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Performance Prediction of a Reservoir Under Gas Injection, Using Output Error Model

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Abstract

The pursuit of knowledge on the future performance of reservoirs is a longstanding subject in the downstream oil and gas industry. Current methods employed are time-consuming and sometimes less reliable. In this study, we propose output error model structure for developing a forecasting model of a reservoir under gas injection. A case study which involves a synthetic reservoir model under gas injection has been utilized to prove the hypothesis that system identification using output error model structure is useful for performance prediction of a reservoir under gas injection. The case study was a medium sized reservoir model with 18017 active cells. It consists of five gas injectors and seven producers. Output error model structure was utilized to develop forecasting models for oil production rate, gas-oil ratio, and water-cut. The models were cross validated using dataset which were not used during modelling. The result of the study confirms that system identification by output error model structure has a very good potential for use in reservoir performance prediction under gas injection.

Keywords: Performance prediction, Output Error, Decline Curve Analysis, Reservoir Simulation, System Identification

1. Introduction

One of the key components in reservoir management is prediction of future behavior of the reservoir. Reservoir simulation using first principles and model calibration, decline curve analysis (DCA) and material balance calculations are approaches currently in use for the purpose (Höök et al., 2014, Rwechungura et al., 2011). These prediction methods, however, have limitations and restrictions. In this study forecasting models for oil production rate, gas-oil ratio, and water-cut are developed using one of the many system identification model structures namely, output error model structure.

2. Literature Review

Various methods have been proposed to assist engineers forecast reservoir performance. One of them is decline curve analysis. The method is in use since it was first introduced by (Asps, 1945). Arps discovered that a plot of the rate of oil production versus time for many wells could be extrapolated into the future to provide an estimate of the future rates of production for a well or a field. The assumption was that, the rate at any preceding date is a constant fraction of production rate. Based on this realization three types of decline trends, namely exponential, harmonic and parabolic decline curves, are recognized. The original decline curve analysis method can only be used as long as the mechanical conditions and reservoir drainage remain constant in a well and the well is produced at capacity. In addition, the decline trends are only valid for boundary dominated flow regions. These conditions put limitation on the applicability of DCA because in most cases the reservoir conditions are subject to change. Moreover, the reservoir could, as it is in most cases, be an infinite acting reservoir. However, decline curve analysis is still popular in the industry due to its simplicity.

Fetkovich (1980), have modified Arp's equation to be able to consider changes in the well pressure and well drainage. However, in both Arps and Fetkovich methods the future rate is extrapolated based on some average production rate per unit time usually per year or per day, which must remain constant. Since development strategies may change with time, this is a significant limitation to the methods. A study (Li and Horne, 2005) stated that the decline curve analysis model is heuristic in a sense that there is no guarantee of optimality. Secondly each of the curves, exponential, harmonic, or parabolic declines, have their respective limitations. The exponential DCA has a tendency to underestimate and overestimate the reserves and production rates. This is due to the unconstrained exponential relation in Arp's decline curve which is only useful for boundary dominated flow region. The harmonic exponential curve has a tendency to over predict reservoir performance. There is no consent on how the decline curve is chosen hence it is difficult for engineers to foresee which type of trend to use.

Reservoir modeling and simulation using conservation laws and constitutive equations is popular and efficient but strongly depends on quality of data. This a significant limitation because the reservoir is a complex subsurface system whose properties are, most often, indirectly inferred from other measurements. Hence the reliability of the information used for modeling and simulation is always in question. Moreover, uncertainties in measured or inferred data are significant because fluid or rock samples can only be considered from a limited number of locations within the vicinity of the reservoir. Further limitations and opportunities are briefly explained in (Zubarev, 2009) and (Tavassoli et al., 2004). A challenge which worth significant attention during reservoir modeling and simulation is the model calibration stage which is referred to as history matching. It is a process whereby reservoir model parameters are adjusted until the model reproduces observed pressure and production data. The process is daunting and in fact consumes a tremendous amount of time and effort. In assisted history matching, data assimilation techniques and optimization algorithms are used to update uncertain reservoir parameters which are originally measured or inferred with uncertainty. The procedure is repeated until an acceptable match is obtained between the model output and observed production and pressure data. Once the updating stage is completed, prediction can be undertaken through running the reservoir simulation model for future times.

The stage of assimilating actual production and data with the reservoir model is an iterative procedure which consumes a significant amount of computational time. This also has a tendency to hamper the performance of any planned reservoir management. (Markovinović and Jansen, 2006) have suggested accelerating the iteration process of such data assimilation through the use of improved initial guess from previous time step. Furthermore, computer assisted updating of a reservoir model may result in geologically inconsistent values (Oliver and Chen, 2011). Besides, history matching is generally an ill-posed inverse problem with an infinite solution space. These types of problems are known to have a uniqueness problem. In other words, different combinations of solution parameters will equally satisfy the governing equation.

