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On the Fractal Characteristics of Loess Subsidence

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Abstract

Loess soils may be considered as a discrete medium. Under increasing humidity subsidence degradation occurs as a result of the slow disintegration of micro-aggregates into free particles, repackaging and porosity changes. The volumetric porosity of the formed new structure and the volumetric porosity changes can be quantified from the position of the fractal theory. The main aim of this paper is to show that the fractal dimension of the particle size distribution function has important role in the description of the subsidence degradation from the standpoint of a new approach to the evaluation of soil subsidence. For approbation of our theoretical propositions we studied subaerial eluvial-diluvial and eolian-diluvial loess soils of Pleistocene age in the outskirts of Dnipropetrovsk. The value of is correlated with the amount of free particles, it is influenced by the genesis because is less in loess horizons than in paleosoil. Minimum of the fractal dimension of the particle size distribution function is obtained with aggregative method of preparation in which decomposition of aggregates is less expressed.

Keywords: loess, fractal, porosity

Introduction

The influence of the organization of a system of elements on its dynamics is one of the fundamental problems of nature sciences. In soil sciences the search for more advanced models which describe in details the structure of complex multiphase ground-water medium has been receiving considerable attention [1]. Design and construction of modern buildings requires improving the quality of subgrade deformations forecasts, including during their operation. Thus, E. T. R. Dean [2] developed a model of a discrete medium as the interaction of discrete particles in a continuous pressure distribution.

Paul G. Joseph [3] describes the process of discrete soil consolidation as the Poisson process of particle motion. Martin Schoenball [4] describes the results of an experiment on modeling of anisotropy of synthetic material made to analyze the permeability of a fractured environment. Yu Ye et al. [5] bind the state of the soil with characteristics of water circulation.

Yongfu Xu [6] managed to find relations between the strength of the coarse medium and its fractal dimension. Some new data are obtained on the properties of loess, J. A. Muñoz-Castelblanco [7] bound water-retaining properties of loess of France with features of their size distribution and micro-aggregate composition. F. Wei and others [8] examined the relationship between the strength and the capacity for absorption of the Chinese loess and Y.-F. Xu [9] connected the swelling pressure of the fractal structure with characteristics of cohesive soils.

The results of A. Russell [10-11] give us the theoretical basis for describing the behavior of fine-grained soil as a discrete structure with fractal characteristics that can shatter under pressure action. It is shown that the fractal dimension of the particle size distribution D_s and pore size distribution D_p (see below for definitions) have an important role in modeling of soil compression. Widespread loess soils may be considered as a discrete medium [12-14]. Under increasing humidity subsidence degradation occurs as a result of the slow disintegration of micro-aggregates into free particles, repackaging and porosity changes. The volumetric porosity of the formed new structure and the volumetric porosity changes can be quantified from the position of the fractal theory. The main aim of this paper is to show that the fractal dimension of the particle size distribution D_s has important role in the description of the subsidence degradation from the standpoint of a new approach to the evaluation of soil subsidence.

Main results

Loess soils is a multiphase heterogeneous system consisting of particles, liquids and air. The pore structure has a significant influence on the process of flooding, while the shape and distribution of particles of certain sizes affect the process of loess subsidence.

In [9, 10] the distribution function $N_s(L > d_s)$ of the particles sizes is defined as the number of particles of the size L such that $L > d_s$, where d_s runs over the real numbers. The fractal dimension of the particle size distribution function is defined as follows

$$D_s = \lim_{d_s \to 0} -\frac{\ln N_s(L > d_s)}{\ln(d_s)} \tag{1}$$

It follows from (1) that $-D_s \ln(d_s) \approx \ln(N_s(d_s))$ hence $\ln(d_s^{-D_s}) \approx \ln(N_s(d_s))$ and finally

$$N_s(L > d_s) \approx A d_s^{-D_s} \tag{2}$$

where D_s is the fractal dimension of the particle size distribution function, A is a coefficient and the sign means "approximately". The particles forming the ground may have only a finite set of sizes. We denote these sizes

$$d_1, d_2, \ldots, d_{n-1}, d_n \tag{3}$$

ranging in decreasing order from the largest.

