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Interconnected Markov Models of Dissimilar Processes of Operation of Complex Technical Systems in Long-Term Autonomous Functioning

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Abstract

The article deals with issues of ensuring reliability and performance of tasks by complex technical systems. Operation of complex technical systems (CTS) is represented as two interconnected sub-process, developing in a different state spaces and times. Interconnected Markov models of dissimilar processes of operation of complex technical systems are considered in long-term autonomous functioning. This article shows a general scheme of two-stage modeling of operation of CTS, allowing to evaluate their effectiveness.

Keywords: Markov models, interconnected processes, operation, reliability, complex technical systems

1. Introduction and Preliminaries

Execution of tasks by the complex technical systems (CTS) is substantially determined by their reliability. One of the most significant factor of reliability changes is the nature of functioning conditions and the strategy of operation, under which it should be understood a set of organizational and technical actions for the maintenance and restoration of systems. [1]. Reducing the intensity and volume of such activities make a negative impact on the indicators of reliability of CTS. This tendency is typical for a class of systems that operate autonomously without the possibility of carrying out of preventive control and remedial actions or with the possibility of their holding in the reduced amount.

These features inherent to the objects of motor transport, offshore facilities, and systems for special and military purposes.

Operation of the CTS of this class includes a number of typical actions. So, periodically CTS through time τ_{baf} leave the stationary points (bases, autoparks, ports, etc.) for execution of tasks during the time T_{af}. At the same time CTS with intensity λ_{mob} functions, for example, in case of relocation from the port to the port (base, destination point, etc.) lasting τ_{em} technical condition of CTS during this time can be controlled only superficially, in a small volume (visual inspection, continuous monitoring of especially important elements, etc.) with the intensity λ_{af} and duration τ_{caf}. Faults and failures occurring are detected during inspections and continuous monitoring with intensities λ_{fau} and λ_{fai} respectively. Problems are solved by the operating staff when finding them outside the points of permanent placement, so CTS passes from a faulty into good condition with the intensity μ_{raf}. Failures leading to loss of efficiency, as a rule, are removed after returning to a stationary (base) for the time τ_{rfai}.

On a stationary point on the CTS in addition to restoration of working capacity is carried out maintenance (M) with periodicity τ_{BM} and duration of τ_M. Maintenance can also be carried out after the fulfillment of n_M routes (outputs). Between maintenance, especially in the case of long-term presence on the base, CTS also can fail with parameter ω_{fai}. Failures are eliminated at the next maintenance or during the inspection before leaving on route lasting τ_{MBR}.

Thus, the process of operation of CTS is accurately divided into two interconnected sub-process developing in a different state space and time. The first sub-process evolving in a continuous operating time outside the stationary base for statuses of movements, expectations of moving to an intermediate point and the eliminations of faults by the operating personnel (crew, team, etc.). Change the state of an object in the subprocess occurs during the time bounded duration of autonomous operation T_{af}. The second component of the CTS operation includes the statuses of the expectation at stationary base in readiness to leave on route or carrying out routine maintenance and unplanned recovery of serviceability or working capacity by specialized units (docks, service stations, etc.). This subprocess develops in an unlimited period of operation CTS. Of course, there is a limit condition in which the operation is terminated, but within considered case such a possibility is not allowed.

Correlation of the considered components of process of operation is carried out through parameters of subprocesses in the form of the appropriate functional dependences. Thus, the interrelation of operation process of the CTS at stationary base and Autonomous functioning it is advisable to take into account through a change of failure rate after leaving the base. This intensity depends on the duration of inspections before leaving on route, i.e $\lambda_{fai}=f(\tau_{MBR})$ and $\lambda_{fai}\geq\omega_{fai}$. Later accepted $\lambda_{fai} = \omega_{fai} \cdot K_\lambda$, $K_\lambda = \tau_M / \tau_{MBR}$ ($\tau_{MBR} < \tau_M$).An inverse interrelation between the sub-processes characterized by a return of serviceable CTS to stationary base or by occurring a failure during autonomous operation, that elimination requires involvement of forces and means of specialized units subdividing.

