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# A Comparative Study of Friction Laws Used in Spur Gear Power Losses Estimation

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## Abstract

In spur geared transmissions, the power loss due to the friction phenomenon is proportional to the coefficient of friction, hence the importance of having a model or an adequate law to calculate the latter. Thus, the objective of this work is initially to present, for a typical spur gear set, a comparison of different frequently used friction laws by specifying their domains of validity and in a second time we will present the variation of the coefficient of friction in function of the meshing position along the path of contact during the engagement between the pinion and the wheel teeth by using a numerical modeling developed in this study.

**Keywords**: Spur gear, friction coefficient, numerical modeling, power losses

## 1. Introduction

Nowadays, power transmitters which are mechanical systems or mechanisms that are used to accommodate the power according to the needs are highly used in many domestic and industrial applications. While the popularity of the latter is closely linked to better performance they offer nevertheless the increase in power to be transmitted at high rotational speeds result in significant power losses in spur geared transmissions and must therefore be taken into account during the different phases especially when designing and choosing the materials of the gears since power losses have direct impact on the lifetime of the power transmitters.

A well designed mechanical system would enable the user to avoid breakage of the mechanism due to thermal expansion and would allow a better design of the cooling systems. Besides energy costs continue to rise and therefore in order to optimize the efficiency of the system, it is very important to estimate the power losses in the transmission in order to reduce it.

In a geared power transmission system, the total power loss can be divided into load- dependent contribution, the losses due to friction phenomenon for example, and into load-independent contribution which includes the power losses related to the lubrication method, to the ventilation on teeth and to the trapping of the air-lubricant between the teeth. In function of the operating conditions, each power loss category controls the overall efficiency of the system. As cited by authors of references [1,2], for instance, the friction phenomenon related power losses are preponderant at low speeds whereas they are low at high-speed in comparison to the power losses due to the ventilation phenomenon and to the trapping phenomenon which was discussed by the authors of references [3, 4].

This work is a preliminary study of the power losses related to the friction phenomenon in spur gear transmissions. The first part includes a comparative study between the empirical laws of friction given in references [5, 6, 7 and 8] and the experimentally validated friction law given in reference [9] and in the second part, we established a numerical modeling of meshing between the pinion and the wheel which is used to study the variation of the instantaneous friction coefficient using an empirical and an experimental friction law for a gear set example of reference [10] the objective being to come up with a suitable friction law which takes into accounts different geometric and operational parameters and to use the latter in the friction power loss calculation process in our future works.

## 2. Literature overview on friction phenomenon between spur gear teeth

In order to understand the friction phenomenon resulting from metal-metal in non-lubricated contacts also known as dry friction, many experiments were conducted. In the case of lubricated contacts there are few models to identify qualitatively and quantitatively the friction phenomenon and its direct influence on power loss in geared power transmissions. According to Diab [1], Velex P. and Cahouet, V. [2], the friction between teeth is one of the major sources of power dissipation and may also be a source of vibrations and noises. The total power losses resulting from this phenomenon is closely linked to the coefficient of friction.

### 2.1. Empirical laws of friction

Some authors have established the friction laws in order to estimate the friction coefficient as a function of geometric and operating parameters. In table 1 below, the friction laws used in this study are presented.

| Author and reference | Friction law | Range of validity |
|---|---|---|
| Benedict and Kelley [5] | $\mu = 0.0127\left(\frac{50}{50-S_{rms}}\right)\log_{10}\left(\frac{3.98\times10^{9} P_h^{2} R}{\upsilon_0 SR (V_r)^3 E'}\right)$ | $\frac{50}{50-S_{rms}} \le 3$ |
| Misharin [6] | $\mu = 0.3865\left(\frac{1}{\upsilon_k V^2{}_r SR}\right)^{0.25}$ | $\frac{V_s}{V_r} \in [0.4;1.3]$ <br> $\mu \in [0.02;0.08]$ |
| O′ Donoghue and Cameron [7] | $\mu = 0.6\left[\frac{S_{cla}+22}{35}\right]\left[\frac{0.756}{\upsilon^{1/8} SR^{1/3} V_r^{1/2} R^{1/2}}\right]$ | |
| Drozdov and Gavrikov [8] | $\mu = \left[\frac{1/V_r}{0.4Rg\sqrt{\upsilon_k} + \phi + 13.4/V_r}\right]$ <br> $\phi = 0.47 - 0.13(10)^{-4} P_{\max} - 0.4(10)^{-4}\upsilon_k$ | $\upsilon_k \in [4;500]$ <br> $V_s \le 15$ <br> $V_r \in [3;20]$ <br> $P_{\max} \in [4000;20000]$ , |