We assume that particles of different sizes are distributed in the soil evenly. Hereby the porous structure A_J formed only by the particles of size d_j is similar to the porous structure A_i formed only by the particles of size d_i, for all $1 \le i, j \le n$. These properties are consistent with the nature of self-similar fractal structures and preserved during and after the subsidence. During the subsidence the particles of the largest size d_1 form a porous structure A_1 with the largest pores. The particles of the next size d_2 are in these pores and will form a porous structure A_2 in whose pores there are smaller particles of size d_3 and so on till the smallest particles.

We use the following denotations: V_s - the volume of all considered particles; $V_s(d_j)$ - the volume of particles of size d_j, which form the structure A_j; $V_p(d_j)$ - the volume of pores of the structure A_j; $V(d_j) = V_p(d_j) + V_s(d_j)$ - the volume of the whole structure A_j.

We do not take into account the pressure in the maximal loose dry state. Then, as the structure A_J is similar to A_i for all $1 \le i, j \le n$, we can assume that

$$V_p(d_j) / V_s(d_j) = V_p(d_i) / V_s(d_i)$$

for all $1 \le i, j \le n$, i.e. the void ratio k_p of A_J does not depend on j. Hereby the porosity

$$V_p(d_j) / V(d_j) = k_p / (1 + k_p) = K$$

of A_j also does not depend on j.

Equation (2) allows us to estimate the number $N_s(d_s)$ of particles of size d_s. Since an increment of the function $N_s(L > d_s)$ is only due the particles of sizes from (3) (i. e. in points $d_1, d_2, ..., d_{n-1}, d_n$), according to equation (2) we can estimate the number of particles of size d_j for each d_j from (3)

$$N_s(d_j) \approx Ad_j^{-D_s} - Ad_{j-1}^{-D_s} = A(d_j^{-D_s} - d_{j-1}^{-D_s}) \quad (4)$$

where $d_0 = 0$ and $1 \le j \le n$.

Following Russell [10, 11], we assume that each particle of size d_j has volume d_j^3. Then, according to (4) can estimate the volume $V_s(d_j)$ of all particles of size d_j as

$$V_s(d_j) = d_j^3 N_s(d_j) \approx A(d_j^{3-D_s} - d_j^3 d_{j-1}^{-D_s}).$$

It implies that

$$V_s(d_1) \approx Ad_1^{3-D_s} \text{ and } V_s(d_j) \approx Ad_{j-1}^{3-D_s}((d_j / d_{j-1})^{3-D_s} - (d_j / d_{j-1})^3) \quad (5),$$

where $2 \le j \le n$.

We use the following denotations (*):

1. $\alpha_1 = 1$ and $\alpha_j = d_j / d_{j-1}$, where $2 \le j \le n$

(we should note that $\alpha_j < 1$ because $d_j < d_{j-1}$, where $2 \le j \le n$);

2. $\beta_1 = 1$ and $\beta_j = 1 - (d_j / d_{j-1})^{D_s} = 1 - (\alpha_j)^{D_s}$, where $2 \le j \le n$.

Then it follows from (5) that

$$V_s(d_j) \approx Ad_j^{3-D_s}\beta_j \quad (6)$$

where $1 \le j \le n$.

It allows us to estimate the volume of all considered particles:

$$V_s \approx A(\sum_{j=1}^{n} \beta_j d_j^{3-D_s}) \quad (7)$$

Consider the simplest case in which soil particles are only of two possible sizes d_1 and d_2, where $d_1 > d_2$. According to (6), $V_s(d_1) \approx Ad_1^{3-D_s}$ and $V_s(d_2) \approx Ad_2^{3-D_s}\beta_2$, where $\beta_2 = 1 - \alpha_2^{D_s}$ and $\alpha_2 = d_2 / d_1$. Then,

$$V_p(d_1) \approx k_p A d_1^{3-D_s}, \; V(d_1) \approx (1 + k_p) A d_1^{3-D_s} \text{ and } V(d_2) \approx (1 + k_p) A d_2^{3-D_s}\beta_2. \quad (8).$$

As it was mentioned above, while the process of loess subsidence the structure A_2 fills the pores of the structure A_1 (i.e. the volume $V(d_2)$ fills the volume $V_p(d_1)$), while creating a new pore structure $A_1 + A_2$, whose volume we denote V.

