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Mathematical Modeling of the System of Drilling Rig

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Abstract

A nonlinear mathematical model of the system of drilling rig, consisting of the rig, the control cabin, the working platform and platforms with engines has been developed. The mathematical model of the rig with six degrees of freedom is represented by a system of nonlinear ordinary differential equations. The methodology for calculation the basic characteristics of oscillation using the method of polynomial transformations was developed. The formulas for the calculation by the method of polynomial transformations were obtained.

Keywords: mathematical modeling, drilling rig, vibration protection system, nonlinear system with six degrees of freedom

1 Introduction

The drilling rig is a set of equipment for drilling of wells and mine shafts. Drilling rigs are intended for the construction of oil and gas wells, mineral exploration and detection groundwater.
Depending on the type of drilling, length and diameter of the trunk, the drilling rig

consists of the following components: rig, control cabin, the working platform, the platform base, drilling winch, hoist, rotary mechanism, internal combustion engines, generating sets, electric motors, pump compressor, tank, piping, control system rigs, control and measuring system.

In the study of the dynamics of vibration isolation systems we need to use non-linear mathematical models [1-5]. Their study is a more complex problem than the study of linear systems. In the theory of nonlinear dynamical systems apply different methods [6-9]. Among the known methods [10,13,14]: the method of averaging, the small parameter method, the method of Krylov-Bogolyubov, method of polynomial transformation, the method of Van der Pol, the method of harmonic balance, the method of Poincare. In the calculation of non-linear system with six degrees of freedom applied the method of polynomial transformations [15]. As a result of polynomial transformation the original system of nonlinear differential equations is transformed to the autonomous form.

In the averaging method and the method of Van der Pol is considered truncated equation and the approximate solution, that does not includes all the components of the non-linear polynomial of high degree. In the method of harmonic balance approximate solution includes only the main frequency components.

For the method of a small parameter and the method of Poincare the approximate solution is found in the form of the power series with a small parameter, if the series is convergent. Here the accuracy of the solution depends essentially on the number of corrections to the zero approximation. As a result of applying the method of polynomial transformations of the equation is converted to autonomous form. In the method of polynomial transformations determined the main system parameters, which characterize the transient and steady-state modes of oscillation.

2 Mathematical model of the system of drilling rig

Consider the scheme rig (Figure 1), which includes control cabin (m_1-weight) mounted on the working platform (m_2-weight), which is mounted on a platform for engines (m_3-weight), which is mounted on a vibrating surface drilling.

During long operation of a construction in a variety of structural elements the vibration loads can cause the growth of microcracks and micropores [11]. On the growth of micro-cracks and micro-pores affects corrosives of environments [12]. Such influence can be taken into account if to consider the time-dependent amplitude of the external load. The process of destruction can occur for a long time even in case of frequent accidents. Therefore, in the calculation model it is ignored. In the calculation model are not considered the occurrence resonances.

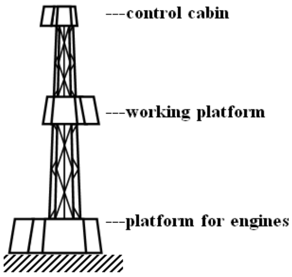

Figure. 1. Scheme the drilling Rig.

Let the horizontal and vertical coordinates the center of mass relative to the equilibrium position: x_1, y_1 - control cabin, x_2, y_2 - working platform, x_3, y_3 - platform for engines. It is assumed that the elastic coupling elements: the control cabin, the working platform, the platform for engines, have the form third-degree polynomial relative to the generalized q-coordinate: $kq + lq^2 + pq^3$; the damping elements have nonlinear relationships cubic characteristic: $b\dot{q} + c\dot{q}^2 + d\dot{q}^3$.
On the surface there are the horizontal and vertical forces in the form of periodic functions: $f_x(t) = A_1 \sin(\omega_1 t) + B_1 \cos(\omega_1 t)$, $f_y(t) = A_2 \sin(\omega_2 t) + B_2 \cos(\omega_2 t)$
For the equations of motion the system apply the Lagrange equations.
$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_{q_i}$, where $L = T_k - T_p$ - Lagrangian, equal to the difference of kinetic and potential energy; Q_{q_i} -generalized force corresponding to the generalized q_i coordinate. Generalized forces are equal:

$$Q_{x_1} = -b_{x1}(\dot{x}_1 - \dot{x}_2) - c_{x1}(\dot{x}_1 - \dot{x}_2)^2 - d_{x1}(\dot{x}_1 - \dot{x}_2)^3 +$$
$$-b_{y1}(\dot{y}_1 - \dot{y}_2) - c_{y1}(\dot{y}_1 - \dot{y}_2)^2 - d_{y1}(\dot{y}_1 - \dot{y}_2)^3,$$
$$Q_{x_2} = -b_{x2}(\dot{x}_2 - \dot{x}_3) - c_{x2}(\dot{x}_2 - \dot{x}_3)^2 - d_{x2}(\dot{x}_2 - \dot{x}_3)^3 - b_{x1}(\dot{x}_2 - \dot{x}_1) +$$
$$-c_{x1}(\dot{x}_2 - \dot{x}_1)^2 - d_{x1}(\dot{x}_2 - \dot{x}_1)^3 - b_{y2}(\dot{y}_2 - \dot{y}_3) - c_{y2}(\dot{y}_2 - \dot{y}_3)^2 +$$
$$-d_{y2}(\dot{y}_2 - \dot{y}_3)^3 - b_{y1}(\dot{y}_2 - \dot{y}_1) - c_{y1}(\dot{y}_2 - \dot{y}_1)^2 - d_{y1}(\dot{y}_2 - \dot{y}_1)^3,$$
$$Q_{x_3} = -b_{x3}(\dot{x}_3 - \dot{f}_x) - c_{x3}(\dot{x}_3 - \dot{f}_x)^2 - d_{x3}(\dot{x}_3 - \dot{f}_x)^3 - b_{x2}(\dot{x}_3 - \dot{x}_2) +$$
$$-c_{x2}(\dot{x}_3 - \dot{x}_2)^2 - d_{x2}(\dot{x}_3 - \dot{x}_2)^3 - b_3(\dot{y}_3 - \dot{f}_y) - c_3(\dot{y}_3 - \dot{f}_y)^2 +$$
$$-d_3(\dot{y}_3 - \dot{f}_y)^3 - b_2(\dot{y}_3 - \dot{y}_2) - c_2(\dot{y}_3 - \dot{y}_2)^2 - d_2(\dot{y}_3 - \dot{y}_2)^3$$

The total kinetic energy of the system:

$$T_k = 0.5m_1\dot{x}_1^2 + 0.5m_1\dot{y}_1^2 + 0.5m_2\dot{x}_2^2 + 0.5m_1\dot{y}_2^2 + 0.5m_3\dot{x}_3^2 + 0.5m_3\dot{y}_3^2$$

The total potential energy of the system:

$$T_p = k_{x1}(x_1 - x_2)^2/2 + l_{x1}(x_1 - x_2)^3/3 + p_{x1}(x_1 - x_2)^4/4 + k_{x2}(x_2 - x_3)^2/2 +$$
$$+l_{x2}(x_2 - x_3)^3/3 + p_{x2}(x_2 - x_3)^4/4 + k_{x3}(x_3 - f_x)^2/2 + l_{x3}(x_3 - f_x)^3/3 + p_{x3}(x_3 - f_x)^4/4$$
$$+k_{y1}(y_1 - y_2)^2/2 + l_{y1}(y_1 - y_2)^3/3 + p_{y1}(y_1 - y_2)^4/4 + k_{y2}(y_2 - y_3)^2/2 + l_{y2}(y_2 - y_3)^3/3$$
$$+p_{y2}(y_2 - y_3)^4/4 + k_{y3}(y_3 - f_y)^2/2 + l_{y3}(y_3 - f_y)^3/3 + p_{y3}(y_3 - f_y)^4/4$$

