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Calculation of Strain-Stress State of Flywheel in Potential Field

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Abstract

This paper focuses on strain-stress state of the flywheel in the kinetic energy storage. The flywheel kinetic storage is based on the flywheel-housing scheme in potential field, at the quasistatic increase of rotor velocity. It is particularly noteworthy that a potential field in flywheel-housing system allows to increase specific energy capacity of energy storage.

Keywords: Flywheel-housing system, Specific energy capacity, Kinetic energy

1 Introduction

With the development of the latest technologies in power and transport industry the problem of energy storage becomes crucial. One of the rapidly developing areas in energy storage and energy transformation is the development of new schemes of kinetic energy storage devices [1-3]. Furthermore, to reduce the aerodynamic losses, which are the result of high angular rate of the principal part of kinetic storage – flywheel, a strong body (which is a vacuum chamber) is used.

Modern technologies and the development of new materials allows us to create structural elements for the kinetic energy storage in such a way that there will be a potential field (e.g., electromagnetic) between the flywheel and the housing, which will give an increase of the stress with the decrease of the gap between the housing and the flywheel. This phenomenon would slightly enhance the kinetic energy of rotation of the rotor, and could lead to higher specific energy capacity of mechanical battery [4].

The paper presents certain results on the evaluation of the flywheel accumulated kinetic energy, and studies changes in the stress-strain state during the quasi-static change of the flywheel angular velocity of rotation for different kinds of the potential field physical parameters behavior. In addition, the article provides the technique for determining the initial gap between the flywheel and the housing.

2 Problem statement and solution

Figure 1 shows the simplest schematic model of the flywheel-housing system. The flywheel (that is basically an aperture disk) rotates with an angular velocity ω inside the housing. Between the housing and the flywheel there exist constant potential field which (at a first approximation) generates excessive pressure q in a gap between a flywheel and the housing. Such approximation is admissible if the potential field is generated by the "lamina" on the external surface of the flywheel and the internal surface of the housing. We will neglect tensile and compression stress arising in the "lamina", and we will consider that housing is not deformable. Let r_0 and r_1 be correspondingly the internal and the external radiuses of the flywheel, and let t_0 be the initial value of a gap size between the flywheel and the housing.

Because of the symmetry stresses arising in the flywheel and in the housing don't depend on the angle of rotation φ, and depend only on current radius r.

For a flywheel radial movements u_r, radial and circumferential stresses σ_{rr} and $\sigma_{\varphi\varphi}$ can be determined by formulas:

$$u_r^m = C_1 r + C_2 r^{-1} - \frac{1-\mu_m^2}{8E_m}\rho_m \omega^2 r^3,$$

$$\sigma_{rr}^m = \frac{E_m}{1-\mu_m}C_1 - \frac{E_m}{1+\mu_m}C_2 r^{-2} - \frac{3+\mu_m}{8}\rho_m \omega^2 r^2,$$

$$\sigma_{\varphi\varphi}^m = \frac{E_m}{1-\mu_m}C_1 + \frac{E_m}{1+\mu_m}C_2 r^{-2} - \frac{1+3\mu_m}{8}\rho_m \omega^2 r^2,$$

where ρ_m is the density of the flywheel material, μ_m and E_m are the Poisson's coefficient and the Young's modulus of material, and the constants C_1 and C_2 are defined from boundary conditions. Static boundary conditions for a flywheel on internal and external surfaces are of the form

$$\sigma_{rr}^m(r_0) = 0, \sigma_{rr}^m(r_1) = -q.$$

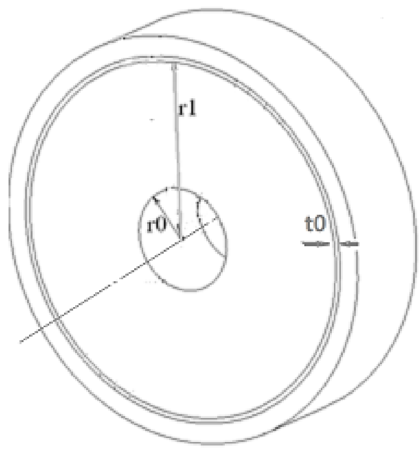

Figure 1. Schematic model of kinetic energy storage

Radial movements in a flywheel can be written as

$$u_r^m = \frac{q r_1^2\left(\left(1-\mu_m\right)r^2+\left(1+\mu_m\right)r_0^2\right)}{E_m r\left(r_0^2-r_1^2\right)}+$$