Hereby there may be two fundamentally different situations:

1*. $V(d_2) < V_p(d_1)$, it means that the structure A_2 is placed in the pores of the structure A_1 and the volume of the soil after the loess subsidence (i.e. the volume of the structure $A_1 + A_2$) is equal to the volume of the structure A_1. Thus, $V = V(d_1)$ and by (8) we have

$$V \approx (1 + k_p) A_1 d^{3-D_s} \tag{9}$$

2*. $V(d_2) > V_p(d_1)$, it means that the structure A_2 completely fills the pores structure of the structure A_1, i.e. pores in the structure A_1 are not contained in the structure $A_1 + A_2$ and hence the structure $A_1 + A_2$ consists of the structure A_2 and the particles of the structure A_1. It implies that $V = V(d_2) + V_s(d_1)$, and hence by (6) and (8) we obtain $V \approx Ad_1^{3-D_s} + (1+k_p)Ad_2^{3-D_s}\beta_2 = Ad_1^{3-D_s}(1+(1+k_p)\beta_2)$, i.e.

$$V \approx (1 + (1+k_p)\beta_2\alpha_2^{3-D_s})Ad_1^{3-D_s} \tag{10}$$

Thus, it is necessary to estimate $V(d_2)$ and $V_p(d_1)$ i.e., by (8), $k_p Ad_1^{3-D_s}$ and $(1+k_p)Ad_2^{3-D_s}\beta_2$. After reductions we see that it is necessary to estimate $k_p/(1+k_p) = K_p$ and $\beta_2\alpha_2^{3-D_s}$, where $\alpha_2 = d_2/d_1$ and $\beta_2 = 1-\alpha_2^{D_s}$.

Thus, to determine which of the situations 1 * or 2 * occurs we have to figure out which of the numbers K_p or $\beta_2\alpha_2^{3-D_s}$ is bigger:

1*. If $K_p > \beta_2\alpha_2^{3-D_s}$ then $V \approx (1+k_p)Ad_1^{3-D_s}$ (11)

2*. If $K_p < \beta_2\alpha_2^{3-D_s}$ then $V \approx (1+(1+k_p)\beta_2\alpha_2^{3-D_s})Ad_1^{3-D_s}$ (12).

Discussion

Consider now the following characteristic number model. We assume that the form of the particles is close to the ball then you can approximately estimate $K_p = 3/7 \approx 0.43$. We also assume that $r_2 = d_2/d_1 = 0.1$. Thus, it is sufficient to compare $K_p \approx 0.43$ and $\beta_2\alpha_2^{3-D_s} = 0.1^{3-D_s} - 0.1^3$. Since the fractal dimension satisfies $2 \le D_s \le 3$, it implies that term -0.1^3 is much smaller than 0.1^{3-D_s} and hence this term can be omitted . So it is necessary to compare 0.1^{3-D_s} and 0.43.

Since $0.43 \approx 0.1^{0.37}$ the function $y = 0.1^x$ is decreasing, if $3 - D_s < 0.37$ then $K_p < \beta_2$ and if $3 - D_s > 0.37$ then $K_p > \beta_2$. Therefore, if $D_s > 2.63$ then $K_p < \beta_2$ and the soil formation is according to scheme 1* and the soil volume is calculated by formula (9). If $D_s < 2.63$ then $K_p > \beta_2$ and the soil formation is according to scheme 2* and the soil volume is calculated by formula (10). This example shows:

the smaller D_s, the bigger the influence of the structure formed by the largest size particles on the value V.

