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Regularization and Recognition Methods of Automata Models of Technical Systems

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Abstract

In paper are presented results of research in area of regularization of partially set automata models of complex technical systems. As a basic way of description of automata models are used geometrical images of automata mappings, offered and developed by V. A. Tverdohlebov. On the basis of interpolation methods are constructed geometrical images of automata models of filed programmable gate arrays, which are realized algorithms of orthogonal transformations: fast Fourier transformation, FWT etc. Are investigated efficiency of interpolation for automata models of FPGA at different configurations of base points.It is considered recognition of partially set automatons with use of interpolation as way of regularization. In form of theorem are constructed conditions of solve existence of recognition of automatons by their geometrical images and on the basis of theorem is offered and developed method of recognition of automatons by their geometrical images.

Subject Classification: 68R01

Keywords: complex technical system, mathematical model of system, discrete determined automaton, geometrical image of automata mapping, interpolation, recognition of automatons

1 Introduction

In the theory of experiments with automatons initial base is decoding of a contained of "black box". Initial data is the information about variants of a contained of black box. Under the general scheme of carrying out of experiment to a black box (to contents of a black box) are put influences, reaction to these influences is observed and on these supervision is constructed logic conclusions. In control problems the automaton and family of automatons are set and it is required to define, contents of a black box is this allocated automaton or the automaton from the set family. In case of diagnosing it is supposed, that contents of a black box is the element of the set family of automatons and it is required to define what it is the automaton. By E.Mure [1], A.Gill [2], T.Hibbard [3] and other authors solve following problems: criteria of existence of the decision of a problem of recognition of a contained of black box are found; the basic method of construction of experiment on automaton recognition in the set family of automatons, including construction minimum on length of simple unconditional experiment (E.Mure, A.Gill) is developed. Further essentially important expansion of approaches and methods of technical diagnosing was representation of laws of functioning of automatons by geometrical images, i.e. numerical mathematical structures in the form of discrete numerical graphics [4]. If the automatons presented for the decision of control and diagnosing problems in their geometrical images to combine with analytically set curves, then search and construction of control and diagnostic experiments can be carried out on the basis of the decision of systems of the equations for the geometrical curves set analytically.

One of the trends in the development of digital technology at present is widely used of programmable logic resources not only for the implementation of separate blocks, but also for designed devices in general, including the creation of systems on chip (SoC). Field programmable gate arrays (FPGAs) and associated design tools allows to perform hard requirements at the time of development, and in a short time to create digital devices and systems with different levels of complexity and degree of integration. Mathematical models of FPGA are basic information for solving circuit design, analysis and technical diagnostics, optimization of structures and laws of functioning of integrated circuits. FPGAs are technical devices with a fairly complex structure, complex laws of functioning and a large dimension of the signals and the memory (at the moment industry produces FPGA models with more than 1,000 pins). In this regard, currently available FPGA mathematical models, in particular, finite determined automatons are not sufficient, not only for solving the problems, but even for explicit representation of the FPGA. Because of these features to define and analyse the laws of the functioning of the FPGA is proposed to use the apparatus of geometrical images of automatons proposed and developed by V.A.Tverdokhlebov [4, 5], including for solving the problems of technical diagnostics. In [7] are offered and developed general methods of regularization of partially set geometrical images of automatons. In these paper are developed methods of interpolation for partially set laws of functioning of the automatons (which considered as mathematical models of field programmable gate arrays), presented by geometrical images.Detailed description of mathematical apparatus of geometrical images see in [4, 5].

2 The method of recognition of automatons by their geometrical images

In these paper are considered initial discrete determined automatons of Mealy type $A = (S, X, Y, \delta, \lambda, s_0)$, where S - set of states, X - set of input signals, Y - set of output signals, δ - next-state function of a kind $\delta : S \times X \to S$, λ - output function of a kind $\lambda : S \times X \to Y$ and $s_0 \in S$ - initial state. Let automaton A_0 is mathematical model of efficient technical system and the family of automatons $\alpha = \{A_i\}_{i\in I}$ represents set I of failures of technical system. We will assume, that these automatons are set by geometrical image γ_0 and family of geometrical images $\beta = \{\gamma_i\}_{i\in I}$. In the developed method of recognition geometrical images γ_0 and $\beta = \{\gamma_i\}_{i\in I}$ rely located on analytically set geometrical curve L_0 and family of analytically set geometrical curves $L = \{L_i\}_{i\in I}$. Then equality $\{L_0\} \cap \bigcup_{i\in I}\{L_i\} = \varnothing$, where $\{L_0\}$ and $\{L_i\}_{i\in I}$ - sets of points of curves, is defined the decision of a problem of the control with use of simple unconditional experiment.

