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The Mathematical Description of the Thermal Combined Laser and Plasma Impact on a Material

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Abstract

The mathematical description of process of the thermal combined laser and plasma impact on a material taking into account ring influence of plasma is developed. Results of modeling of process of heating of Steel 45 by laser, ring plasma and combined laser and plasma energy streams.

Keywords: laser, plasma, heat treatment, temperature distribution

1 Introduction

One of the most effective methods of treatment of metal alloys is the use of the plasma flow and the laser radiation. Selection of the optimal technological modes of such processing is carried out on the basis of solving the appropriate problems theory of heat conduction. However, many known calculation formulas [1, 2, 5] are obtained for the separate use of these sources of concentrated energy. Hybrid laser-plasma processing of materials treated as a model with the joint point and one-dimensional ring sources of energy.
Above shows, necessity of carrying out theoretical studying the process of heating with a laser beam at the center of the annular plasma source (Fig. 1).

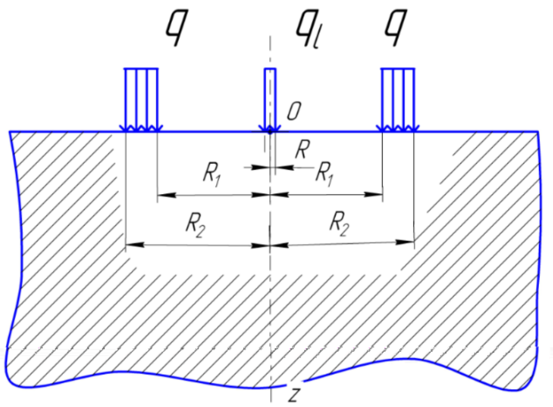

Figure 1: Model of a semibounded body heated by hybrid heat source: q, q_l - the specific heat flux from the plasma and the laser source, respectively; R - radius of the laser radiation; R1, R2 - the inner and outer radii of the plasma source.

In the case of a stationary combined source the task is reduced to the solution of two separate problems with the application of the theory of superposition: body heating by circular energy source - the laser beam and body heating by annular source - plasma flow [3]. Processing of the material is conducted in the field of laser radiation exposure, temperature calculation possible to conduct only for the joint axis laser-plasma source.

2 The solution

We will consider the pattern that occupies the halfspace $z \geq 0$.
For the first problem, there is a ready solution of the form [1]:

$$\theta_0(0,z,\tau_l) = \frac{2Aq_{l0}\sqrt{\tau_l}}{b}\left(ierfc\left(\frac{z^2}{2\sqrt{a\tau_l}}\right) - ierfc\left(\frac{\sqrt{R^2+z^2}}{2\sqrt{a\tau_l}}\right)\right),$$

wherein: A - the coefficient of absorption of the laser radiation, q_{l0} – specific heat flux from the laser source, τ_l – exposure time of the laser source, b – coefficient of thermal activity, z – coordinate along the axis, a – the thermal diffusivity, R - radius of the laser beam on the surface of a semi-infinite body.

In the second problem, we assume semi-bounded body with a uniform temperature distribution over the entire volume, equal T_0. At the initial time $\tau > 0$ on the bounding surface ($z = 0$) in the annular plasma flow $R_1 > r > R_2$ starts acting time-varying flat heat source specific power $q(\tau)$. In the other regions on the surface the body ($0 \le r < R_1$, $R_2 < r \le \infty$), the temperature gradient along the normal z to the boundary of the body is missing, i.e. there is ideal thermal insulation. Take the origin of coordinates at the center of the circle to match the first task, ($0 \le r < R_1$) on the surface $z = 0$. Changing the temperature occurs in the two directions r and z.