Table1: Empirical laws of friction used to determine the coefficient of friction

Where $S_{rms}$ and $S_{cla}$ are the surface roughness parameters in μm defined as $S_{rms}$=1.25 $S_{cla}$, $\upsilon_0$ and $\upsilon_k$ respectively the dynamic viscosity in centipoise (cPo) and cinematic viscosity in centistokes (cSt), Vr and Vs respectively the rolling and sliding velocities in (m/s), $P_h$ the maximum pressure of Hertz in pascal (Pa), R the effective radius of curvature in meters (m) and SR the slide to roll ratio defined as the ratio between the sliding velocity and the rolling velocity.

### 2.2. Experimental law of friction

To establish a friction law that takes into account several meshing factors including lubrication, material type and surface roughness, the authors H.Xu, A. Kahraman, N. Anderson and D. Maddock [9] considered the contact between teeth as an elastohydrodynamic (EHD) contact because the latter is characterized by low contact areas, significant surface deformation, high contact pressure and the presence of the oil film between the contacting asperities. They came to the following friction law:

$$\mu = e^{f(SR,P_h,\upsilon_0)} P_h^{b_2} \left|SR\right|^{b_3} V_e^{b_6} \upsilon_0^{b7} R^{b8}$$

Where:

$$f(SR,P_h,\upsilon_0) = b_1 + b_4 \left|SR\right| P_h \log_{10}(\upsilon_0) + b_5 e^{-\left|SR\right| P_h \log_{10}(\upsilon_0)} + b_9 e^{S}$$

Ve is the entraining velocity in m/s and the constants bi are given as:

bi = -8.916465 ; 1.03303 ; 1.036077 ; -0.354068 ; 2.812084 ; -0.100601 ; 0.752755 ; -0.390958 ; 0.620305 for i = 1 to 9.

## 3. Comparative study of friction laws

The comparative study of empirical and experimental laws of friction was made using the data in table 2 below:

| Parameter | Value |
|---|---|
| Effective radius of curvature R (mm) | 5 |
| Maximum pressure of Hertz $P_h$ (Gpa) | 2 |
| Cinematic viscosity $\upsilon_k$ (cSt) | 13 |
| Dynamic viscosity $\upsilon_0$ (cPo) | 10 |
| Surface roughness parameter (Srms (µm) | 0.07 |

Table 2: Comparison data of the friction laws

In the case of lubricated contacts the coefficient of friction varies with the load, the entrainment speed, the sliding velocity, material properties of surfaces in contact and the properties of the used lubricants. Figures 1, 2, 3 and 4 illustrate the variations of the friction coefficient from empirical laws during meshing.

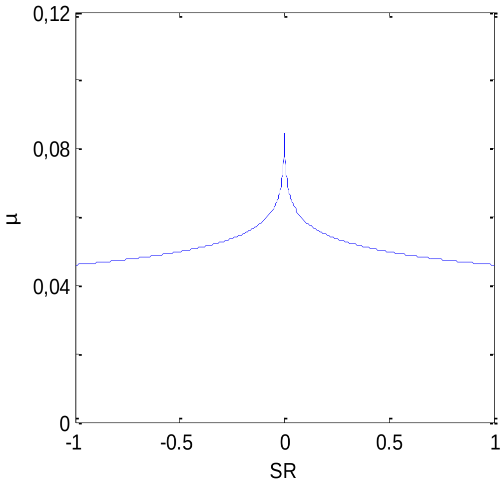


Fig.1: Friction coefficient by Benedict and Kelley [5]