Substituting the expression for T, Q in the Lagrange equations, we obtain the system of six nonlinear differential equations of the second order:

$$\begin{cases} m_1\ddot{x}_1 + b_{x1}(\dot{x}_1 - \dot{x}_2) + c_{x1}(\dot{x}_1 - \dot{x}_2)^2 + d_{x1}(\dot{x}_1 - \dot{x}_2)^3 + \\ +k_{x1}(x_1 - x_2) + l_{x1}(x_1 - x_2)^2 + p_{x1}(x_1 - x_2)^3 = 0, \\ m_1\ddot{y}_1 + b_{y1}(\dot{y}_1 - \dot{y}_2) + c_{y1}(\dot{y}_1 - \dot{y}_2)^2 + d_{y1}(\dot{y}_1 - \dot{y}_2)^3 + \\ +k_{y1}(y_1 - y_2) + l_{y1}(y_1 - y_2)^2 + p_{y1}(y_1 - y_2)^3 = 0, \\ m_2\ddot{x}_2 + b_{x1}(\dot{x}_2 - \dot{x}_1) + c_{x1}(\dot{x}_2 - \dot{x}_1)^2 + d_{x1}(\dot{x}_2 - \dot{x}_1)^3 + k_{x1}(x_2 - x_1) + \\ -l_{x1}(x_2 - x_1)^2 + p_{x1}(x_2 - x_1)^3 + b_{x2}(\dot{x}_2 - \dot{x}_3) + c_{x2}(\dot{x}_2 - \dot{x}_3)^2 + \\ +d_{x2}(\dot{x}_2 - \dot{x}_3)^3 + k_{x2}(x_2 - x_3) + l_{x2}(x_2 - x_3)^2 + p_{x2}(x_2 - x_3)^3 = 0, \\ m_2\ddot{y}_2 + b_{y1}(\dot{y}_2 - \dot{y}_1) + c_{y1}(\dot{y}_2 - \dot{y}_1)^2 + d_{y1}(\dot{y}_2 - \dot{y}_1)^3 + k_{y1}(y_2 - y_1) + \\ -l_{y1}(y_2 - y_1)^2 + p_{y1}(y_2 - y_1)^3 + b_{y2}(\dot{y}_2 - \dot{y}_3) + c_{y2}(\dot{y}_2 - \dot{y}_3)^2 + \\ +d_{y2}(\dot{y}_2 - \dot{y}_3)^3 + k_{y2}(y_2 - y_3) + l_{y2}(y_2 - y_3)^2 + p_{y2}(y_2 - y_3)^3 = 0, \\ m_3\ddot{x}_3 + b_{x2}(\dot{x}_3 - \dot{x}_2) + c_{x2}(\dot{x}_3 - \dot{x}_2)^2 + d_{x2}(\dot{x}_3 - \dot{x}_2)^3 + k_{x2}(x_3 - x_2) + \\ -l_{x2}(x_3 - x_2)^2 + p_{x2}(x_3 - x_2)^3 + b_{x3}(\dot{x}_3 - \dot{f}_x) + c_{x3}(\dot{x}_3 - \dot{f}_x)^2 + \\ +d_{x3}(\dot{x}_3 - \dot{f}_x)^3 + k_{x3}(x_3 - f_x) + l_{x3}(x_3 - f_x)^2 + p_{x3}(x_3 - f_x)^3 = 0, \\ m_3\ddot{y}_3 + b_{y2}(\dot{y}_3 - \dot{y}_2) + c_{y2}(\dot{y}_3 - \dot{y}_2)^2 + d_{y2}(\dot{y}_3 - \dot{y}_2)^3 + k_{y2}(y_3 - y_2) + \\ -l_{y2}(y_3 - y_2)^2 + p_{y2}(y_3 - y_2)^3 + b_{y3}(\dot{y}_3 - \dot{f}_y) + c_{y3}(\dot{y}_3 - \dot{f}_y)^2 + \\ +d_{y3}(\dot{y}_3 - \dot{f}_y)^3 + k_{y3}(y_3 - f_y) + l_{y3}(y_3 - f_y)^2 + p_{y3}(y_3 - f_y)^3 = 0 \end{cases}$$