Definition 2.1 *Let L - a geometrical curve and Δ - a piece on an axis of abscisses, on which the part of curve L (or all curve L) is defined. This part of a curve we will designate $L(\Delta)$.*

Theorem 2.2 *Let initial automaton $A_0 = (S, X, Y, \delta, \lambda, s_0)$ have a geometrical image γ_0 , located on curve L_0 and $\alpha = \{A_i\}_{i\in I}$ - a family of initial automatons, where $A_i = (S_i, X, Y, \delta_i, \delta_i, s_{0_i})$,$s_{0_i} \in S_i$, and $\beta = \{\gamma_i\}_{i\in I}$ - a family of their geometrical images, located accordingly on curves from family $L = \{L_i\}_{i\in I}$. If $L_0(\Delta) \cap L_i(\Delta) = \varnothing$, $i \in I$, is carried out and in a piece Δ of abscissa axis are defined some points of geometrical image γ_0 and geometrical images from family $\beta = \{\gamma_i\}_{i\in I}$, then piece Δ contains the decision of a problem of recognition of the automaton concerning of family α by simple unconditional experiment.*

The proof. *Let $I = 1, 2, ..., k$. We will present system of equalities $\{L_i(\Delta)\} \cap \{L_j(\Delta)\} = \varnothing$,$i, j \in I$, $i \neq j$, as conjunction of separate statements: $(\{L_1(\Delta)\} \cap \{L_2(\Delta)\} = \varnothing)$ & $(\{L_1(\Delta)\} \cap \{L_3(\Delta)\} = \varnothing)$ & $(\{L_1(\Delta)\} \cap \{L_k(\Delta)\} = \varnothing)$ & ... & $(\{L_{k-2}(\Delta)\} \cap \{L_{k-1}(\Delta)\} = \varnothing)$ & $(\{L_{k-1}(\Delta)\} \cap \{L_k(\Delta)\} = \varnothing)$. If on construction of geometrical images as binary relations p is the first coordinate of some points in all geometrical images $\{\gamma_i\}_{i\in I}$. From equality $\{L_1(\Delta)\} \cap \{L_2(\Delta)\} = \varnothing$ follows, that at any choice $p \in X^*$, at which*

$r_1(p) \in \Delta$, $\gamma_1(p) \neq \gamma_2(p)$ and the observable behaviour of automatons A_1 and A_2 is recognized by simple unconditional experiment. Similar conclusions take place for all equalities from conjunction of separate statements, i.e. all pairs of automatons of a kind (A_i, A_j), $i, j \in I$, $i \neq j$, are recognized by output sequences on the general input sequence p , i.e. by simple unconditional experiment.

On the basis of the theorem 1 is offered the method (with 4 stages) of recognition of the automaton, which laws of functioning are set by the geometrical images, located on analytically set curves.

Stage 1. Construction (choice) of family $L = \{L_i\}_{i \in I}$ of geometrical curves and an arrangement on them of geometrical images of laws of functioning of automatons from family of automatons $\alpha = \{A_i\}_{i \in I}$.

Stage 2. For system of equalities $\{L_i(\Delta)\} \cap \{L_j(\Delta)\} = \varnothing$,$i, j \in I$, $i \neq j$, is defined the family of decisions $\{\Delta_{ij}\}$,$i, j \in I$, $i \neq j$.

Stage 3. Piece $\Delta = \bigcap\limits_{i \neq j} \Delta_{ij}$, which on construction satisfies to following conditions, is defined:

1. If $\Delta \neq \varnothing$, then each point of a piece Δ, which is the first coordinate of points of geometrical images of automatons from family of automatons α , defines the decision of a problem of recognition of the automaton in family of automatons by simple unconditional experiment.

2. If $\Delta = \varnothing$ then for chosen concrete geometrical curves L_i, $i \in I$, and the placing of geometrical images of laws of functioning of automatons on these curves, the decision of a problem of recognition of the automatons in family of automatons by simple unconditional experiment does not exist.

Stage 4. According to conditions $\Delta \neq \varnothing$ or $\Delta = \varnothing$ is defined the concrete decision of a problem of recognition of the automaton in family of automatons by simple unconditional experiment or the conclusion becomes, that for family α of automatons, and the chosen family of geometrical curves L and the chosen arrangement of geometrical images on curves the decision of a problem of recognition of the automaton in family of automatons by simple unconditional experiment does not exist.