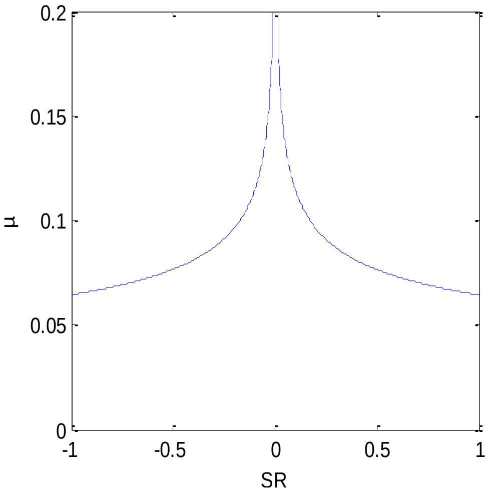


Fig.2: Friction coefficient by Misharin [6]

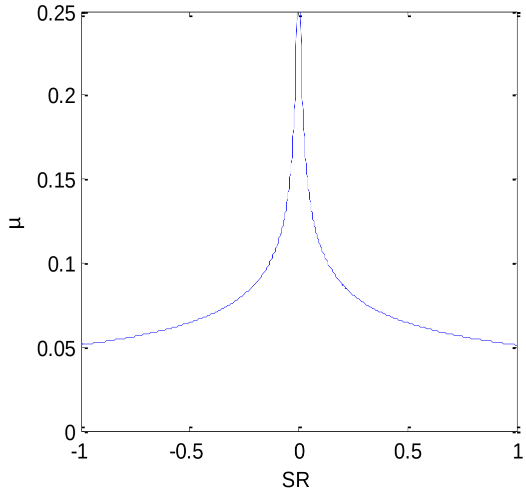


Fig.3: Friction coefficient by O' Donoghue and Cameron [7]

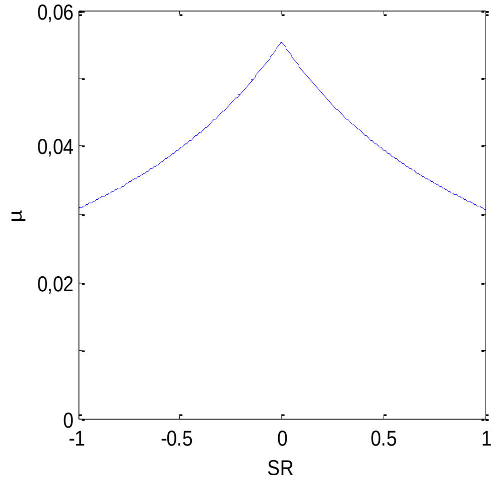


Fig.4: Friction coefficient by Drozdov and Gavrikov [8]

The figure 5 below shows the variation of the friction coefficient during the engagement based on an experimental law of friction.

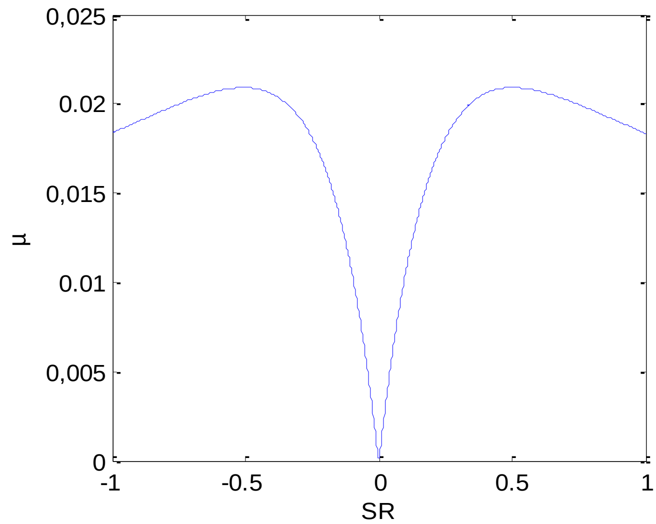


Fig.5: Friction coefficient by Kahraman and Xiu [9]

The figure 6 below shows a comparison of the evolution of these different laws of friction versus the slide to roll ration (SR).