Approbation

For approbation of our theoretical propositions we studied subaerial eluvial-diluvial and eolian-diluvial loess soils of Pleistocene age in the outskirts of Dnipropetrovsk. Determination of particle size distribution in three ways sample preparations were performed [15], that allowed us to find out the number and size of particles which are forming aggregates.

The aggregate units are composed mostly of fine clay (dofinovskiy *ePIIIdf*, udayskiy *vdPIIIud*, Dnieper *vdPIIdn* horizons) and large dust particles (vitachevskiy horizon *ePIIIvt*), which are released in the destruction of aggregates during the subsidence [15].

By (8) of [10], the volume of particles larger than d_s obeys $V_s(L > d_s) \approx Bd_s^{3-D_s}$ and hence the mass of particles larger than d_s obeys $M_s(L > d_s) \approx \rho Bd_s^{3-D_s}$, where B is a coefficient and ρ is the density of the stuff of the particles. It easily implies that $D_m = 3 - D_s$ is the fractal dimension of the function $M_s(L > d_s)$.

Particle size mass distribution $M_s(L > d_s)$ curves were constructed in the double logarithmic scale, then under different degrees of disintegration with dispersed, aggregative or semi disperse specimen preparation methods there was found the fractal dimension D_m of $M_s(L > d_s)$. Then we have found $D_s = 3 - D_m$ (Fig. 1).

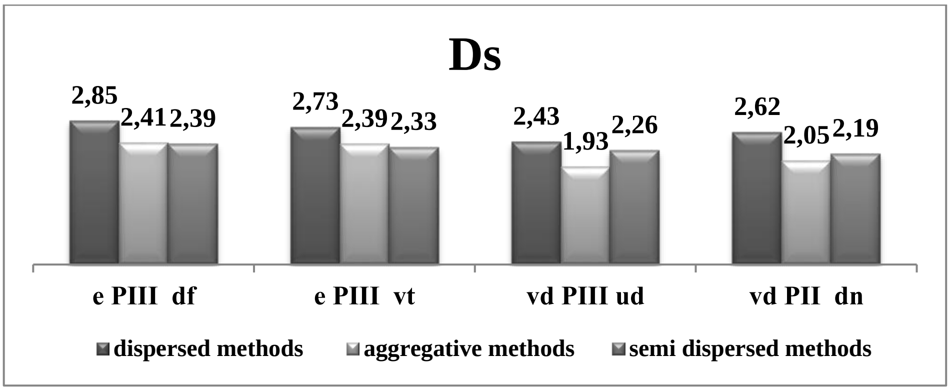

Fig. 1. Values of the fractal dimension D_s of the particle size distribution function for fractions of paleosoil and loess horizons with dispersed, aggregative, and semi dispersed sample preparation methods

Denotations in Fig. 1:

1. e PIII df – values of D_s in dofinovskiy horizon with dispersed, aggregative and dispersed semi preparation methods.
2. $e\ P_{III}\ vt$; $vd\ P_{III}\ ud$; $vd\ P_{II}\ dn$ –the same for Vitachevskiy, Udayskiy and Dnieprovskiy horizons.

The value of D_s is correlated with the amount of free particles, it is influenced by the genesis because D_s is less in loess horizons than in paleosoil. Minimum of D_s is obtained with aggregative method of preparation in which decomposition of aggregates is less expressed. Formation of a new structure in the state after the subsidence by the first variant is possible only in paleosoil horizons under conditions of maximum disintegration of aggregates.

For all other options the environment porosity in the state after subsidence is determined by the size of the largest fractions. This consistent pattern is most evident in subaerial detritus, which is indicated by D_s values 1.93 and 2.05 in the Dnieper and Udayskiy horizons.

