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Approximation of Aerodynamic Coefficients in the Flight Dynamics Simulator

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Abstract

The annex to the development of simulators flight dynamics of flight simulators modular architecture addresses current issues of approximation of table-graphically given functions. The example of implementation is given.

Keywords: asymptotic polynomials, approximation of functions of several variables

Introduction

At present, the development of flight simulators [1] is based on the concept of modularity (the possibility of merging, separation and modification of individual elements without their impact on the system as a whole). Modules are created independently and then combined into blocks to obtain the required results. Exchange large data flows (all operations in real time) carried out on the basis of parallelizing computational processes. In particular, the dynamics of flight simulators in the integration of the equations of motion numerically at each step requires prior update aerodynamic coefficients. To save time you can use the pre-upgrade approximation of aerodynamic coefficients.

The asymptotic polynomials of I. Eterman

Let the function $F(u,v)$ specified in tabular and graphic on the rectangle $\{a \le u \le b, \quad c \le v \le d\}$ (a finite set of graphs showing the dependence of the function $F(u,v)$ of the first variable u in each of the given values of the second variable v).

Let us its polynomial approximation

$$R(u,v) = A_{00} + A_{10}u + A_{01}v + A_{20}u^2 + A_{11}uv + A_{02}v^2 + A_{21}u^2v + A_{12}uv^2 + A_{22}u^2v^2.$$

For this we use asymptotic polynomials, which were used in [2] for the approximation of functions $\varphi(z)$ on the interval $-1 \le z \le 1$

$$Q_n(z) = b_0T_0(z) + \sum_{r=1}^{n} b_rT_r(z),$$

$$b_0 = \frac{1}{n+1}\sum_{j=0}^{n+1}{}'\varphi\left(\eta_j^{(n)}\right), \quad b_r = \frac{2}{n+1}\sum_{j=0}^{n+1}{}'\varphi\left(\eta_j^{(n)}\right)T_r\left(\eta_j^{(n)}\right) \;\; (r = \overline{1,n});$$

symbol $\sum'$ means: $\sum_{j=0}^{n+1}{}'a_j = \frac{a_0}{2} + \sum_{j=1}^{n} a_j + \frac{a_{n+1}}{2}$;

$T_r(z) = \cos r \arccos z$ $(r = 0,1,\ldots)$ – Chebyshev polynomials of the first kind:

$$T_0(z) = 1, \quad T_1(z) = z, \quad T_2(z) = 2z^2 - 1, \ldots;$$

Points $\eta_j^{(n)} = \cos\frac{j\pi}{n+1}\ (j = 0,1,\ldots,n+1)$ at $n \to \infty$ are asymptotically close to the points of alternance $\xi_j^{(n)}$ (the point at which the difference between the continuous function and its the best polynomial approximation of a given degree n turn reach values $\pm E_n(\varphi)$, where $E_n(\varphi) = \min_{P_n}\max_{-1 \le z \le 1}|\varphi(z) - P_n(z)|$ - the best approximation by polynomials P_n).

Rightly:

$$|\varphi(z) - Q_n(z)| \le E_n(\varphi)\left(E_n(\varphi) + 9 + \frac{2}{\pi}\ln(n+1)\right),$$

$$\left|\varphi\left(\eta_j^{(n)}\right) - Q_n\left(\eta_j^{(n)}\right)\right| \le \frac{1}{2^n(n+1)!}\varphi^{(n+1)}(\theta) \quad (-1 < \theta < 1, j = 0,1,\ldots,n+1).$$

When approximating (bounded by a polynomial of degree $n = 2$) function of two variables we use successive approximation asymptotic polynomial for each of the two variables (eventually obtain the required form of the asymptotic polynomial $R(u,v)$ for two variables).

We introduce $x = ru - q, y = sv - h\,; r = \dfrac{2}{b-a}, q = \dfrac{b+a}{b-a}, s = \dfrac{2}{d-c}, h = \dfrac{d+c}{d-c}$; ($-1 \le x \le 1, -1 \le y \le 1$ at $a \le u \le b, \quad c \le v \le d$). After the change of variables we obtain $F(u,v) = f(x,y)$. The problem reduces to the approximation given in tabular and graphical functions $f(x,y)$ (are given $f(x_i, y_i)$, $i = \overline{0,m}$). Instead continue $\eta_j^{(2)}$ to use the notation η_j ($\eta_0 = 1, \eta_1 = \dfrac{1}{2}, \eta_2 = -\dfrac{1}{2}, \ \eta_3 = -1$). In the future, the necessary values $f_{kl} = f(\eta_k, \eta_l)$, $k,l = 0,1,2,3$. Setpoints y_i ($i = 0,1,\ldots,m$), does not coincide with η_l ($l = 0,1,2,3$). If $m = 4$ the points y_i are equidistant, it is sufficient to remove $y_2 = 0$, then $y_0 = -1 = \eta_3$, $y_1 = -\dfrac{1}{2} = \eta_2$, $y_3 = \dfrac{1}{2} = \eta_1$, $y_4 = 1 = \eta_0$. In other cases, values $f_{kl} = f(\eta_k, \eta_l)$ can be determined approximately in the form

$$f(\eta_k, \eta_l) \approx \sum_{i=0}^{m} f(\eta_k, y_i) \prod_{j \ne i} \frac{\eta_l - y_j}{y_i - y_j}$$

(Lagrange interpolation polynomial of degree m). Of course, it is inevitable some will reduce the accuracy of the final result.