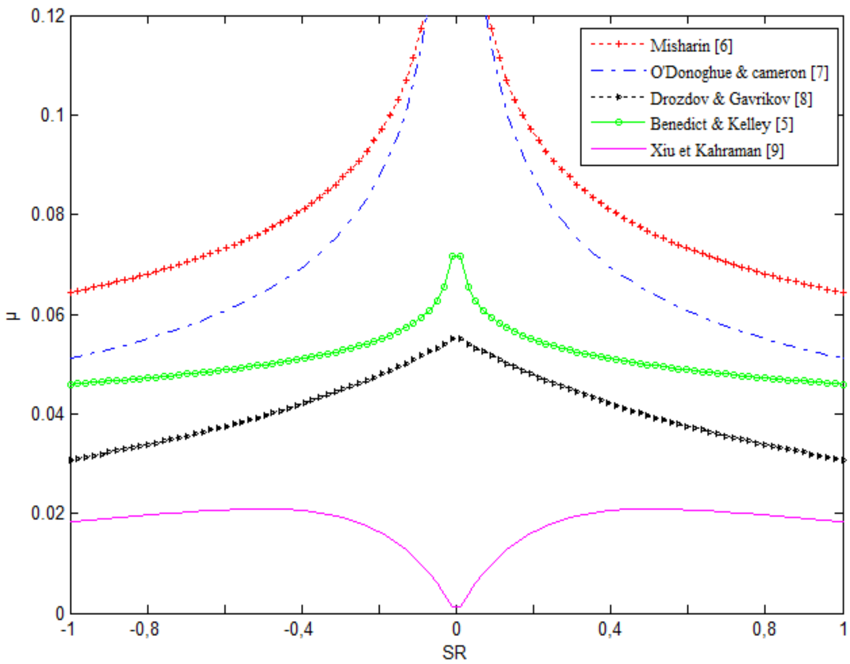


Fig.6: Comparison of the laws of friction

We remark that when the slide to roll ratio (SR) value is zero, the empirical friction laws of Misharin [6], O 'Donoghue and Cameron [7] and Drozdov and Gavrikov [8], predict their greatest values of the coefficient of friction at the pitch point where the sliding velocity is zero whereas the Xiu and Kahraman's friction law [9] predicts a minimum value of the friction coefficient at that same point, aware of that, some authors have established limits of validity of their friction laws for example the Misharin's law [6] is only valid for the slide to roll ratios ranging between 0.4 and 1.3.
Moreover, the empirical law of friction of Benedict and Kelley [5] predicts a value of the friction coefficient which approaches 0.1; this value was used by the authors N. Anderson and S. Loewenthal, [10], in their works.
Finally, the friction coefficient found with Xiu and Kahraman's law [9] is in agreement with the theoretical friction curve.

## 4. Meshing modeling

By studying the impact of different geometric parameters and operation on the power loss due to friction and by exploiting the documentation on the gears in this case that of the reference [11], we developed a numerical model that allows us to know the variation of different meshing parameters during the engagement between pinion and wheel teeth.
To evaluate variation of the instantaneous friction coefficient along the line of action, we chose the law of Xiu and Kahraman [9] since it has been experimentally validated and the empirical law of Benedict and Kelley [5] since it

predicts a friction coefficient with maximum value tends towards 0.1 unlike other empirical laws for which the friction coefficient tends to infinity at the pitch point where the sliding velocity is zero. Those two laws are presented in table 3 below:

| Author and reference | Law of friction |
|---|---|
| Benedict and Kelley [5] | $\mu = 0.0127\left(\frac{50}{50 - S_{rms}}\right)\log_{10}\left(\frac{3.98\times10^{9} P_h{}^{2} R}{\upsilon_0 SR(V_r)^{3} E'}\right)$ |
| H.Xu, Donald R. Houser, A. Kahraman [9] | $\mu = e^{f(SR, P_h, \upsilon_0)} P_h{}^{b_2} \lvert SR \rvert^{b_3} V_e{}^{b_6} \upsilon_0{}^{b7} R^{b8}$ <br> $f(SR, P_h, \upsilon_0) = b_1 + b_4 \lvert SR \rvert P_h \log_{10}(\upsilon_0) + b_5 e^{-\lvert SR \rvert P_h \log_{10}(\upsilon_0)} + b_9 e^{S}$ |

Table 3: Empirical and experimental laws of friction

The study was conducted using the gear data from taken in the work of authors N. Anderson and S. Loewenthal [10]. The table 4 below shows the data.