Here x_1, x_2, x_3 - the absolute horizontal coordinates the center of mass, and y_1, y_2, y_3 - the absolute vertical coordinates the center of mass relative to the equilibrium position of the system

We enter the relative coordinates relative to vibrations the surface.

$$z_1 = x_1 - f_x,\ z_2 = x_2 - f_x,\ z_3 = x_3 - f_x,\ z_4 = y_1 - f_y,\ z_5 = y_2 - f_y,\ z_6 = y_3 - f_y,$$

We write the system of nonlinear equations in the new variables:

$$
\begin{cases}
m_1\ddot{z}_1 + b_{x1}(\dot{z}_1-\dot{z}_2) + c_{x1}(\dot{z}_1-\dot{z}_2)^2 + d_{x1}(\dot{z}_1-\dot{z}_2)^3 + \\
+k_{x1}(z_1-z_2) + l_{x1}(z_1-z_2)^2 + p_{x1}(z_1-z_2)^3 = -m_1\ddot{f}_x, \\
m_2\ddot{z}_2 + b_{x1}(\dot{z}_2-\dot{z}_1) + c_{x1}(\dot{z}_2-\dot{z}_1)^2 + d_{x1}(\dot{z}_2-\dot{z}_1)^3 + k_{x1}(z_2-z_1) + \\
-l_{x1}(z_2-z_1)^2 + p_{x1}(z_2-z_1)^3 + b_{x2}(\dot{z}_2-\dot{z}_3) + c_{x2}(\dot{z}_2-\dot{z}_3)^2 + \\
+d_{x2}(\dot{z}_2-\dot{z}_3)^3 + k_{x2}(z_2-z_3) + l_{x2}(z_2-z_3)^2 + p_{x2}(z_2-z_3)^3 = -m_2\ddot{f}_x, \\
m_3\ddot{z}_3 + b_{x2}(\dot{z}_3-\dot{z}_2) + c_{x2}(\dot{z}_3-\dot{z}_2)^2 + d_{x2}(\dot{z}_3-\dot{z}_2)^3 + k_{x2}(z_3-z_2) - l_{x2}(z_3-z_2)^2 + \\
+p_{x2}(z_3-z_2)^3 + b_{x3}\dot{z}_3 + c_{x3}\dot{z}_3{}^2 + d_{x3}\dot{z}_3{}^3 + k_{x3}z_3 + l_{x3}z_3{}^2 + p_{x3}z_3{}^3 = -m_3\ddot{f}_x, \\
m_1\ddot{z}_4 + b_{y1}(\dot{z}_4-\dot{z}_5) + c_{y1}(\dot{z}_4-\dot{z}_5)^2 + d_{y1}(\dot{z}_4-\dot{z}_5)^3 + \\
+k_{y1}(z_4-z_5) + l_{y1}(z_4-z_5)^2 + p_{y1}(z_4-z_5)^3 = -m_1\ddot{f}_y, \\
m_2\ddot{z}_5 + b_{y1}(\dot{z}_5-\dot{z}_4) + c_{y1}(\dot{z}_5-\dot{z}_4)^2 + d_{y1}(\dot{z}_5-\dot{z}_4)^3 + k_{y1}(z_5-z_4) + \\
-l_{y1}(z_5-z_4)^2 + p_{y1}(z_5-z_4)^3 + b_{y2}(\dot{z}_5-\dot{z}_6) + c_{y2}(\dot{z}_5-\dot{z}_6)^2 + \\
+d_{y2}(\dot{z}_5-\dot{z}_6)^3 + k_{y2}(z_5-z_6) + l_{y2}(z_5-z_6)^2 + p_{y2}(z_5-z_6)^3 = -m_2\ddot{f}_y, \\
m_3\ddot{z}_6 + b_{y2}(\dot{z}_6-\dot{z}_5) + c_{y2}(\dot{z}_6-\dot{z}_5)^2 + d_{y2}(\dot{z}_6-\dot{z}_5)^3 + k_{y2}(z_6-z_5) - l_{y2}(z_6-z_5)^2 + \\
+p_{y2}(z_6-z_5)^3 + b_{y3}\dot{z}_6 + c_{y3}\dot{z}_6{}^2 + d_{y3}\dot{z}_6{}^3 + k_{y3}z_6 + l_{y3}z_6{}^2 + p_{y3}z_6{}^3 = -m_3\ddot{f}_y
\end{cases}
$$