3 Development of automata models of FPGA based on interpolation

In paper are constructed mathematical models in form of geometrical image of FPGA Spartan II from company Xilinx (FPGA - field programmable gate array) including development of geometrical images based on classical interpolation methods. Are research FPGA models, realized algorithms of digital signal processing - algorithms, based on orthogonal transformations: fast

Fourier transformation (FFT), discrete Hartley transformation, fast Walsh - Hadamard transform etc. In paper are analysed transformations on 256, 512, 1024 and 2048 points. FPGA is programmed to realization of these algorithms with use of system Xilinx ISE ([8]). After development of VHDL programm are used modules ModelSim and Timing Analyzer which are components of Xilinx ISE. With use of these modules is developed model of functioning of FPGA which is programmed to realization of concrete algorithms and carried out the construction of the set of pairs (p, q) , where p - sequence of input signals and q - sequence of output signals. Development of geometric image of the laws of the Mealy type automaton $A = (S, X, Y, \delta, \lambda, s_0)$ for FPGA, that implements the Fast Fourier Transform algorithm, based on the classical methods of interpolation is carried out under the following restrictions. As input "pins" (channels) of FPGA in this example of implementation of the FFT algorithm are used 28 channels, and as output - 40.

Hence, power of the input alphabet of the automaton, being the mathematical model of FPGA realizing described above algorithm is equal 2^{28} (i.e. the input signal x_1 corresponds to application on all 28 input channels of 28 logical "0" (0000000000000000000000000000), and to last input signal x_m where $m = 2^{28}$ corresponds application on 28 input channels of 28 logical "1" (1111111111111111111111111111)), and power of the output alphabet is 2^{40}. In these example it is not considered, that connectors can be in a Z-condition (or high-resistance state). Analysis of laws of FPGA functioning even only on input sequences of a length 10 will demand application of $2^{28^{10}}$ sequences of input signals and supervision of reactions on them. The volume of the watched information will make more than 10^{72} Tb. In practice from set of sequences of input signals of huge dimension are used only finite subset of small dimension defined by functions which realized by FPGA. The obvious behavior of FPGA at application of input sequences of the signals which are not containing in set of input sequences, defined by functional applicability of FPGA, is not known and basically cannot be full and compactly presented. In these paper is carried out development of full mathematical model of FPGA in the form of a geometrical image of the automaton on the basis of use of classical methods of interpolation.

Before construction of a full geometrical image it is necessary to set a partial geometrical image of automaton $A = (S, X, Y, \delta, \lambda, s_0)$. For this purpose the known information about connection of input signals on FPGA and observable reactions on the basis of used coding signals and replacement of sequences of input signals and output signals by their numbers on linear orders ω_1 on set X^* and ω_2 on set Y of output signals is transformed to set of pairs of numbers of a kind $(r_1(p), r_2(y_{j_k}))$, where $r_1(p)$ - number of an input sequence $p = x_{i_1}x_{i_2}...x_{i_k}$ by linear order ω_1 on set X^* , and $r_2(y_{j_k})$ - number of last signal in output sequence $q = y_{j_1}y_{j_2}...y_{j_k} = \lambda(s_0, p)$ by linear order ω_2 on set

Y. The received set of pairs of numbers of a kind $(r_1(p), r_2(y_{j_k}))$ is considered as set of points of partially set geometrical image of the automaton and are used as base points of interpolation. Efficiency of the specified methods of interpolation for regularization of partially set geometrical images of laws of functioning is analyzed at a various arrangement and number of base points of interpolation. Whereas in a considered example of construction of a full geometrical image of the automaton of capacity of set of input signals and set of output signals have huge dimensions, number of points in the image of a geometrical image even on the initial piece including all sequences till length 5 makes $2^28 + 2^{28^2} + 2^{28^3} + 2^{28^4} + 2^{28^5}$. Therefore the full and obvious form of a geometrical image in the analytical form - the equation is resulted.

As a result of use of modules ModelSim and Timing Analyzer the information on connection of input signals on FPGA and observable reactions is taken. The set consisting more than from 1000 pairs of a kind (input sequence, output sequence) is constructed. After coding sets of signals of the automaton and replacement of sequences of input signals and observable output signals by their numbers on orders ω_1 and ω_2 these set is transformed to set H of 1000 pairs of numbers of a kind $(r_1(p), r_2(y_{j_k}))$. The pairs from H are interpreted as a point of partially set geometrical image and considered as base points of interpolation at regularization on the basis of used methods of interpolation. As an example for the following 5 base points of interpolation below are resulted interpolation polynoms of Newton and Lagrange. Base points of interpolation: (2120534, 809736530), (52632095, 30746053), (68772096, 695693412), (87339717, 730868786), (91199536, 354395489). Interpolation Newton's polynom at different distance between points after calculation of the difference quotients $\sigma^k_{y_i} = \frac{\sigma^{k-1}_{y_{i+1}} - \sigma^{k-1}_{y_i}}{x_{i+k} - x_i}$, i = 0, 1, 2, ..., n-1; k = 1, 2, ..., n, n+1 - the number of base point of interpolation, $x_0, x_1, ..., x_n$ - points of interpolation, $y_0, y_1, ..., y_n$ - values of function in these points, looks like:

$y_n(x) = 809736530 + \sigma_0 y_0 \cdot (x - 2120534) + \sigma^2 y_0 \cdot (x - 2120534)(x - 52632095) + \sigma^3 y_0 \cdot (x - 2120534)(x - 52632095)(x - 68772096) + \sigma^4 y_0 \cdot (x - 2120534)(x - 52632095)(x - 68772096)(x - 87339717)$ where values of $\sigma^i y_i$ are presented in table 1. Whereas the description of a polynom even for 5 points of interpolation bulky enough, the description of polynoms for greater number of points in an obvious form is not presented.