Note that most of the described CTS operational activities are random (the occurrence of faults and malfunctions, the duration of the recovery, maintenance, technical state control, etc.). Both analytical and statistical methods are used for their description [2-11]. Despite the fact that in terms of closest approach to the reality statistical methods favourably differ from analytical methods, it is reasonable to use them at researches for replacement of natural experiment or in the absence of analytical models, at the same time recognizing the difficulty of obtaining optimal solutions. The mathematical apparatus of the theory of Markov processes is the easiest and most convenient analytical method in terms of possibilities of analyses of STC's operation [12]. But it must be kept in mind that the processes that transform the CTS from one status to another, should be described by exponential functions.

Here is a brief reasons for objectivity of a choice of exponential distribution laws for the description of groups of flows, characteristic for CTS: the occurrence of failures, their restoration, the output of the CTS of various types of control and maintenance(for more details, see [13,14]).

It is known that the description of the event streams by exponential functions is possible, if these streams are the simplest, i.e. the conditions of stationarity, ordinary and absence of aftereffect are satisfied.

From the point of view of stationarity of failure flows the period of operation of CTS is usually subdivided into three sections: running-in period, period of normal operation and period of intensive wear. The main reasons that cause the violation of the stationarity of flow failures are the presence of a running-in period after entering the CTS in operation and the effect of gradual failures. Glitches arise at any time after input of CTS in operation, gradual failure - as a rule, after a sufficiently long operating time due to wear and ageing of materials.

Implementation of preventive measures including the replacement of aging components can ensure that the total flow of CTS's failures will be determined by the flow of glitches, which is well described by exponential functions.

It should be noted that the CTS is composed of a large number of elements with high reliability, generating quite a lot of low intensity streams of failures. The probability of simultaneous occurrence of two or more failures over a short time interval is close to zero, it indicates the ordinary of this flow. Since the failure rate of Complex Technical System is incommensurable with any of the failure rate included in the CTS elements, we can believe that the failure rate of CTS's elements are independent In addition, there is no aftereffect in the failure rate, i.e., the appearance of failures in the one area of the operating time practically does not change the probability of occurrence of any number of failures in the other area.

Using the Palm Khinchin theorem, which confirm that if there is a sufficiently large number of independent streams of low intensity, each of them is stationary and ordinary, the sum of these flows forms a stream, close to simple, so you can accept the assumption of the elementary failure rate of CTS.

As for the restore threads, with the rapid recovery of the failed elements, which is typical for CTS, the reliability function is practically independent of distribution’s form of the recovery time, and the replacement of the real exponential laws leads to small errors, so they can be neglected.
This fully applies to the flows characterized by the periodicity and duration of maintenance and inspections. In this way, with some assumptions flows (threads) converted complex technical system (CTS) from one state to another can be described by exponential relationships, and consequently, for constructing of math model, you can use the Markov-process theory, in particular, widely used processes with continuous time and discrete phase space.

The study of interrelated random processes is considered in [15]. However in our case one of considered processes is not ergodic, that is why in direct position that approach is not able to use. As an alternative approach it could use to divide phase space states of basic process into two subsets, one of which is non-ergodic and so the process inside expands as unstable. Imagine that the phase space of complex technical system operation is 9 states with nonconductor Markov chain (fig. 1).