Common optimization techniques during assisted history matching are expectation minimization, minimization of deviations from goals, minimization of maximum costs, and optimization over soft constraints (Rwechungura et al., 2011). One such method gaining popularity in recent years is the Ensemble Kalman filter (EnKF). It is a Bayesian approach for determining updated *a posterior* distribution using a likelihood function and *a prior* information. Currently, EnKF has a lot of publicity in the research community, but is less used in practice. Currently the most commonly used approaches for production forecasting are those that are based on proxy and genetic algorithm (Zubarev, 2009). A recent study by (Sun et al., 2014) published a significant improvement for history matching method in heated heavy oil reservoir. A simulation study of heavy oil reservoir is conducted whereby high viscosity of the oil limits the viability and also the recovery factor. By the assistance of electromagnetic radiation, heat loss issue from thermal heating was solved but the fluid displacement and production history is hardly understood. Katter Bauer et al. introduced cross-well seismic imaging to help the conventional history matching overcome this problem. Combining with the ensemble Kalman filter, they manage to decrease the uncertainty and significantly enhanced the accuracy of forecasting.

Forecasting of systems with uncertainties is not a unique concept in petroleum engineering; it is there in many fields of other engineering disciplines, social sciences and economics. A number of model structures are in use for different applications. Some of the application areas include process control, image reconstruction, GPS tracking, prediction of future demand and value for a product and so on. The nature of these problems is very much similar but probably the complexity of these problems is less compared to the problem in forecasting reservoir performance. A reason being, reservoirs are subsurface systems whose dimensions and properties can only be inferred from secondary measurements. One of the common dynamic model structure used and found effective in other fields of study is the output error (OE) model structure.

In this paper the application of OE model for forecasting future reservoir performance, in particular forecasting oil production rate, gas-oil ratio, and water cut, is studied.

3. Methodology

3.1 Output Error Model Structure

A discrete input and output system can be described using the following general, linear polynomial model, Equation 1.

$$y(k) = q^{-n}G(q^{-1}, \theta)u(k) + H(q^{-1}, \theta)e(k) \tag{1}$$

where:

$u(k)$ and $y(k)$ are input and output of the system, respectively.

$e(k)$ is a random white noise signal (representing measurement noise and other random disturbances).

$G(q^{-1}, \theta)$ is transfer function of the deterministic part of the model.

$H(q^{-1}, \theta)$ is transfer function of the stochastic part.

q^{-1} is a backward shift operator where $q^{-1}u(k) = u(k-1)$

θ is the set of model parameters to be determined by optimization

$G(q^{-1}, \theta)$ is a deterministic transfer function and it represents relationship between the output and the input signal. $H(q^{-1}, \theta)$ is a stochastic transfer function and represents how a random disturbance affects the output signal. A combination of deterministic and stochastic part in this manner is referred to as linear polynomial model. $G(q^{-1},\theta)$ and $H(q^{-1}, \theta)e(k)$ are rational polynomials as defined in Equations 2 and 3:

$$G(q^{-1}, \theta) = \frac{B(q^{-1},\theta)}{A(q^{-1},\theta)F(q^{-1},\theta)} \tag{2}$$

$$H(q^{-1}, \theta) = \frac{C(q^{-1},\theta)}{A(q^{-1},\theta)D(q^{-1},\theta)} \tag{3}$$

The vector θ is the set of model parameters, and the terms A, B, C, D, and F are polynomials of some degree, with respect to the shift operator q^{-1}. Choice of the degree of polynomial is one aspect of the system identification process. A detailed discussion of polynomial models and other related system identification methods can be found in (Ljung, 1998) and (Nelles, 2001). An important characteristic of linear polynomial models is that when the internal behavior of the order of the system is of no (or minor) importance and the underlying system dynamics is (close to) linear, low order dynamic model can often be identified that approximate (and predict) the input-output behavior of the system very accurately.
The output error model, presented in Equation 4, is a special case of the general linear polynomial model where A (q^{-1}, θ), C (q^{-1}, θ) and D (q^{-1}, θ) are all equal to one. The noise is assumed to disturb the process additively at the output, not somewhere inside the process.

$$y(k) = \frac{B(q^{-1})}{F(q^{-1})} u(k) + e(k) \tag{4}$$

The coefficients $B(q^{-1})$ and $F(q^{-1})$, shown in Equations 5 and 6, are polynomials with respect to the backward shift operator q^{-1} and are defined as:

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \cdots + b_{k_{b-1}} q^{-(k_b - 1)} \tag{5}$$

$$F(q^{-1}) = 1 + f_1 q^{-1} + \cdots + f_{k_{f-1}} q^{-k_f} \tag{6}$$

The stochastic term $e(k)$ is estimated as the difference between the actual data and model prediction, $\hat{y}(k)$, (Equation 7).

$$e(k) = y(k) - \hat{y}(k) \tag{7}$$

Substituting Equation 7 into Equation 4 and rearranging results:

$$\hat{y}(k) = \frac{B(z)}{F(z)} u(k) \tag{8}$$

Equation 8 is a one step ahead predictor of the output error model. Generally, the identification process follows the procedure outlined in Figure 1.

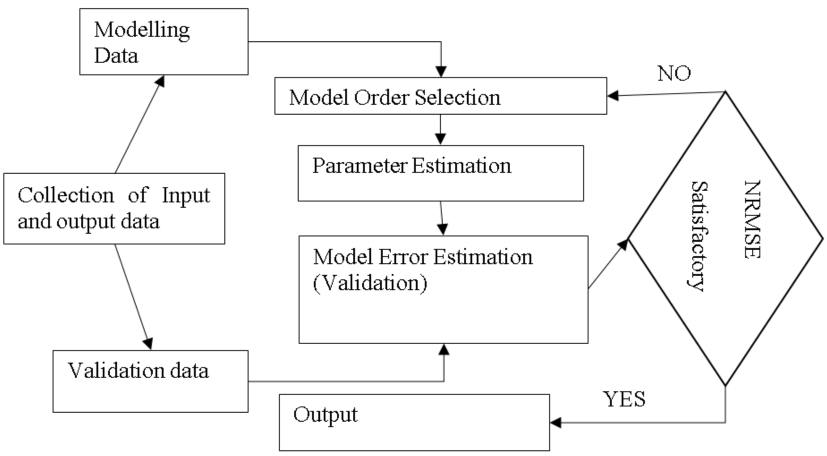

Figure 1: General procedure for system identification

Efficacy of an empirical model, such as the output error model structure, is determined through the use of some misfit measurement technique. The misfit measures the difference between model output and observed value. In this research, the normalized root mean squared error (NRMSE) is used. NRMSE measures the deviation between measured and model output and is useful to compare deviations from different models. Normalization can be done using mean value, difference between maximum and minimum values or the difference between observed and mean. With no specific reason we have considered the later one as shown in Equation 9.

$$NRMSE = \left(\frac{\| y - \hat{y} \|}{\| y - y_{mean} \|} \right) x100\% \qquad (9)$$

3.2 Case Study

An intermediate sized synthetic reservoir model with dimensions 43 x 28 x 28, of which 18017 grid block cells are active, is built using a commercial reservoir simulator. The result from dynamic simulation of the synthetic reservoir is used to identify a forecasting model based on output error model structure. The reservoir model consists of five gas injector and seven producer wells as shown in Figure 2. The initial reservoir properties are shown in Table 1. The model is run for a period of 33 years and Oil production rate (OPR), gas injection rate (GIR), water-cut (WC) and gas-oil ratio (GOR) data are recorded at a time interval of one month. The data are presented in Figure 3.

Table 1: Initial reservoir properties

Equilibrium Reservoir properties		**units**
Datum depth	2001	ft
Datum pressure	1364.8	psia
Water/oil contact	2162.07	ft
Oil density	57.494	lbm/ft3
Water density	62.899	lbm/ft3
Gas density	0.0548	lbm/ft4
Compressibility	3.44E-06	rb/stb
Oil formation volume factor	1.01	
Oil viscosity	0.59	Cp