For write the periodic functions we introduce the complex variables:
$z_7 = \exp(i\omega_1 t), z_8 = \exp(i\omega_2 t)$
In the new variables the periodic functions of the right parts of system can be written:

$$m\ddot{f}_x(t) = -mA_1\omega_1^2\sin(\omega_1 t) - mB_1\omega_1^2\cos(\omega_1 t) = imA_1\omega_1^2(z_7-\overline{z}_7)/2 - mB_1\omega_1^2(z_7+\overline{z}_7)/2$$
$$m\ddot{f}_y(t) = -mA_2\omega_2^2\sin(\omega_2 t) - mB_2\omega_2^2\cos(\omega_2 t) = imA_2\omega_2^2(z_8-\overline{z}_8)/2 - mB_2\omega_2^2(z_8+\overline{z}_8)/2$$

3 The methodology for the calculation of the mathematical model of the system

In contrast to the methods of calculation which proposed in [16], following method of polynomial transformations is applied [15]. We write the system of equations in matrix form: $\dot{Z}_1 = NZ_1 + R_1$, where R_1 - nonlinear vector system.
$Z_1 = [z_1, z_2, z_3, z_4, z_5, z_6, \dot{z}_1, \dot{z}_2, \dot{z}_3, \dot{z}_4, \dot{z}_5, \dot{z}_6, z_7, \overline{z}_7, z_8, \overline{z}_8]^T \equiv [z_1, z_2, ..., z_{16}]^T$ - column.
As a result, a linear change of variables: $Z_2 = LZ_1$, obtain a system with a linear diagonal matrix of the form: $\dot{Z}_2 = \Lambda Z_2 + R_2$.
According to the method [15] to make substitution of variables: $z_2^j = z_3^j + \sum_{|\nu|=2}^{4} p_\nu^j Z_3^\nu$, ($j$=1…12)
As a result polynomial transformation we obtain autonomous differential system up to terms of the fourth order:

$$\begin{cases} \dot{z}_1 = q_1 z_1, \;\; z_7 = \overline{z}_1, \;\; \dot{z}_2 = q_2 z_2, \;\; z_8 = \overline{z}_2, \\ \dot{z}_3 = q_3 z_3, \;\; z_9 = \overline{z}_3, \;\; \dot{z}_4 = q_4 z_4, \;\; z_{10} = \overline{z}_4, \\ \dot{z}_5 = q_5 z_5, \;\; z_{11} = \overline{z}_5, \;\; \dot{z}_6 = q_6 z_6, \;\; z_{12} = \overline{z}_6. \end{cases}$$

The solution of an autonomous system of equations can be written as:

$$z_1 = \rho_{01}\exp(q_1 t), z_2 = \rho_{02}\exp(q_2 t), z_3 = \rho_{03}\exp(q_3 t),$$
$$z_4 = \rho_{01}\exp(q_4 t), z_5 = \rho_{01}\exp(q_5 t), z_6 = \rho_{01}\exp(q_6 t)$$