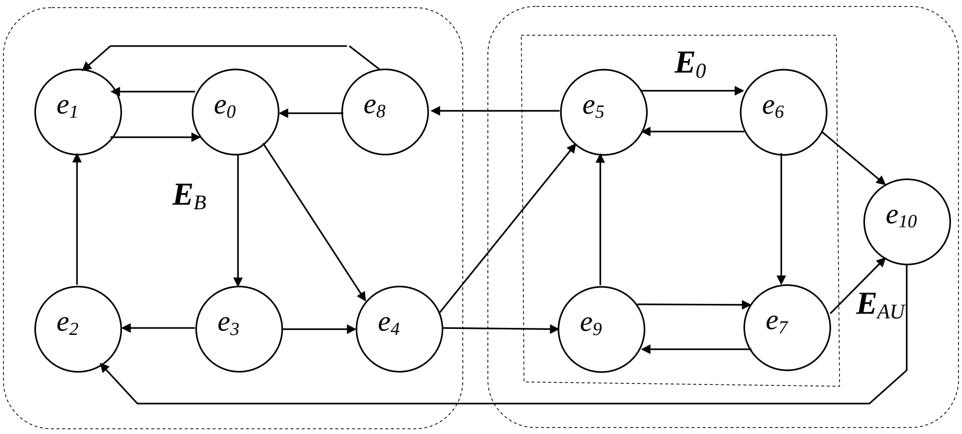

Fig.1 Graph of model’s statuses and transitions when CTS in the process of operation

2. Main results

The phase space is divided into two disjoint subsets: subset $\boldsymbol{E_B}$ includes the process state $\boldsymbol{E_B}=\{e_0...e_4, e_8\}$ when the object is on the stationary base, and $\boldsymbol{E_{AU}}=\{e_5...e_7, e_9, e_{10}\}$– when the object is outside the stationary base for autonomous execution of tasks. From the set of statuses of the phase space of status from $\boldsymbol{E_0} \subseteq \boldsymbol{E_{AU}}$ $\boldsymbol{E_0}=\{e_5, e_6, e_7, e_9\}$ is determined the efficiency of the process at an autonomous operation outside the stationary base, because when the object is in these statuses, it is provided the actual effect of its intended use (income, etc.). An indicator of efficiency is the sum of the probabilities of staying at these statuses PE = P5 + P6 + P7 + P9.

Table 1 - Physical sense of model's statuses when CTS in the process of operation

Status	Physical sense
e_0	The object is located at the base in a serviceable condition
e_1	Maintenance of serviceable or reconstructed object is carried out
e_2	Restoring of the failed object
e_3	The object is in a state of hidden failure (detected during maintenance)
e_4	Preparation to living on route with the elimination of failures
e_5	Being in readiness for use outside the stationary point
e_6	Functioning (checking) of the serviceable object outside the stationary
e_7	Functioning (checking) of the faulty object outside the stationary
e_8	Returning to the stationary
e_9	Being in readiness for use outside the stationary point with the presence of a fault
e_{10}	State of nonoperability, requiring restoration works on a stationary point

The transitions between subsets of States $\boldsymbol{E}_B$ and $\boldsymbol{E}_{AU}$ are quite rare, that allows to consider them as relatively independent phase state spaces of the object. In considering subset $\boldsymbol{E}_{AU}$ as an independent phase space excluding transition to $\boldsymbol{E}_B$ status e_{10} becomes absorbing (fig.2), he study process is provided to this status for the final time.

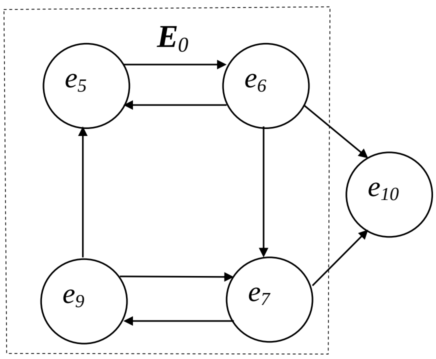

Fig.2 The graph of states and transitions of the process at the autonomous functioning of the CTS

In addition, a feature of the model is the presence of two starting statuses (e_5, e_9), where the vector of probabilities of being in these statuses is determined by the amount of inspections before leaving the base.

The dynamics of this process is described by the Kolmogorov-Chapman, for the graph of states and transitions (fig.2) they have a following form:

$$\begin{cases} \frac{dP_5(t)}{dt} = -\lambda_{mob}P_5(t) + \lambda_{af}P_6(t) + \mu_{af}P_9(t); \\ \frac{dP_6(t)}{dt} = -(\lambda_{af} + \lambda_{fau} + \lambda_{fai})P_6(t) + \lambda_{mob}P_5(t), \qquad \lambda_{af} = 1/\tau_{bm}; \\ \frac{dP_7(t)}{dt} = -\lambda_{af}P_7(t) + \lambda_{fau}P_6(t) + \lambda_{mob}P_9(t); \\ \frac{dP_9(t)}{dt} = -(\lambda_{mob} + \mu_{af})P_9(t) + \lambda_{af}P_7(t); \\ \frac{dP_{10}(t)}{dt} = \lambda_0[P_6(t) + P_7(t)]. \end{cases} \tag{1}$$

with the initial conditions $P_5(0) = K_\lambda^{-1}$, $P_9(0) = 1 - P_5(0)$; $P_6(0) = P_7(0) = P_{10}(0) = 0.$