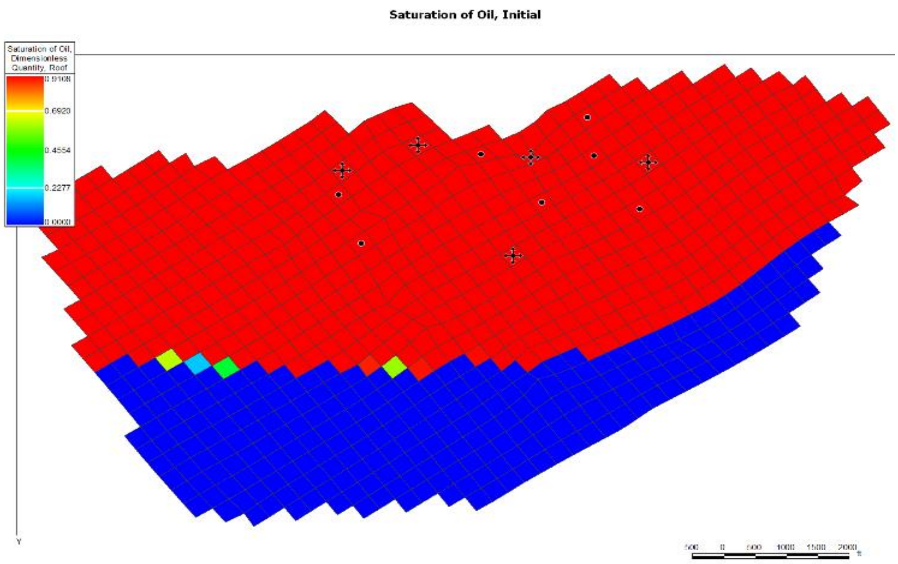

Figure 2: top structure and initial oil saturation profile of the synthetic model (●= producers, ┼ injectors)

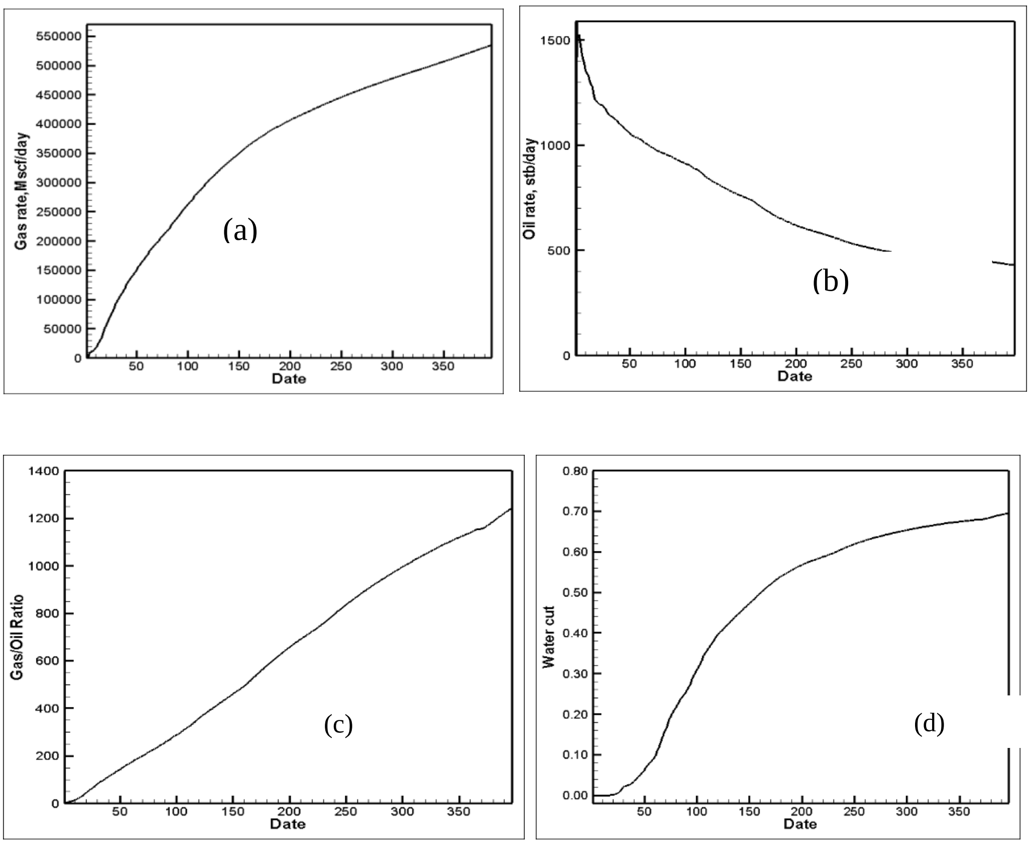

Figure 3: (a) Gas injection rate (b) Oil production rate (c) Gas-oil ratio (d) Water-cut

4. Result and Discussion

The synthetic reservoir model has been simulated for a period of 33 years and the outcome is reported on a monthly basis. As such, a total of 396 discrete data of oil production rate (OPR), gas-oil ratio (GOR), water-cut (WC) and gas injection rate (GIR) are obtained. Each of the data sets are then split into modeling and validation with a 60% and 40% ratio, respectively. Three OE relationships that model OPR, GOR, and WC as a function of GIR are developed and validated.