Conclusions

The value of D_s is correlated with the amount of free particles, it is influenced by the genesis because D_s is less in loess horizons than in paleosoil. Minimum of D_s is obtained with aggregative method of preparation in which decomposition of aggregates is less expressed. Formation of a new structure in the state after the subsidence by the first variant is possible only in paleosoil horizons under conditions of maximum disintegration of aggregates.

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References

[1] V.I. Osipov, V.N. Sokolov and N.A. Rumjanctheva, *Mikrostruktura Glinistuch Porod*, Nedroa, Moskow, 1989.

[2] E.T.R. Dean, Particle mechanics approach to continuum constitutive modelling, *Geotechnical Research*, **2** (2015), 3 - 34. http://dx.doi.org/10.1680/gr.14.00018

[3] Paul G. Joseph, Viscosity and secondary consolidation in one-dimensional loading, *Geotechnical Research*, **1** (2014), 90 - 98.

http://dx.doi.org/10.1680/gr.14.00008

[4] M. Schoenball, M. Selzer, N. Kühnle, B. Nestler, J. Schmittbuhl and T. Kohl, Flow anisotropy in sheared fractures with self-affine surfaces, European Geothermal Congress, PS2-05б, 2013.

[5] Y. Ye, G. Chiogna, O. Cirpka, P. Grathwoh and M. Rolle, Experimental investigation of compound-specific dilution of solute plumes in saturated porous media: 2-D vs. 3-D flow-through systems, *Journal of Contaminant Hydrology*, **172** (2015), 33 - 47. http://dx.doi.org/10.1016/j.jconhyd.2014.11.002

[6] Y.-F. Xu, Fractals in soil mechanics, Yantu congcheng xuebao, *Chinese Jornal of Geotechnical Engineering*, **37** (2015), 16 - 20.

[7] J.A. Muñoz-Castelblanco, J.M. Pereira, P. Delage and Y.J. Cui, The water retention properties of natural unsaturated loess from Northern France, *Géotechnique*, **62** (2012), 95 - 106. http://dx.doi.org/10.1680/geot.9.p.084

[8] F. Wei, Z.-H. Yao, Z.-H. Chen, L.-H. Su, L.-L. Bao and J.-G. Li, Influence of structural properties on strength and yielding characteristics of unsaturated Q3 loess, *Rock and Soil Mechanics*, **36** (2015).

[9] Y. Xu, X. Feng, H. Zhu and F. Chu, Fractal model for rockfill shear strength based on particle fragmentation, *Granular Matter*, **17** (2015), 753 - 761. http://dx.doi.org/10.1007/s10035-015-0591-z

[10] A.R. Russell, A compression line for soils with evolving particle and pore size distributions due to particle crushing, *Geotechnique Letters*, **1** (2011), 5 - 9. http://dx.doi.org/10.1680/geolett.10.00003

[11] A.R. Russell, How water retention in fractal soils depends on particle and pore sizes, shapes, volumes and surface areas, *Geotechnique*, **64** (2014), 379 - 390. http://dx.doi.org/10.1680/geot.13.p.165

[12] T.P. Mokritskaya, A.V. Tushev and E.V. Nikulchev, The Study of Chaotic States of Dispersive Grounds, *Applied Mathematical Sciences*, **9** (2015), no. 87, 4341- 4350. http://dx.doi.org/10.12988/ams.2015.55370

[13] E.V. Nikulchev and A.P. Kondratov, Method of Generation of Chaos Map in the Centre Manifold, *Advanced Studies in Theoretical Physics*, **9** (2015), 795 - 800. http://dx.doi.org/10.12988/astp.2015.51097

[14] E.V. Nikulchev, Geometric Method of Reconstructing Systems from Experimental Data, *Technical Physics Letters*, **33** (2007), 267 - 269.

http://dx.doi.org/10.1134/S1063785007030248

[15] T.G. Ryashchenko, V.V. Akulova and M.A. Erbaeva, Loessial soils of Priangaria, Transbaikalia, Mongolia and northwestern China, *Quaternary International*, **179** (2008), 90 - 95.
http://dx.doi.org/10.1016/j.quaint.2007.06.035

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