$$+\frac{\rho_m\omega^2}{8E_m r}\left(\left(-1+\mu_m^2\right)r^4+\left(3+\mu_m\right)\left(\left(1+\mu_m\right)r_1^2 r_0^2+\left(1-\mu_m\right)r^2\left(r_0^2+r_1^2\right)\right)\right),$$

while radial and circumferential stresses can be written as

$$\sigma_{rr}^m = -\frac{q r_1^2\left(r^2-r_0^2\right)}{r^2\left(r_1^2-r_0^2\right)}-\frac{\left(3+\mu_m\right)\left(r^2-r_1^2\right)\rho_m\omega^2}{8r^2},$$

$$\sigma_{\varphi\varphi}^m = -\frac{q r_1^2\left(r^2+r_0^2\right)}{r^2\left(r_1^2-r_0^2\right)}+\frac{\left(-\left(1+3\mu_m\right)r^4+\left(3+\mu_m\right)r_1^2 r_0^2+\left(3+\mu_m\right)r^2\left(r_0^2+r_1^2\right)\right)\rho_m\omega^2}{8r^2}.$$

Maximum circumferential stresses in a flywheel arise on the internal surface $\sigma_{\varphi\varphi}^m\left(r_0\right)$, while maximum (in absolute value) radial stresses arise on the external surface of a flywheel $\left(\sigma_{rr}^m\left(r_1\right)=-q\right)$. In the extremely loaded flywheel these stresses (in absolute value) must be equal:

$$\sigma_{\varphi\varphi}^m\left(r_0\right)=-\frac{q r_1^2\left(r_0^2+r_0^2\right)}{r_0^2\left(r_1^2-r_0^2\right)}+\frac{\left(-\left(1+3\mu_m\right)r_0^4+\left(3+\mu_m\right)r_1^2 r_0^2+\left(3+\mu_m\right)r_0^2\left(r_0^2+r_1^2\right)\right)\rho_m\omega^2}{8r_0^2}=q$$

.

From this ratio we can find limit angular velocity of a flywheel in housing

$$\omega_{\max}^k = 2\sqrt{\frac{q\left(3r_1^2-r_0^2\right)}{\left(r_1^2-r_0^2\right)\left(\left(1-\mu_m\right)r_0^2+\left(3+\mu_m\right)r_1^2\right)\rho_m}} = 2\sqrt{\frac{\sigma_y^m\left(3r_1^2-r_0^2\right)}{\left(r_1^2-r_0^2\right)\left(\left(1-\mu_m\right)r_0^2+\left(3+\mu_m\right)r_1^2\right)\rho_m}}$$

,

since in the limiting case $\left(q=\sigma_y^m\right)$.

Defining the limiting angular velocity of the flywheel without a housing as

$$\omega_{\max} = 2\sqrt{\frac{\sigma_y^m}{\left(\left(1-\mu_m\right)r_0^2+\left(3+\mu_m\right)r_1^2\right)\rho_m}},$$

we can find their ratio:

$$\frac{\omega_{\max}^k}{\omega_{\max}} = \sqrt{\frac{3r_1^2-r_0^2}{r_1^2-r_0^2}} = \sqrt{\frac{3-k^2}{1-k^2}}$$

where $k=r_0/r_1$. Current value of the gap between the flywheel and the housing can be found using formulas:

$$t = t_0 - u_r^m\left(r_1\right) = t_0 + q\frac{r_1^2\left(\left(1-\mu_m\right)r_1^2+\left(1+\mu_m\right)r_0^2\right)}{E_m r_1\left(r_1^2-r_0^2\right)} -$$

$$-\frac{\rho_m\omega^2}{8E_m r_1}\left(\left(-1+\mu_m^2\right)r_1^4+\left(1+\mu_m\right)\left(3+\mu_m\right)r_1^2r_0^2+\left(1-\mu_m\right)\left(3+\mu_m\right)r_1^2\left(r_0^2+r_1^2\right)\right).$$

We can represent this ratio as

$$t=A+qB,$$

where:

$$A=t_0-\frac{\rho_m\omega^2 r_1}{4E_m}\left(\left(3+\mu_m\right)r_0^2+\left(1-\mu_m\right)r_1^2\right)=t_0+A_1,\ B=\frac{r_1^2\left(\left(1-\mu_m\right)r_1^2+\left(1+\mu_m\right)r_0^2\right)}{E_m r_1\left(r_1^2-r_0^2\right)},$$

$$A_1=-\frac{\rho_m\omega^2 r_1}{4E_m}\left(\left(3+\mu_m\right)r_0^2+\left(1-\mu_m\right)r_1^2\right).$$

Depending on physical properties of a potential field $q=\gamma_n/t^n$, where $n=1,2,3,4\ldots$ For some n we can get accurate values of stresses in the flywheel and analyze changes of stresses in the flywheel during quasistatic overspeeding.

