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Calculation of Stresses in Superflywheel

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Abstract

In this work it was obtained the exact solution of a problem of rotation of the superflywheel made by a packing of family of concentric cylinders at each other with a constant tightness. It was noted an increase in power capacity of a superflywheel while reducing of relative radius of an internal opening in a flywheel and with increasing the coefficient of Poisson of superflywheel material.

Keywords: flywheel, relative energy capacity, kinetic energy

1 Introduction

The development of modern technology dictates the appearance of many different types of mobile devices in industry and transport. Thus the problem of energy storage is becoming increasingly important. Design and creation of new kinds of materials allows to project energy storage devices, working on entirely new principles. Also it can be attributed to them the so-called kinetic energy storage devices in which energy is stored in the form of rotational kinetic energy [1, 2].
A characteristic feature of such energy storages is a very high speed of the drive rotor (flywheel) rotation, which is mostly limited by its strength qualities. In this regard, the design of dynamic energy storage preference flywheel made by winding or layering tapes (so-called superflywheel). During the spin up of superflywheel separation of the rotor part appear, so the layers should be glued or welded. One option is to manufacture superflywheels by concentric flywheel system at each other with tension and their following gluing or welding. Calculation of the strength of such designs often produced numerically [3-11] and is associated with some difficulties, but sometimes by the introduction of a number of simplifications possible to obtain accurate solutions [12-14].
The paper presents the calculation of stresses in superflywheels produced by a packing of the family of concentric thin-walled cylinders at each other with a constant tightness. The effect of the relative radius of the inner holes in the flywheel and the Poisson ratio of the material on the specific energy superflywheels was investigated.

2 Problem statement and solution

Consider the problem of determining the stress in rotating at a constant angular velocity ω of a uniform disk density ρ, Young's modulus E and Poisson's ratio μ [15]. For a disc with an outer radius r_1 and the central opening of radius r_0 expressions for stresses will take the form

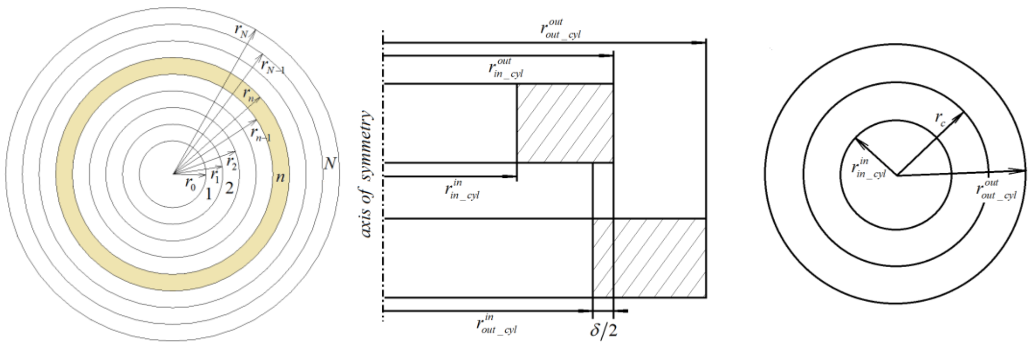
$$\sigma_{rr}\left(r\right)=\frac{3+\mu}{8}\rho\omega^2 r_1^2\left(1+k^2\left(1-r_1^2 r^{-2}\right)-r^2 r_1^{-2}\right),$$

$$\sigma_{\varphi\varphi}(r)=\frac{3+\mu}{8}\rho\omega^2 r_1^2\left(1+k^2\left(1+r_1^2 r^{-2}\right)-\frac{1+3\mu}{3+\mu}r^2 r_1^{-2}\right),$$

where $k=r_0/r_1$.

Belt flywheel with an acceptable degree of accuracy can be represented as a system of concentric cylinders, dressed at each other with some tension (Figure 1). In this case, before the start of loading in flywheel there some stress-strain state, which can be determined by calculating of so-called composite cylinder [16]. In calculating of the composite cylinder (to be specific consisting of two cylinders) is major to establish the basic pressures on their contact surface p_c by a predetermined interference pull δ, which is a difference between the outer diameter of the inner cylinder and the inner diameter of the outer cylinder (Figure 2). Obviously, the reduction of the outer radius of the inner cylinder u_r^{in} and an increase of the inner radius of the outer cylinder u_r^{out} equal to half the interference (tension?)

$$\left|u_r^{in}\right|+\left|u_r^{out}\right|=\delta/2.$$



Figures 1 and 2: Scheme a super flywheel and scheme of concentric packing of two cylinders