Mathematically formulated the problem reduces to solving the following system of differential equations [4]: for the region $0 \le r < R_1$, $z \ge 0$, $\tau > 0$ after the change of variable $\theta_1(r,z,\tau) = T_1(r,z,\tau) - T_0$

$$\frac{\partial^2 \theta_1(r,z,\tau)}{\partial r^2} + \frac{1}{r}\frac{\partial \theta_1(r,z,\tau)}{\partial r} + \frac{\partial^2 \theta_1(r,z,\tau)}{\partial z^2} = \frac{1}{a}\frac{\partial \theta_1(r,z,\tau)}{\partial \tau};$$

for the region $R_1 > r > R_2$, $z \ge 0$, $\tau > 0$ after the change of variable $\theta_2(r,z,\tau) = T_2(r,z,\tau) - T_0$

$$\frac{\partial^2 \theta_2(r,z,\tau)}{\partial r^2} + \frac{1}{r}\frac{\partial \theta_2(r,z,\tau)}{\partial r} + \frac{\partial^2 \theta_2(r,z,\tau)}{\partial z^2} = \frac{1}{a}\frac{\partial \theta_2(r,z,\tau)}{\partial \tau};$$

for the region $R_2 < r \le \infty$, $z \ge 0$, $\tau > 0$ after the change of variable $\theta_3(r,z,\tau) = T_3(r,z,\tau) - T_0$

$$\frac{\partial^2 \theta_3(r,z,\tau)}{\partial r^2} + \frac{1}{r}\frac{\partial \theta_3(r,z,\tau)}{\partial r} + \frac{\partial^2 \theta_3(r,z,\tau)}{\partial z^2} = \frac{1}{a}\frac{\partial \theta_3(r,z,\tau)}{\partial \tau}/$$

The initial conditions for the described system will have the form

$$\theta_1(r,z,0) = \theta_2(r,z,0) = \theta_3(r,z,0) = 0.$$

The boundary conditions for the ring-shaped plasma treatment are of the form:

$$\frac{\partial \theta_1(r,0,\tau)}{\partial z} = 0 \quad (0 \le r < R_1),$$

$$\frac{\partial \theta_1(0,z,\tau)}{\partial z} = 0,$$

$$\frac{\partial \theta_2(r,0,\tau)}{\partial z} = -\frac{q(\tau)}{\lambda} \quad (R_1 > r > R_2),$$

$$\frac{\partial \theta_3(r,0,\tau)}{\partial z} = 0 \quad (r > R_2),$$

$$\theta_1(r,\infty,0) = \theta_2(r,\infty,0) = \theta_3(r,\infty,0) = 0,$$

$$\frac{\partial\theta_1(r,\infty,\tau)}{\partial z}=\frac{\partial\theta_2(r,\infty,\tau)}{\partial z}=\frac{\partial\theta_3(r,\infty,\tau)}{\partial z}=0,$$

$$\theta_3(\infty,z,\tau)=0,$$

$$\frac{\partial\theta_3(\infty,z,\tau)}{\partial r}=0,$$

wherein: λ – coefficient of thermal conductivity.

Conjugation conditions for the three areas have the form:

$$\theta_1(R_1,z,\tau)=\theta_2(R_1,z,\tau),$$

$$\frac{\partial\theta_1(R_1,z,\tau)}{\partial r}=\frac{\partial\theta_2(R_1,z,\tau)}{\partial r},$$

$$\theta_2(R_2,z,\tau)=\theta_3(R_2,z,\tau),$$

$$\frac{\partial\theta_2(R_2,z,\tau)}{\partial r}=\frac{\partial\theta_3(R_2,z,\tau)}{\partial r}.$$

Solution of the system of differential equations with given boundary conditions is carried out using the infinite integral transforms Fourier and Laplace. Applying successively the cosine Fourier transform and Laplace transform, the original system of differential equations takes the form:

$$\frac{\partial^2\bar\theta_{1c}(r,p,s)}{\partial r^2}+\frac{1}{r}\frac{\partial\bar\theta_{1c}(r,p,s)}{\partial r}-\left(p^2+\frac{s}{a}\right)\bar\theta_{1c}(r,p,s)=0,$$

$$\frac{\partial^2\bar\theta_{2c}(r,p,s)}{\partial r^2}+\frac{1}{r}\frac{\partial\bar\theta_{2c}(r,p,s)}{\partial r}-\left(p^2+\frac{s}{a}\right)\bar\theta_{2c}(r,p,s)+\frac{\bar q(s)}{\lambda}=0,$$

$$\frac{\partial^2\bar\theta_{3c}(r,p,s)}{\partial r^2}+\frac{1}{r}\frac{\partial\bar\theta_{3c}(r,p,s)}{\partial r}-\left(p^2+\frac{s}{a}\right)\bar\theta_{3c}(r,p,s)=0.$$