If we assume $q=\gamma_1/t$, then to determine t we need to solve the equation:

$$t=A+B\gamma_1/t.$$

The solution is defined as:

$$t=\frac{1}{2}\left(A+\sqrt{A^2+4B\gamma_1}\right).$$

The second root of quadratic equation is negative, so we do not consider it. Then the stresses in the flywheel are determined by formulas:

$$\sigma_{rr}^m=-\frac{2\gamma_1 r_1^2\left(r^2-r_0^2\right)}{\left(A+\sqrt{A^2+4B\gamma_1}\right)r^2\left(r_1^2-r_0^2\right)}-\frac{\left(3+\mu_m\right)\left(r^2-r_1^2\right)\rho_m\omega^2}{8r^2},$$

$$\sigma_{\varphi\varphi}^m=-\frac{2\gamma_1 r_1^2\left(r^2+r_0^2\right)}{\left(A+\sqrt{A^2+4B\gamma_1}\right)r^2\left(r_1^2-r_0^2\right)}+\frac{\left(-\left(1+3\mu_m\right)r^4+\left(3+\mu_m\right)r_1^2r_0^2+\left(3+\mu_m\right)r^2\left(r_0^2+r_1^2\right)\right)\rho_m\omega^2}{8r^2}.$$

If we assume $q=\gamma_2/t^2$, then to determine t we need to solve the equation:

$$t=A+B\gamma_2/t^2.$$

$$t=\frac{1}{3}\left(A+\frac{2^{2/3}A^2+\left(2A^3+27B\gamma_2+3\sqrt{3}\sqrt{4A^3B\gamma_2+27B^2\gamma_2^2}\right)^{2/3}}{2^{1/3}\left(2A^3+27B\gamma_2+3\sqrt{3}\sqrt{4A^3B\gamma_2+27B^2\gamma_2^2}\right)^{1/3}}\right),$$

other roots of the cubic equation will be complex. Then stresses in a flywheel are determined by formulas.

$$\sigma_{rr}^{m}=-\frac{3\gamma_2 r_1^2\left(r^2-r_0^2\right)}{\left(A+\dfrac{2^{2/3}A^2+\left(2A^3+27B\gamma_2+3\sqrt{3}\sqrt{4A^3B\gamma_2+27B^2\gamma_2^2}\right)^{2/3}}{2^{1/3}\left(2A^3+27B\gamma_2+3\sqrt{3}\sqrt{4A^3B\gamma_2+27B^2\gamma_2^2}\right)^{1/3}}\right)r^2\left(r_1^2-r_0^2\right)}-$$

$$-\frac{\left(3+\mu_m\right)\left(r^2-r_1^2\right)\rho_m\omega^2}{8r^2},$$

$$\sigma_{\varphi\varphi}^{m}=-\frac{3\gamma_2 r_1^2\left(r^2+r_0^2\right)}{\left(A+\dfrac{2^{2/3}A^2+\left(2A^3+27B\gamma_2+3\sqrt{3}\sqrt{4A^3B\gamma_2+27B^2\gamma_2^2}\right)^{2/3}}{2^{1/3}\left(2A^3+27B\gamma_2+3\sqrt{3}\sqrt{4A^3B\gamma_2+27B^2\gamma_2^2}\right)^{1/3}}\right)r^2\left(r_1^2-r_0^2\right)}+$$

$$+\frac{\left(-\left(1+3\mu_m\right)r^4+\left(3+\mu_m\right)r_1^2r_0^2+\left(3+\mu_m\right)r^2\left(r_0^2+r_1^2\right)\right)\rho_m\omega^2}{8r^2}.$$