The numerical solution of the differential equation system performed in MATHCAD system [16], taking into account the fact that the intensity of failures during the operation and during inspections increases 10…100 times [17]. Value ranges of parameters of the researched process of operation are selected as the most characteristic for considered class of CTS. It should be noted that the system of equations (1) is stiff, so its solution is obtained by Rosenbrock methods, which allows us to get the right (adequate) results in contrast to the traditionally applied in these cases, the Runge-Kutta method. The results of solving the system of equations (1) are shown in fig 3, which shows a decrease of efficiency with the increase in the duration of the autonomous functioning of the CTS.

The general tendency of changes in the efficiency is quite clear and does not contradict the logic of the process, but the degree of decrease in the value of index P_E depends significantly on the level of intensity λ_{fai} and on adopted strategy of movements (control) of the objects in the autonomous functioning. So, under intense inclusions (inspections) about 1 times a week, there is a sharp decline in the efficiency during Autonomous operation. Decreasing of intensity λ_{af} to the level of two times a month increases the efficiency on 7…8% during autonomous operation within six months. Further reduction of intensity λ_{af} to 1 times per month significant impact on the change in the index does not have, in this case the efficiency is increased not more than on 1…2%.

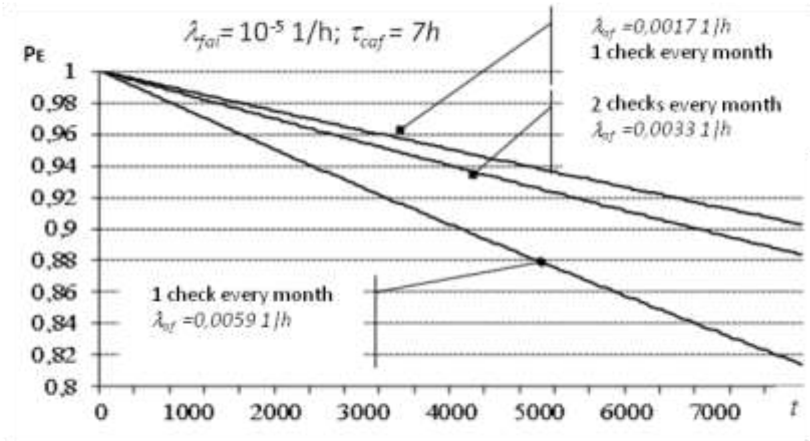

Fig.3. Change of efficiency of use CTS in case of autonomous functioning

In general, the non-stationary process model in the form of (1) links the value of the indicator of efficiency of P_E time of the autonomous functioning T_{af} for a given level of reliability and periodicity of monitoring. Unambiguity of such compliance allows to define the possible (reached) efficiency at the known duration of autonomous functioning or to set the duration for the requested level of efficiency. Changes in these indicators of efficiency and for different values of failure rate in the period of Autonomous functioning of the CTS are obtained using the developed model and are summarized in Table 2. The parameter value adopted in the calculations is λ_{af} =0,0033 1/h, i.e. the object function (checked) outside of the stationary base 2 times a month (λ_{mob}=0.0047 1/h.) with a duration of τ_{caf}=7 h. Based on the results for accepted initial data to ensure the effectiveness of $P_E \geq 0{,}8$ is possible if the failure rate is (3…7)·10^{-5} 1/h. during the autonomous functioning not more than 4,5 months. The increase in requirements to efficiency to level $P_E \geq 0{,}9$ reduces the autonomy to 3 months for the same reliability or demands to reduce the failure rate to λ_{fai}< (1…2)·10$^{-5)}$ 1/h.