The rest transformed boundary conditions and matching conditions for the variable *r* take the form:

$$\frac{\partial\bar\theta_{1c}(0,p,s)}{\partial r}=0,$$

$$\bar\theta_{3c}(\infty,p,s)=0,$$

$$\frac{\partial\bar\theta_{3c}(\infty,p,s)}{\partial r}=0,$$

$$\bar\theta_{1c}(R_1,p,s)=\bar\theta_{2c}(R_1,p,s),$$

$$\frac{\partial\bar\theta_{1c}(R_1,p,s)}{\partial r}=\frac{\partial\bar\theta_{2c}(R_1,p,s)}{\partial r},$$

$$\bar\theta_{2c}(R_2,p,s)=\bar\theta_{3c}(R_2,p,s),$$

$$\frac{\partial\bar\theta_{2c}(R_2,p,s)}{\partial r}=\frac{\partial\bar\theta_{3c}(R_2,p,s)}{\partial r}.$$

General equations modified zero-order Bessel equations for the first area $0\le r<R_1$, $\operatorname{Re}p>0$, $\operatorname{Re}s>0$ take the form:

$$\bar{\theta}_{1c}(r,p,s)=A_1I_0\left(\sqrt{p^2+\frac{s}{a}}r\right)+B_1K_0\left(\sqrt{p^2+\frac{s}{a}}r\right);$$

for the second region: $R_1>r>R_2$, $\operatorname{Re}p>0$, $\operatorname{Re}s>0$:

$$\bar{\theta}_{2c}(r,p,s)-\frac{\bar{q}(s)}{\left(p^2+\frac{s}{a}\right)\lambda}=A_2I_0\left(\sqrt{p^2+\frac{s}{a}}r\right)+B_2K_0\left(\sqrt{p^2+\frac{s}{a}}r\right);$$

for the third region $r>R_2$, $\operatorname{Re}p>0$, $\operatorname{Re}s>0$:

$$\bar{\theta}_{3c}(r,p,s)=A_3I_0\left(\sqrt{p^2+\frac{s}{a}}r\right)+B_3K_0\left(\sqrt{p^2+\frac{s}{a}}r\right),$$

wherein: A_1, A_2, A_3, B_1, B_2, B_3 – constants of integration, I_0 – modified Bessel function of the first kind of order zero, K_0 – modified Bessel function of the second kind of order zero.

By the symmetry condition, constant of integration $B_1=0$. In accordance with the boundary conditions, the constant of integration $A_3=0$. To determine the constant of integration A_1 is possible to use the conjugation conditions areas on the spatial boundaries isotropic semi-infinite body:

$$A_1=\frac{1}{\lambda}\frac{\bar{q}(s)}{\sqrt{p^2+\frac{s}{a}}}\left(R_1K_1\left(R_1\sqrt{p^2+\frac{s}{a}}\right)-R_2K_1\left(R_2\sqrt{p^2+\frac{s}{a}}\right)\right),$$

wherein: K_1 - modified Bessel function of the second kind of first order.

Then the corresponding solution for the image of function $\bar{\theta}_{1c}(r,p,s)$ takes the form:

$$\bar{\theta}_{1c}(r,p,s)=\frac{\bar{q}(s)I_0\left(r\sqrt{p^2+\frac{s}{a}}\right)}{\lambda\sqrt{p^2+\frac{s}{a}}}\left(R_1K_1\left(R_1\sqrt{p^2+\frac{s}{a}}\right)-R_2K_1\left(R_2\sqrt{p^2+\frac{s}{a}}\right)\right).$$

To determine the redundant temperature at the axis $z\geq 0$ from the impact of the annular plasma source, taking into account the values of the modified Bessel function $I_0(0)=1$, can be written:

$$\bar{\theta}_1(0,z,s)=\frac{2R_1\bar{q}(s)}{\pi\lambda}\int_0^\infty\frac{\cos pz}{\sqrt{p^2+\frac{s}{a}}}K_1\left(R_1\sqrt{p^2+\frac{s}{a}}\right)dp-\frac{2R_2\bar{q}(s)}{\pi\lambda}\int_0^\infty\frac{\cos pz}{\sqrt{p^2+\frac{s}{a}}}\times$$

$$\times K_1\left(R_2\sqrt{p^2+\frac{s}{a}}\right)dp$$

Considering the relation:

$$\cos px = \frac{\sqrt{\pi}}{\sqrt{2}}\sqrt{px}J_{-\frac{1}{2}}(px),$$

wherein: $J_{-\frac{1}{2}}$ - Bessel function of half-integral order.