For example, writing of an interpolation polynom at use of 100 base points of interpolation borrows more than 10 pages. With use of a computer interpolation polynomials are calculated at use from 10 up to 1000 base points of interpolation. The interpolation polynom of Lagrange for pointed above five base points of interpolation looks like:

$$y_n(x) = 3.16833 \frac{(x-52632095)\cdot(x-68772096)\cdot(x-87339717)\cdot(x-91199536)}{10^{23}} -$$
$$-2.81741 \frac{(x-2120534)\cdot(x-68772096)\cdot(x-87339717)\cdot(x-91199536)}{10^{23}} +$$

Table 1: Table of values of difference relations for specific interpolation points

x_i	y_i	σy_i	$\sigma^2 y_i$	$\sigma^3 y_i$	$\sigma^4 y_i$
2120534	809736530	-15,422023	8,49504E-07	-2,3257E-14	-6,99757E-22
52632095	30746053	41,1987185	-1,13244E-06	-8,55906E-14	
68772096	695693412	1,89444700	-4,43345E-06		
87339717	730868786	-97,536515			
91199536	354395489				

$$+1.55299\frac{(x-2120534)\cdot(x-52632095)\cdot(x-87339717)\cdot(x-91199536)}{10^{21}}-$$
$$-3.44789\frac{(x-2120534)\cdot(x-52632095)\cdot(x-68772096)\cdot(x-91199536)}{10^{21}}+$$
$$+1.19164\frac{(x-2120534)\cdot(x-52632095)\cdot(x-68772096)\cdot(x-87339717)}{10^{21}}.$$

The disadvantage of the Gauss method of interpolation unlike the considered methods of interpolation of Newton and Lagrange is limiting of a possibility of its use only for a case of equidistant points of interpolation. These property superimposes additional limitings on extraction of base points of interpolation. After construction of completely preset geometrical image of the automaton on the basis of classical methods of interpolation was constructed first 2000 points of a geometrical image on the basis of extraction of the information about connection of input signals applied to FPGA (programmed on realization of algorithm of a FFT) and watched reactions with use of modules ModelSim and Timing Analyzer. Obtained data were compared with data received as a result of interpolation of partially preset geometrical images. On the basis of the led computing experiment it is shown, that for the considered algorithm of the FFT realized on FPGA, a method of interpolation of the Gauss yields the best outcomes, than methods of interpolation of Newton and Lagrange. But, as already it has noted been above a method of the Gauss has limiting on a disposition of points of interpolation which at practical use can become to difficulties. For each of the considered algorithms the most effective method of interpolation is certain.

4 Conclusions

In paper on the basis of use of the apparatus of geometrical images of automatons are offered method and the algorithm, developed for recognition of laws of functioning of discrete determined dynamic systems (automatons), set by the automaton mappings, placed on analytically set geometrical curves. Method is proved by corresponding theorem. In paper are stated models and the methods developed for constructed of mathematical model of FPGA in form of geometrical image of automaton. Are investigated efficiency of regularization of partially set geometrical images of automatons by different interpolation methods.

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References

[1] E. Moore, Gedanken-experiments on sequential machines, *Automata Studies*, ed. C. E. Shannon and J. McCarthy [Russian translation], IL, 1956.

[2] A. Gill, *Introduction to the Theory of Finite-State Machines*, McGraw-Hill, 1962.

[3] T. N. Hibbard, Exact upper bounds for lengths of minimal experiments to determine the final state, for two classes of sequential machines, *Cybernetics Collection*, no.2, In.Lit., Moscow, 1966.

[4] V.A. Tverdohlebov, Geometrical images of laws of functioning of automatons, *Publishing house Scientific book*, Saratov, 2008.

[5] V.A.Tverdohlebov, A.S.Epifanov, Representation of automatons by geometrical structures, *Publishing Center "Science"*, Saratov, 2013.

[6] V.Rokhlin, M.Tygert, Fast algorithms for spherical harmonic expansions, *SIAM J. Sci. Computing*, **27** (2006), no. 6, 1903 - 1928. http://dx.doi.org/10.1137/050623073

[7] A.S.Epifanov, Methods of interpolation of automatons models of systems, *Applied Mathematical Sciences*, **8** (2014), no. 81, 4025 - 4030. http://dx.doi.org/10.12988/ams.2014.45324

[8] www.xilinx.com

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