For each fixed value $y\,(-1 \le y \le 1)$

$$f(x,y) \approx b_0(y) T_0(x) + \sum_{p=1}^{2} b_p(y) T_p(x),$$

$$b_0(y) = \frac{1}{3}\sum_{k=0}^{3}{}' f(\eta_k, y), \quad b_p(y) = \frac{2}{3}\sum_{k=0}^{3}{}' f(\eta_k, y) T_p(\eta_k) \quad (p = 1,2).$$

In turn, each time $k = 0,1,2,3$ we get an approximation

$$f(\eta_k, y) \approx b_{k0} T_0(y) + \sum_{q=1}^{2} b_{kq} T_q(y), \quad b_{k0} = \frac{1}{3}\sum_{l=0}^{3}{}' f_{kl}, \quad b_{kq} = \frac{2}{3}\sum_{l=0}^{3}{}' f_{kl} T_q(\eta_l) \quad (q = 1,2).$$

As a result, we arrive at an approximation

$$f(x,y) \approx Q(x,y),$$

$$Q(x,y) = \frac{1}{9}\sum_{k=0}^{3}{}'\sum_{l=0}^{3}{}' f_{kl}\left(1 + 2\sum_{p=1}^{2} T_p(\eta_k) T_p(x) + 2\sum_{q=1}^{2} T_q(\eta_l) T_q(y) + 4\sum_{p=1}^{2}\sum_{q=1}^{2} T_p(\eta_k) T_q(\eta_l) T_p(x) T_q(y)\right)$$

Is there $T_1(z) = z$, $T_2(z) = 2z^2 - 1$; $\eta_0 = 1, \eta_1 = \dfrac{1}{2}, \eta_2 = -\dfrac{1}{2}, \ \eta_3 = -1$.

After similar terms shall have:

$$Q(x,y) = \frac{1}{9}\sum_{k=0}^{3}{}'\sum_{l=0}^{3}{}' f_{kl}(1 + +2\eta_k x + 2(2\eta_k^2 - 1)(2x^2 - 1) + 2\eta_l y + 2(2\eta_l^2 - 1)(2y^2 - 1) +$$
$$+ 4\eta_k\eta_l xy + 4\eta_l(2\eta_k^2 - 1)y(2x^2 - 1) + 4\eta_k(2\eta_l^2 - 1)x(2y^2 - 1) + 4(2\eta_k^2 - 1)(2\eta_l^2 - 1)(2x^2 - 1)(2y^2 - 1)).$$

Thus, we obtain an approximation of the function $f(x,y)$ on a rectangle $\{-1 \le x \le 1, -1 \le y \le 1\}$ asymptotic polynomial in two variables:

$$Q(x,y) = a_{00} + a_{10}x + a_{01}y + a_{20}x^2 + a_{11}xy + a_{02}y^2 + a_{21}x^2y + a_{12}xy^2 + a_{22}x^2y^2 .$$

Coefficients $a_{ij} = \sum_{k=0}^{3}\sum_{l=0}^{3} c_{kl}^{ij} f_{kl} \quad (i, j = 0,1,2)$ is the sum of products of elements of the matrix $f = [f_{kl}]$ on the relevant elements of the matrix $c^{ij} = [c_{kl}^{ij}]$. Returning to the original variables u, v, we finally obtain approximating a given function $F(u,v)$ on a rectangle $\{a \le u \le b, \quad c \le v \le d\}$ asymptotic polynomial in two variables

$$R(u,v) = Q(ru - q, sv - h); \quad r = \frac{2}{b-a}, q = \frac{b+a}{b-a}, s = \frac{2}{d-c}, h = \frac{d+c}{d-c}.$$