If we assume $q=\gamma_3/t^3$, then to determine t we need to solve the equation:

$$t=A+B\gamma_3/t^3.$$

The solution is defined as:

$$t=\frac{A}{4}+\frac{1}{2}\left(\frac{A^2}{4}+\frac{-2^{2/3}3^{1/3}4B\gamma_3+\left(-9A^2B\gamma_3+\sqrt{3}\sqrt{27A^4B^2\gamma_3^2+256B^3\gamma_3^3}\right)^{2/3}}{2^{1/3}3^{2/3}\left(-9A^2B\gamma_3+\sqrt{3}\sqrt{27A^4B^2\gamma_3^2+256B^3\gamma_3^3}\right)^{1/3}}\right)^{1/2}+$$

$$+\frac{1}{2}\left(\frac{A^2}{4}+\frac{2^{2/3}3^{1/3}4B\gamma_3+\left(-9A^2B\gamma_3-\sqrt{3}\sqrt{27A^4B^2\gamma_3^2+256B^3\gamma_3^3}\right)^{2/3}}{2^{1/3}3^{2/3}\left(-9A^2B\gamma_3+\sqrt{3}\sqrt{27A^4B^2\gamma_3^2+256B^3\gamma_3^3}\right)^{1/3}}+\right.$$

$$\left.+A^3/4\left(\frac{A^2}{4}+\frac{-2^{2/3}3^{1/3}4B\gamma_3+\left(-9A^2B\gamma_3+\sqrt{3}\sqrt{27A^4B^2\gamma_3^2+256B^3\gamma_3^3}\right)^{2/3}}{2^{1/3}3^{2/3}\left(-9A^2B\gamma_3+\sqrt{3}\sqrt{27A^4B^2\gamma_3^2+256B^3\gamma_3^3}\right)^{1/3}}\right)^{1/2}\right)^{1/2}.$$

Because of space limitation, ratios for circumferential and radial stresses will not be considered here. For *n≥4* we can get the current size of a gap *t* only by numerical solution of the equation:

$$t=A+B\gamma_n/t^n.$$

To determine the initial value of the gap t0, we can use the following formulas:

$$t_0=t-A_1-B\sigma_y^m=\left(\gamma_n/\sigma_y^m\right)^{1/n}-A_1-B\sigma_y^m=\left(\gamma_n/\sigma_y^m\right)^{1/n}-\frac{r_1\left(\left(1-\mu_m\right)r_1^2+\left(1+\mu_m\right)r_0^2\right)}{E_m\left(r_1^2-r_0^2\right)}\sigma_y^m+$$

$$+\frac{\rho_m\omega^2r_1}{4E_m}\left(\left(3+\mu_m\right)r_0^2+\left(1-\mu_m\right)r_1^2\right)$$

3 Evaluation results

We calculated values for a steel flywheel with the following geometrical and mechanical characteristics: $r_0 = 0.05$ m, $r_0 = 0.1$ m, $E_m = 2.1*10^{11}$ MPa, $\sigma_y^m = 1.155*10^9$ MPa, $\mu_m = 0.3$, $\rho_m = 7860$ kg/m^3, $\gamma_n = 1$. In this case $\omega_{\max} = 4112.75$ rad/s, $\omega_{\max}^k = 7875.32$ rad/s.

Figure 2 shows the dependency diagram of $\omega_{\max}^k / \omega_{\max}$ and k.