The transient and steady oscillation modes rig with the following parameters is calculated:

$$m_1 = 2.24, m_2 = 6.36, m_3 = 18.12, b_{x1} = 0.20, c_{x1} = 0.01, d_{x1} = 0.01, b_{x2} = 0.60,$$
$$c_{x2} = 0.02, d_{x2} = 0.02, b_{x3} = 1.72, c_{x3} = 0.02, d_{x3} = 0.08, k_{x1} = 1.20, l_{x1} = 0.03,$$
$$p_{x1} = 0.01, k_{x2} = 3.10, l_{x2} = 0.10, p_{x2} = 0.05, k_{x3} = 9.10, l_{x3} = 0.35, p_{x3} = 0.15,$$
$$b_{y1} = 0.22, c_{y1} = 0.02, d_{y1} = 0.01, k_{y1} = 1.22, l_{y1} = 0.04, p_{y1} = 0.02,$$
$$b_{y2} = 0.62, c_{y2} = 0.01, d_{y2} = 0.03, k_{y2} = 3.12, l_{y2} = 0.12, p_{y2} = 0.06,$$
$$b_{y3} = 1.74, c_{y3} = 0.03, d_{y3} = 0.09, k_{y3} = 9.11, l_{y3} = 0.38, p_{y3} = 0.18$$

On the surface there are the horizontal and vertical forces:

$$f_x(t) = 0.4\sin(2t) + 0.1\cos(2t), \; f_y(t) = 0.5\sin(2t) + 0.1\cos(2t).$$

According to the method of polynomial transformations [15] was found the coefficients of the transformed for autonomous system:

$$q_1 = -0.085 + 0.012i, q_2 = -0.156 + 0.014i,$$
$$q_3 = -0.045 + 0.001i, q_4 = -0.082 + 0.010i,$$
$$q_5 = -0.153 + 0.011i, q_6 = -0.043 + 0.001i$$

The method of polynomial transformations obtained polyharmonic steady oscillation mode of the system:

$$x_1 = 0.253812\sin(t\omega_1 + 0.081) - 0.000003\sin(2t\omega_1 + 0.884) - 0.0037,$$
$$x_2 = 0.283867\sin(t\omega_1 + 0.234) - 0.000026\sin(2t\omega_1 + 0.898) - 0.0038,$$
$$x_3 = 0.314533\sin(t\omega_1 + 0.362) - 0.000186\sin(2t\omega_1 + 0.708) - 0.0028,$$
$$y_1 = 0.335612\sin(t\omega_2 + 0.074) - 0.000005\sin(2t\omega_2 + 0.785) - 0.0057,$$
$$y_2 = 0.370212\sin(t\omega_2 + 0.211) - 0.000045\sin(2t\omega_2 + 0.968) - 0.0056,$$
$$y_3 = 0.412611\sin(t\omega_2 + 0.392) - 0.000291\sin(2t\omega_2 + 0.678) - 0.0041$$

Some numerical results are presented in Fig. 2–5. Figure 2 and 3 show graphs of amplitudes of oscillations in time. Fluctuations occur with a frequency of the external force. Figure 4 shows the absolute horizontal and vertical displacement in establishing oscillation. Forced oscillations in the beginning of the establishment are quasi-periodic. Figure 5 presents absolute displacement in the plane XY.

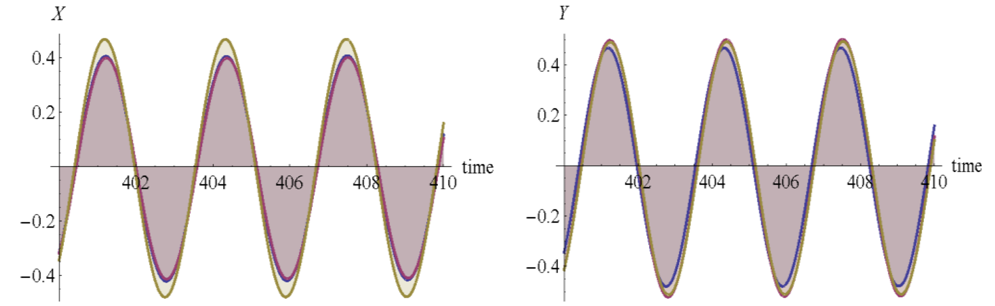

Figure.2. The relative horizontal and vertical displacement of steady-state oscillation the system. Legend: — M1 — M2 — M3

