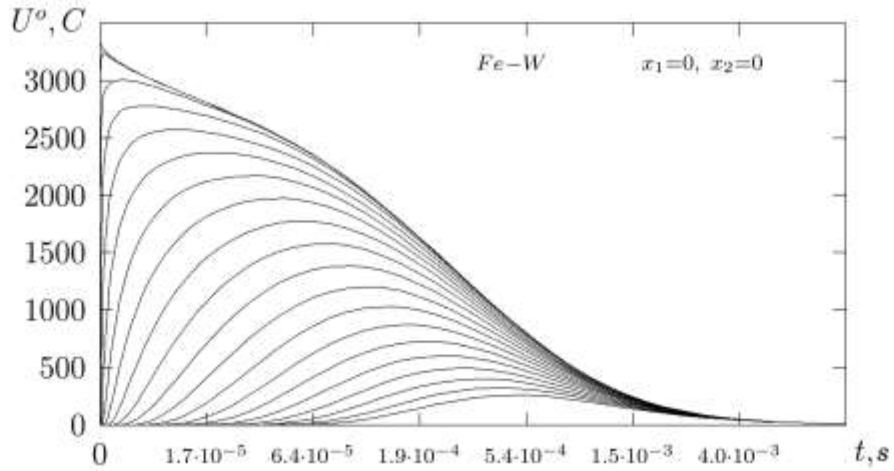

Figure 4. Changing of the temperature of the points on the axis $0x_3$ with the time in case *Fe – W*

While heat transferring the important role is taken by the basic temperature of the drop, equal to material's melting point. The temperature of the same nodes of substrate increases in the materials of the drop *Ti – Ta – W* counting from titanium *Ti* to wolfram *W*. The closer the spot is to the surface $x_3 = 0$ or to the outer bound of drop on the surface $x_3 = 0$ the sharper increases the temperature in the beginning and the earlier it stabilizes and begins to decrease.

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