| Parameter | Value |
|---|---|
| Number of teeth (pinion) | 48 |
| Number of teeth (Wheel) | 80 |
| Module (mm) | 3.175 |
| Tooth width (mm) | 10 |
| Cinematic viscosity $\upsilon_0$ (cSt) | 60 |
| Pressure angle (degres) | 20 |
| Torque (Nm) | 271 |
| Rotational speed (rpm) | 2000 |
| Young's modulus (N/m²) | 2,276 x $10^{11}$ |

Table 4: Geometric data used in the study

In an orthonormal base where the origin is the point of intersection of the line of action with the base circle of the pinion, we plotted the variation of the friction coefficient in function of the meshing position using the Benedict and Kelley's friction law [5] and that of Xiu and Kahraman [9]. The curves are shown in figure 7 below:

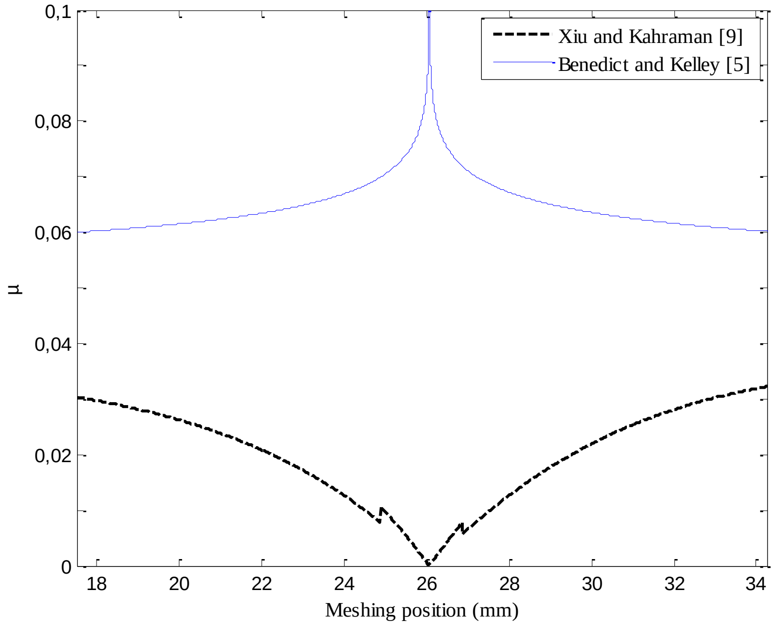


Fig.7: Variation of the friction coefficient along the line of action

It is noted that with the friction law established by Xiu and Kahraman [9], the friction coefficient reaches its maximum values at the points of beginning and ending of engagement and it reaches its minimum value at the pitch point, this remark is in agreement with the results of studies obtained by authors in the reference [9] when studying the variation of the friction coefficient in function of the slide to roll ratio.

With the law of Benedict and Kelley [5], the friction coefficient reaches its minimum value at the points of beginning and end of engagement and reaches its maximum value of 0.1 at the pitch point. The authors N. Anderson and S. Loewenthal [10] came to the same conclusion after having studied the variation of the friction coefficient in function of the meshing position.

## 5. Conclusion

Using data from a typical example of gear meshing, the obtained friction coefficient expression leads to a nonlinear form, therefore a numerical modeling was established in order to determine the average coefficient of friction using numerical integration. The developed program is used to study the instantaneous variation of the coefficient of friction along the line of action in involute spur gears.

Comparing the obtained results by using the empirical law of Benedict and Kelley [5] with the experimentally validated Xiu and Kahraman's frictional law [9], we noticed that for the empirical law, the friction coefficient reaches its minimum value

at the meshing starting and ending points and its maximum value at the pitch point while with the experimentally validated frictional law, the friction coefficient reaches its maximum value at the starting and ending points of meshing and its minimum value at the pitch point. These remarks are in agreement with the published results of the cited authors.
We can assume that the results found by using the Xiu and Kahraman's friction law [9] are adequate since the latter predicts its lowest friction coefficient value at the pitch point where there is no relative sliding whereas those found by using the Benedict and Kelley's law [5] aren't because their formula overestimates the friction coefficient. This can be explained by the fact that the law of reference [5] has the sliding velocity as a denominator and so when the latter tends towards zero, the friction coefficient tends towards infinite which has no tangible physical explanation. Therefore, in our future studies the Kahraman and Xiu's friction law will be used to calculate the instantaneous power losses due to the friction phenomenon in involute spur gears.

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