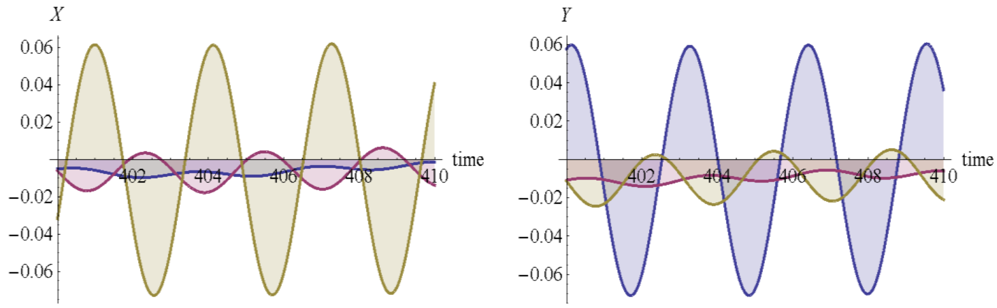

Figure.3. The absolute horizontal and vertical displacement of steady-state oscillation the system. Legend: — M1 — M2 — M3

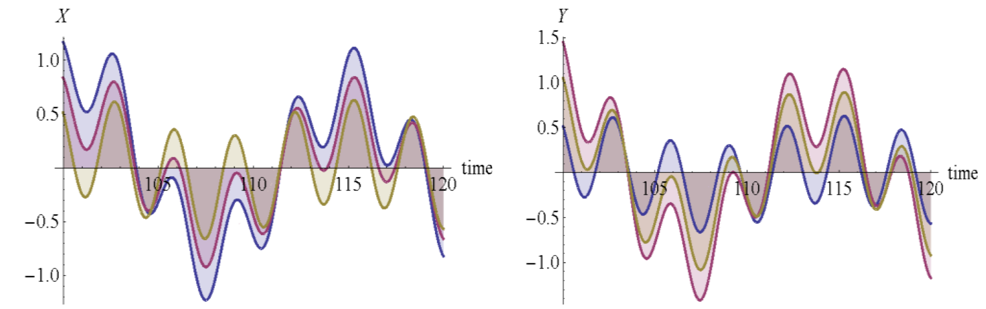

Figure.4. The absolute horizontal and vertical displacement when establishing oscillation the system. Legend: — M1 — M2 — M3

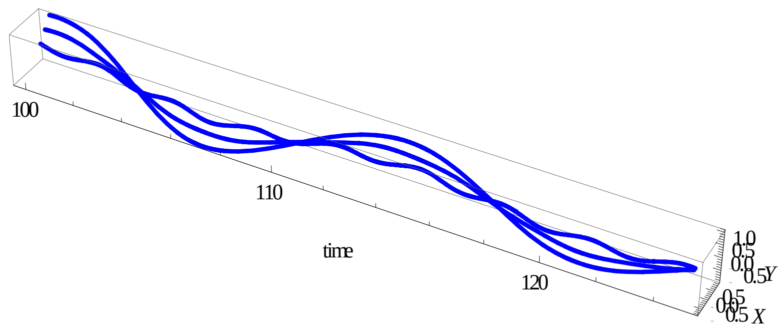

Figure.5. The oscillations in the XY plane.

The comparison of the results was performed with the numerical results that are obtained by solving the Cauchy problem for systems of differential equations using numerical method of Dormand-Prince [17], which considers structural features of the equations [18], and can be successfully used for solving differential equations in partial derivatives [19-21]. The difference between the maximum deviations is less than 2%.

4 Conclusion

The mathematical model of the system of drilling rig in the form system of nonlinear differential equations allows us to find the frequency responses of the system. Application the method of polynomial transformations enables us to find the steady oscillation mode with mathematical method less laborious than the methods of solution for nonlinear differential equations.

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