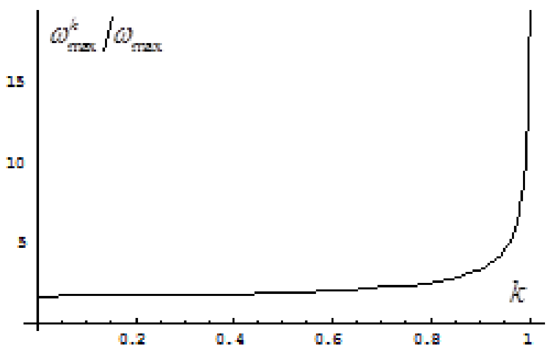

Figure 2. Ratio of $\omega_{max}^k/\omega_{max}$ from *k*

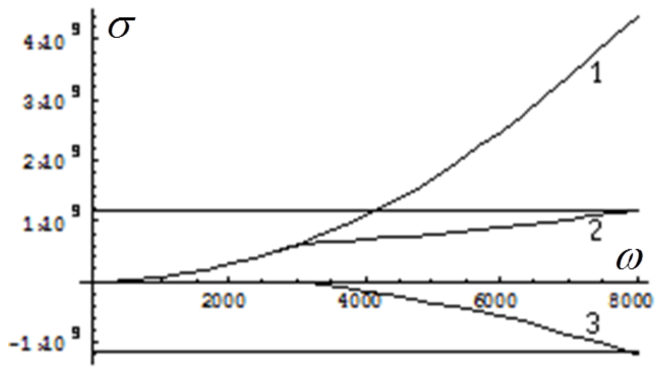

Figure 3. Maximal circumferential stresses in the flywheel (without a housing) – “1”, maximal circumferential stresses in the flywheel (on the external surface) in potential field – “2”, maximal radial stresses in the flywheel (on the external surface) in potential field – “3”, for n equal to 1.

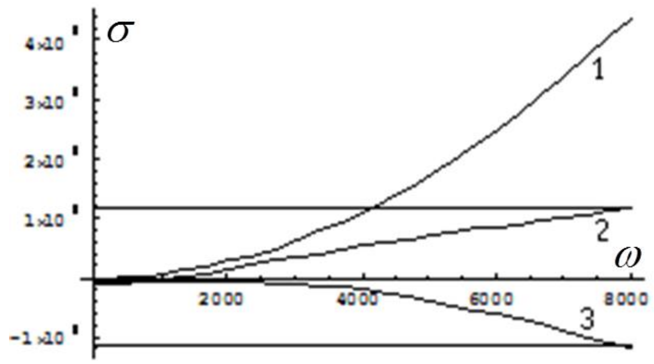

Figure 4. Maximal circumferential stresses in the flywheel (without a housing) – “1”, maximal circumferential stresses in the flywheel (on the external surface) in potential field – “2”, maximal radial stresses in the flywheel (on the external surface) in potential field – “3”, for n equal to 2.

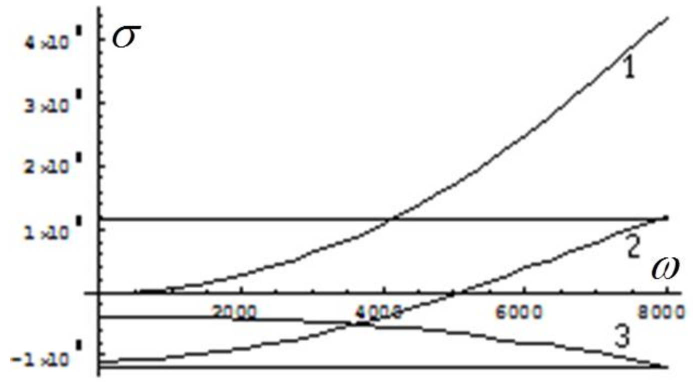

Figure 5. Maximal circumferential stresses in the flywheel (without a housing) – “1”, maximal circumferential stresses in the flywheel (on the external surface) in potential field – “2”, maximal radial stresses in the flywheel (on the external surface) in potential field – “3”, for n equal to 3.

The values of the initial gap between the flywheel and the housing for various n are shown in table 1.

Table 1. The gap between the flywheel and the housing

n	1	2	3	4
t_0 (м)	0.000133346	0.00016277	0.00108645	0.00555778

Figures 3-5 illustrate ratios: maximal circumferential stresses in the flywheel (without a housing) – “1”, maximal circumferential stresses in the flywheel (on the external surface) in potential field – “2”, maximal radial stresses in the flywheel (on the external surface) in potential field – “3”, for n equal to 1, 2 and 3 respectively.

4 Analysis of results and conclusion

This paper presents the estimation of change of angular velocity of the flywheel placed in an inextensible housing in a potential field. We study the changes in the intense deformed condition of the flywheel, representing an apertured disk, in the course of its quasistatic overspeeding.

It is significant that the ratio of angular velocity limits of the flywheel rotation (for a design with a housing and without it) does not depend on mechanical characteristics of a flywheel material, and nature of changes of the potential field physical parameters. The ratio only depends on relative geometrical parameters of a flywheel.

The ultimate strength σ_y^m of a flywheel material and parameter γ_n, which characterizes the intensity of a potential field, determine the size of the initial gap between a flywheel and the housing.

However, it should be noted that the choice of parameter γ_n should be chosen carefully, since too large γ_n may lead to initial stresses in a flywheel overstepping the limiting stresses.

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