Given that the tightness is very small compared to the radius of the contact surface, it can be assumed that the outer radius of the inner cylinder $r_{in\_cyl}^{out}$ and the outer cylinder $r_{out\_cyl}^{in}$ inner radius equal to the radius of the contact surface r_c

$$r_{in\_cyl}^{out}=r_{out\_cyl}^{in}=r_c.$$

The contact pressure p_c is outer to the inner cylinder and the inner to the outer cylinder. Thus, to determine the stress-strain state will require a decision Lame problem of the deformation of the cylinder under the influence of internal and external pressure. Formulas for the definition of radial movements and district and radial tension in the cylinder with an internal and external radius which is under internal and external pressure in case of a flat tension have an appearance

$$u_r = \frac{p_1 r_1^2 \left[(1-\mu) r + (1+\mu) r_2^2 r^{-1}\right]}{E\left(r_2^2 - r_1^2\right)} - \frac{p_2 r_2^2 \left[(1-\mu) r + (1+\mu) r_1^2 r^{-1}\right]}{E\left(r_2^2 - r_1^2\right)},$$

$$\sigma_{rr} = \frac{p_1 r_1^2 \left(1 - r_2^2 r^{-2}\right)}{\left(r_2^2 - r_1^2\right)} - \frac{p_2 r_2^2 \left(1 - r_1^2 r^{-2}\right)}{\left(r_2^2 - r_1^2\right)},$$

$$\sigma_{\varphi\varphi} = \frac{p_1 r_1^2 \left(1 + r_2^2 r^{-2}\right)}{\left(r_2^2 - r_1^2\right)} - \frac{p_2 r_2^2 \left(1 + r_1^2 r^{-2}\right)}{\left(r_2^2 - r_1^2\right)},$$

where E and μ - the Young's modulus and Poisson's coefficient of material. Then radial displacement of a contact surface of the internal cylinder is determined by a formula

$$u_r^{in}\left(r_c\right) = -\frac{p_c r_c^2 \left[(1-\mu) r_c + (1+\mu)\left(r_{n\_cyl}^{in}\right)^2 r_c^{-1}\right]}{E\left(r_c^2 - \left(r_{n\_cyl}^{in}\right)^2\right)},$$

and radial displacement of a contact surface of the internal cylinder is determined by a formula

$$u_r^{out}\left(r_c\right) = \frac{p_c r_c^2 \left[(1-\mu) r_c + (1+\mu)\left(r_{n\_cyl}^{out}\right)^2 r_c^{-1}\right]}{E\left(\left(r_{n\_cyl}^{out}\right)^2 - r_c^2\right)}.$$

If the inner and outer cylinders are made of the same material, the connection between the contact and pressure tightness is obtained in the form of

$$p_c = \frac{\delta E}{2 r_c} \frac{\left(1-\left(r_{in\_cyl}^{in}/r_c\right)^2\right)\left(1-\left(r_c/r_{out\_cyl}^{out}\right)^2\right)}{\left(1+\left(r_{in\_cyl}^{in}/r_c\right)^2\right)\left(1-\left(r_c/r_{out\_cyl}^{out}\right)^2\right)+\left(1-\left(r_{in\_cyl}^{in}/r_c\right)^2\right)\left(1+\left(r_c/r_{out\_cyl}^{out}\right)^2\right)}.$$

It seems that the head of cylinders on each other occur by increasing radius. Then the tension in the inner cylinder (cylinder 1 in Figure 1) can be defined as superposition of making all the cylinders heads, i.e. by actually contact pressure from nozzles into cylinders with an inner radius r_0 and an outer radius r_i cylinder with an inner radius r_i and an outer radius r_{i+1} (i changes from 1 to $n-1$). Each subsequent voltage (except the last) cylinder It can be defined as superposition of decisions all external nozzle to j-th cylinders, i.e., by actually contact pressure from nozzles into cylinders with an inner radius r_0 and an outer radius of a cylinder with an inner radius r_i and an outer radius r_{i+1} (i changes from $j+1$ to $n-1$). By this decision, one needs to add stress on the contact pressure on the inner edge j-th already formed on the cylinder before the cylinder (with inner radius r_0 and an outer radius r_{i-1}). In the outer (largest) cylinder stresses occur only on the contact pressure on the inner edge of the cylinder heads for all other. Then for n-th cylinder in the case of permanent interference voltage will be determined by the formulas.

$$\sigma_{rr}^{n}=\frac{\delta E}{2}\frac{\left(r_{n-1}^2-r_0^2\right)r_{n-1}}{\left(r_{n-1}^2+r_0^2\right)\left(r_n^2-r_{n-1}^2\right)+\left(r_{n-1}^2-r_0^2\right)\left(r_n^2+r_{n-1}^2\right)}\frac{r^2-r_n^2}{r^2}-$$

$$-\frac{\delta E}{2}\sum_{i=n}^{N-1}\frac{\left(r_{i+1}^2-r_i^2\right)r_i}{\left(r_i^2+r_0^2\right)\left(r_{i+1}^2-r_i^2\right)+\left(r_i^2-r_0^2\right)\left(r_{i+1}^2+r_i^2\right)}\frac{r^2-r_0^2}{r^2},$$