Table 2-indicator of efficiency P_E

Failure rate λ_{fai} [1/h]	The duration of autonomous functioning CTS in route					
	2 months (1440 h.)	2,5 m (1800 h.)	3 m (2160 h.)	3,5 m (2520 h.)	4 m (2880 h.)	4,5 m (3240 h.)
$5\cdot10^{-4}$	0.33083	0.24877	0.18707	0.14067	0.10578	0.079547
$1\cdot10^{-4}$	0.79637	0.75103	0.70827	0.66795	0.62992	0.59406
$7\cdot10^{-5}$	0.85235	0.81801	0.78506	0.75343	0.72308	0.69395
$5\cdot10^{-5}$	0.89200	0.86614	0.84104	0.81666	0.79299	0.77000
$3\cdot10^{-5}$	0.93363	0.91727	0.90120	0.88541	0.86989	0.85465
$1\cdot10^{-5}$	0.97733	0.97158	0.96587	0.96018	0.95453	0.94892
$5\cdot10^{-6}$	0.9886	0.98568	0.98278	0.97988	0.97699	0.97411
$1\cdot10^{-6}$	0.99771	0.99712	0.99653	0.99594	0.99535	0.99477

Note that the interdependence of indicators of autonomy and operational effectiveness n the general model (fig.1) creates limitation on the development of process of operation of CTS's in a subset of $\boldsymbol{E}_B$ of the phase state space. Modeling of considered interrelated processes of CTS's operation is necessary to realize sequentially in two stages (fig.4):

1. Modeling of non-stationary process of operation of CTS out of basic point with subsequent determination of the intensities of the transitions of the integrated model for known or specified values of indicators of autonomy and efficiency;

2. Modeling of the overall process, taking into account the received limitations and estimation of indicators of efficiency of process of CTS's operation.

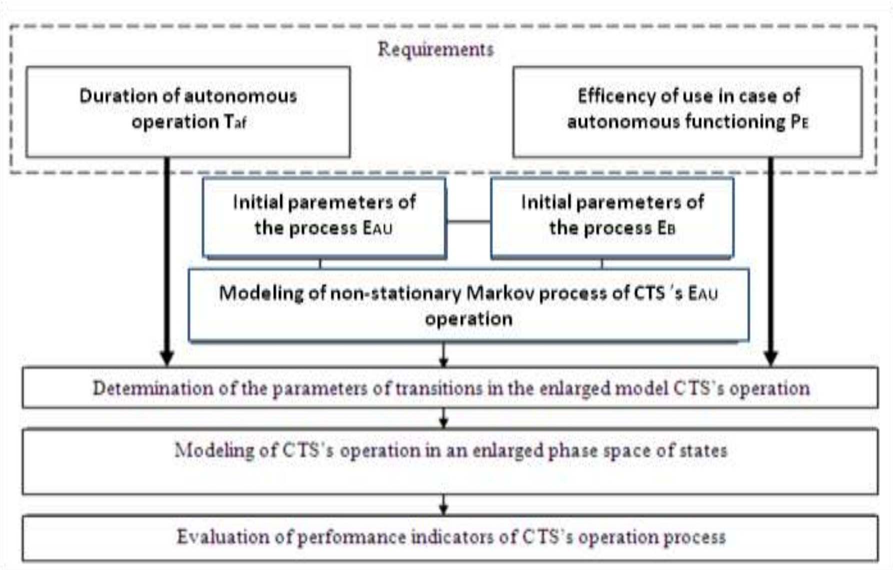

Fig.4 The general scheme of modeling of CTS's operation process

Based on the general scheme of modeling, for the probability received at the first stage P_E modeling of the process in a subset of $\boldsymbol{E}_B$ statuses can be carried out in case of enlargement of the initial graph of model (fig.5). In the presented model e_A status characterizes the presence of CTS outside of stationary point, but in general the physical meaning of the enlarged model is similar to the status of the initial. The features of the process under conditions of Autonomous functioning are expressed through the intensity of transitions $\{e_A, e_2\}$ and $\{e_A, e_8\}$, are equal to $\lambda_{A2} = {(1-P_E)}/{T_{af}}$, $\lambda_{A8} = {P_E}/{T_{af}}$.