We emphasize that it is more expedient to leave this polynomial arranged in powers $x = ru - q, y = sv - h$. Representation

$$R(u,v) = A_{00} + A_{10}u + A_{01}v + A_{20}u^2 + A_{11}uv + A_{02}v^2 + A_{21}u^2v + A_{12}uv^2 + A_{22}u^2v^2$$

requires the calculation of new coefficients A_{ij}:

$$A_{00} = a_{00} - qa_{10} - ha_{01} + q^2a_{20} + qha_{11} + h^2a_{02} - q^2ha_{21} - qh^2a_{12} + q^2h^2a_{22},$$
$$A_{10} = ra_{10} - 2rqa_{20} - rha_{11} + 2rqha_{21} + rh^2a_{12} - 2rqh^2a_{22},$$
$$A_{01} = sa_{01} - sqa_{11} - 2sha_{02} + sq^2a_{21} + 2sqha_{12} - 2sq^2ha_{22},$$
$$A_{20} = r^2a_{20} - r^2ha_{21} + r^2h^2a_{22}, \quad A_{11} = rsa_{11} - 2rsqa_{21} - 2rsha_{12} + 4rsqha_{22},$$
$$A_{02} = s^2a_{02} - s^2qa_{12} + s^2q^2a_{22}, \quad A_{21} = r^2sa_{21} - 2r^2sha_{22},$$
$$A_{12} = rs^2a_{12} - 2rs^2qa_{22}, \quad A_{22} = r^2s^2a_{22}$$

(it causes the accumulation of new errors).

Increased degree n approximating polynomial above $n = m - 1$ only complicates the calculation, but does not increase the accuracy of the approximation. Therefore, we made the choice of the approximating polynomial of degree $n = 2$ assumes that $m + 1 \ge 4$.

Example approximation of functions defined on a rectangle

We give an approximation of the aerodynamic coefficients in a simulator of flight dynamics of transport aircraft [1]. We approximate the function $F(u,v)$ defined on a rectangle $\{0 \le u \le 10; 0{,}4 \le v \le 0{,}9\}$ for four graphs (Table1); $F(u,v) = f(x,y)$, where $x = 0{,}2u - 1$, $y = 4v - 2{,}6$.

Table1. The values of the aerodynamic coefficient $F(u,v)$.

-0,030	-0,250	-0,325	-0,375	1	10
-0,237	-0,390	-0,444	-0,461	0,5	7,5
-0,582	-0,555	-0,533	-0,498	-0,5	2,5
-0,640	-0,587	-0,547	-0,500	-1	0
1	0,2	-0,2	-1	x / y	u
0,9	0,7	0,6	0,4	v	

Two middle column corresponding to the values $v = 0{,}6; 0{,}7$, that is, values $y = -0{,}2; 0{,}2$, replace the columns corresponding to the value $y = -0{,}5; 0{,}5$. For this purpose we interpolation Lagrange polynomial of the third degree:

$$f(\eta_k, \eta_l) = \sum_{i=0}^{3} f(\eta_k, y_l) \prod_{j \neq i} \frac{\eta_l - y_j}{y_i - y_j} \quad (k = 0,1,2,3;\ l = 1,2),$$

where $y_0 = -1, y_1 = -0{,}2, y_2 = 0{,}2, y_3 = 1;\quad \eta_0 = 1, \eta_1 = 0{,}5, \eta_2 = -0{,}5, \eta_3 = -1$. Multiplying the elements of the matrix $\mathbf{f} = [f_{kl}]$ on the relevant elements of the matrix $c^{ij} = [c_{kl}^{ij}]$, we obtain the asymptotic coefficients of the polynomial $Q(x,y)$, so that:

$$Q(x,y) = -0{,}503 + 0{,}141x + 0{,}030y + 0{,}074x^2 + 0{,}140xy + 0{,}035y^2 + \\ + 0{,}019x^2y + 0{,}051xy^2 + 0{,}002x^2y^2.$$

Returning to the original variables, we obtain the desired polynomial

$$R(u,v) = Q(0{,}2u - 1,\ 4v - 2{,}6),$$

approximating a given function.

Decomposition $R(u,v)$ in powers of u, v (is less accurate) has the form:

$$R(u,v) = -0{,}455 + 0{,}010u - 0{,}073v + 0{,}002u^2 - 0{,}114uv - 0{,}224v^2 + \\ + 0{,}001u^2v + 0{,}150uv^2 + 0{,}001u^2v^2.$$

We finally obtain

$$F(u,v) \approx Q(0{,}2u - 1,\ 4v - 2{,}6),$$

$$Q(x,y) = -0{,}503 + 0{,}141x + 0{,}030y + 0{,}074x^2 + 0{,}140xy + 0{,}035y^2 + \\ + 0{,}019x^2y + 0{,}051xy^2 + 0{,}002x^2y^2.$$

The proposed approach has been successfully used in the mathematical modeling of kinetic processes of formation of physical and mechanical properties of composite materials and analytical description of the quality criteria [3…5].

Conclusion

We propose a method of approximation of functions of two variables (aerodynamic coefficients) in the annex to the development of the simulator flight dynamics transport aircraft. The effectiveness of the method is confirmed by a concrete example.

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