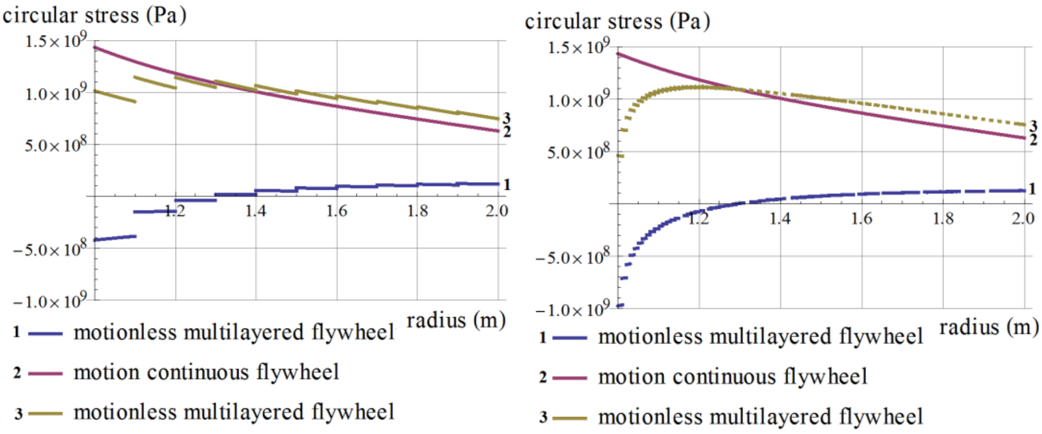
$$\sigma_{\varphi\varphi}^{n}=\frac{\delta E}{2}\frac{\left(r_{n-1}^2-r_0^2\right)r_{n-1}}{\left(r_{n-1}^2+r_0^2\right)\left(r_n^2-r_{n-1}^2\right)+\left(r_{n-1}^2-r_0^2\right)\left(r_n^2+r_{n-1}^2\right)}\frac{r^2+r_n^2}{r^2}-$$

$$-\frac{\delta E}{2}\sum_{i=n}^{N-1}\frac{\left(r_{i+1}^2-r_i^2\right)r_i}{\left(r_i^2+r_0^2\right)\left(r_{i+1}^2-r_i^2\right)+\left(r_i^2-r_0^2\right)\left(r_{i+1}^2+r_i^2\right)}\frac{r^2+r_0^2}{r^2}.$$

3 Results of the solution

Series of calculations was performed for super flywheel with the following geometrical and mechanical properties: Young's modulus $E=200000\,MPa$, outer radius $r_1=2\,m$, temporary resistance $\sigma_{вр}=1100\,MPa$, material density $\rho=7860\;kg/m^3$. Poisson's ratio μ, the inner radius r_0 and the number of layers N varied.

Figures 3-4 shows the diagram of circular strains in a solid flywheel ($N=1$) and super flywheel (both before and after his promotion) for $r_0=1\,m$ and $\mu=0.3$ for each option the corresponding angular speed $N=10$ and $N=100$ respectively for the same case of interference (interference was selected in each case). Table 1 shows the estimated corresponding to each case the number of layers, the angular velocity, the coefficient of increase in specific energy of the flywheel equal to the square of the limiting angular super flywheel rotation speed ω_{rot}^{N} to limit the angular velocity of a solid flywheel (single-layer) ω_{rot}^{1}.



Figures 3 and 4: District stress cylinder superflywheel before and after his promotion to $N = 10$ and $N = 100$ respectively

Table 1. The dependence of the change in energy intensity $\left(\omega_{rot}^{N} / \omega_{rot}^{1}\right)^2$ from the geometrical and mechanical parameters of superflywheel

$r_0\ [m]$	The number N of layers	Poisson's ratio μ	$\left(\omega_{rot}^{N} / \omega_{rot}^{1}\right)^2$
0.1	10	0.3	1.689
1.0	10	0.01	1.224
1.0	10	0.3	1.288
1.0	10	0.49	1.328
1.9	10	0.3	1.028
0.1	100	0.3	1.763
1.0	100	0.01	1.249
1.0	100	0.3	1.313
1.0	100	0.49	1.353
1.9	100	0.3	1.031

4 Analysis of results and conclusion

Analysis of the results shows that the production of superflywheels via a concentric nozzle with interference thin-walled cylinders with each other (with their subsequent gluing or welding) with an appropriate choice of interference can lead to an increase in construction of super flywheel energy intensity. The relative reduction in internal radius super flywheel holes, increasing the number of layers (planted on each other thin-walled cylinder) and an increase in the Poisson ratio of the material, which is made of super flywheel, also leads to increased specific energy capacity. The exact solutions of distribution of stresses in the layers of super flywheel.

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