The OE models developed are presented in Equations 10, 11, and 12.
Oil Production rate [OE (1-1-1)]

$$q(k) = \alpha * GIR(k) + e_1(k) \qquad \text{where} \quad \alpha = \frac{3.047x10^{-6}\, q^{-1}}{1-0.9951q^{-1}} \tag{10}$$

Gas-oil ratio [OE (3-3-1)]

$$GOR(k) = \beta * GIR(k) + e_2(k) \quad \text{where}\ \beta = \frac{0.0002866\,q^{-1} - 0.0005842q^{-2} + 0.0002855q^{-3}}{1-1.976\,q^{-1} - 0.9524\,q^{-2} + 0.02378q^{-3}} \tag{11}$$

Water-cut [OE (3-3-1)]

$$WC(k) = \gamma * GIR(k) + e_3(k) \quad \text{where:} \quad \gamma = \frac{-0.006017q^{-1} - 0.01324q^{-2} - 0.00722q^{-3}}{1-2.967\,q^{-1} + 2.936q^{-2} - 0.9687q^{-3}} \tag{12}$$

Outcomes of the validation process for the three models is presented in a crossplot of estimated versus validation data, Figure 4. The normalized root mean square error exhibited by the output error models is also displayed within the figures.

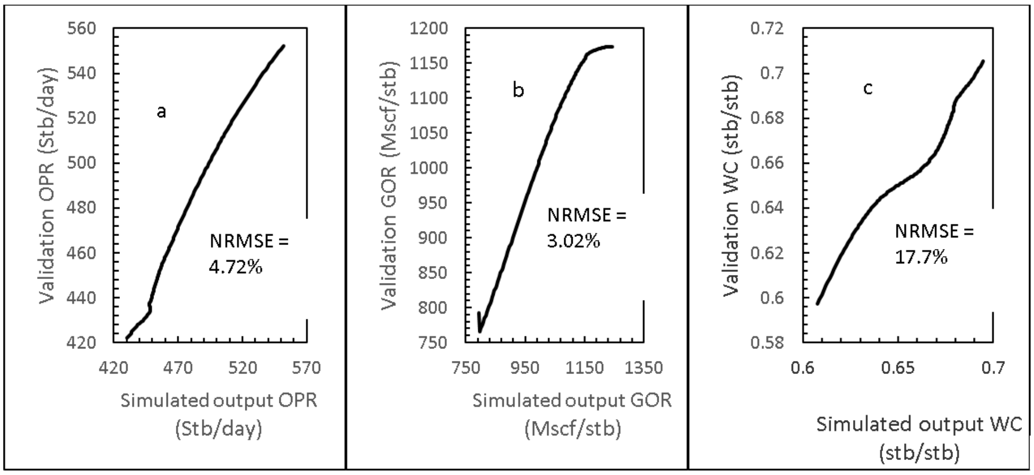

Figure 4: Cross plot of estimated and validation (a)oil production rate data (b) Gas-oil ratio data (c)water-cut data

The figures demonstrate the efficacy of each model in predicting future performance of the reservoir. It can be observed from the angle of the plot and the NRMSE calculated that the first model which represents oil production rate, has shown a very good performance when compared with the validation data set. This implies that the model can be used for mid-term to long-term prediction of OPR. The second case has displayed almost straight 45^0 line at the beginning and for a few more number of time steps. This indicates the high efficacy of the model. However, as time goes on, especially at the end of the validation data set, the model slightly has deviated from the expected. Therefore, the model is only useful for short-term to mid-term prediction of GOR. All the results indicate that OE model can be utilized as an efficient and reliable forecasting tool. The use of OE model, however, is limited to forecasting future performance of time dependent variables, such as those demonstrated in this work. In cases where the main objective is to forecast pressure related terms such as saturation, reservoir simulation should be considered.

5. Conclusion

System identification with output error model structure is proposed for constructing dynamic models of a reservoir. A case study involving gas injection in synthetic reservoir model, is used to demonstrate the usefulness of the proposed approach. Output error model structure is employed to capture the dynamics of fluid flow and predict future performance which is expressed in terms of oil production rate, gas-oil ratio and water-cut. Forecasting models for all the three performance measures are developed. A normalized mean square error (NRMSE) of 4.72% is obtained during validation of the oil production rate forecasting model. Similarly output error models that represent gas-oil ratio and water-cut are also cross validated and have resulted in a NRMSE of 3.07 and 17.7%, respectively. The results indicate that the proposed approach has a very good potential for use in performance prediction and hence improve reservoir management practices. In addition, a significant amount of computational time and effort could be spared, when compared to reservoir simulation methods which uses first principle modeling.

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