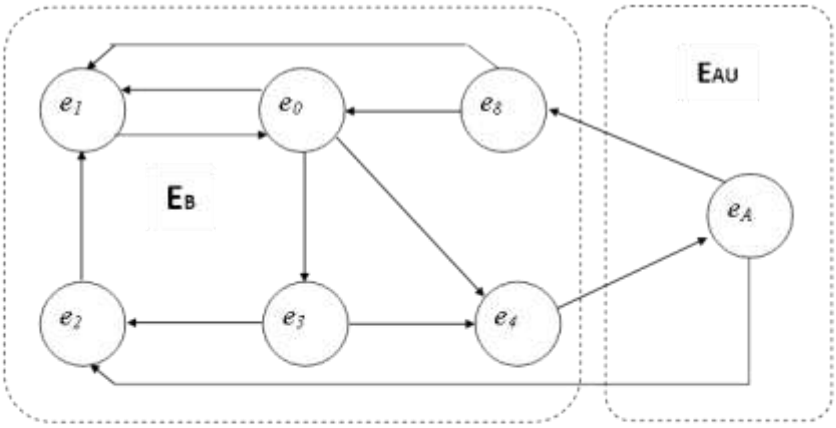

Fig.5 The enlarged state graph of CTS's operation process

Since the process of operation involves the periodic implementation of a set of organizational and technical activities, and the graph of model (fig.5) has an irreducible Markov chain, the analytical dependence for the probabilities of states can be constructed in a stationary mode. This mode is described by a system of algebraic equations, which for the particular process has the following form:

$$\begin{cases} -(\lambda_{01}+\lambda_{03}+\lambda_{04})\cdot P_0+\lambda_{10}\cdot P_1+\lambda_{80}\cdot P_8 = 0; \\ \lambda_{01}\cdot P_0-\lambda_{10}\cdot P_1+\lambda_{21}\cdot P_2+\lambda_{81}\cdot P_8 = 0; \\ -\lambda_{21}\cdot P_2+\lambda_{32}\cdot P_3+\lambda_{A2}\cdot P_A = 0; \\ \lambda_{03}\cdot P_0-(\lambda_{32}+\lambda_{34})\cdot P_3 = 0; \\ \lambda_{04}\cdot P_0+\lambda_{34}\cdot P_3-\lambda_{4A}\cdot P_4 = 0; \\ \lambda_{4A}\cdot P_4-(\lambda_{A2}+\lambda_{A8})\cdot P_A = 0; \\ \lambda_{A8}\cdot P_A-(\lambda_{80}+\lambda_{81})\cdot P_8 = 0; \\ \sum_{i=0}^{6} P_i = 1. \end{cases} \quad (2)$$

In the system of equations (2) nonzero elements of the generating matrix are equal

$$\lambda_{01}=\lambda_{32}=1/\tau_{BM};\ \lambda_{03}=\omega_0;\ \lambda_{04}=\lambda_{34}=\lambda_{mob};$$

$$\lambda_{10}=1/\tau_{BM};\qquad \lambda_{21}=1/\tau_{rfai};\qquad \lambda_{4A}=1/\tau_{BM};$$

$$\lambda_{80}=1/n_M\cdot\tau_{r.};\quad \lambda_{81}=(n_{TO}-1)/n_M\cdot\tau_r.$$

The calculation of the intensity of the transition from the return from the route status (e_8) is made under condition of non-zero, even though negligibly small residence time (if τ_r=1-2 h). The solution of the system (P vector) can be obtained by standard methods of solving algebraic equations [16].

In the phase state space of the enlarged model (fig.5) when CTS is at the stationary base the efficiency of operation is determined by the probability P_0 (taking into account rating on the amount of probabilities of statuses $\boldsymbol{E}_B$), when the object is ready to use without the hidden failure.

Thus, the modeling of the operation of CTS in the form of two interconnected dissimilar phase spaces of states, allows to evaluate the effectiveness of management decisions as the activities maintenance of working capacity of CTC and reliability of its autonomous functioning during the execution of the tasks outside the stationary base over a long period.

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