Function takes the form:

$$\bar{\theta}_1(0,z,s) = \frac{\sqrt{2}}{\sqrt{\pi}}\frac{R_1\sqrt{z}\bar{q}(s)}{\lambda}\int_0^{\infty}\frac{J_{-1/2}(pz)K_1\left(R_1\sqrt{p^2+\frac{s}{a}}\right)\sqrt{p}}{\sqrt{p^2+\frac{s}{a}}}dp - \frac{\sqrt{2}}{\sqrt{\pi}}\times$$

$$\times\frac{R_2\sqrt{z}\bar{q}(s)}{\lambda}\int_0^{\infty}\frac{J_{-1/2}(pz)K_1\left(R_2\sqrt{p^2+\frac{s}{a}}\right)\sqrt{p}}{\sqrt{p^2+\frac{s}{a}}}dp\,.$$

Recorded values of the integrals are particular type integrals Sonin-Gegenbauer. With into account expression of the modified Bessel function of the second kind of half-integer order in terms of elementary functions can be written:

$$\bar{\theta}_1(0,z,s) = \frac{1}{b}\bar{q}(s)\frac{1}{\sqrt{s}}\exp\left(-\frac{\sqrt{R_1^2+z^2}}{\sqrt{a}}\sqrt{s}\right) - \frac{1}{b}\bar{q}(s)\frac{1}{\sqrt{s}}\exp\left(-\frac{\sqrt{R_2^2+z^2}}{\sqrt{a}}\sqrt{s}\right).$$

Using the inverse Laplace transform proceed to the original function that describes the distribution of the temperature field along the axis under the influence of only the annular heat source - plasma ring:

$$\theta_1(0,z,\tau) = \frac{1}{b\sqrt{\pi}}\int_0^{\tau}\exp\left(-\frac{z^2}{4a(\tau-\xi)}\right)\times$$

$$\times\left(\exp\left(-\frac{R_1^2}{4a(\tau-\xi)}\right) - \exp\left(-\frac{R_2^2}{4a(\tau-\xi)}\right)\right)\frac{q(\xi)}{\sqrt{\tau-\xi}}d\xi\,.$$

If the heat flow density $q(\tau) = q_0 = const$, the redundant temperature at the axis r = 0 of the annular plasma source can be found:

$$\theta_1(0,z,\tau) = \frac{2q_0\sqrt{\tau}}{b}\left(ierfc\left(\frac{\sqrt{R_1^2+z^2}}{2\sqrt{a\tau}}\right) - erfc\left(\frac{\sqrt{R_2^2+z^2}}{2\sqrt{a\tau}}\right)\right).$$

Turning to the general problem of temperature field along the axis of the laser radiation, taking into account the annular stream of the plasma, will be calculated by the following relationship:

$$\theta(0,z,\tau) = \frac{2q_0\sqrt{\tau}}{b}\left(ierfc\left(\frac{\sqrt{R_1^2+z^2}}{2\sqrt{a\tau}}\right) - ierfc\left(\frac{\sqrt{R_2^2+z^2}}{2\sqrt{a\tau}}\right)\right) +$$

$$+\frac{2Aq_{l0}\sqrt{\tau}}{b}\left(ierfc\left(\frac{z^2}{2\sqrt{a\tau}}\right) - ierfc\left(\frac{\sqrt{R^2+z^2}}{2\sqrt{a\tau}}\right)\right).$$

Have calculated temperature distribution on the axis of the combined influence for

Steel 45 using program PTC Mathcad v14.0 (license №PTC60602CD140.004 of 18.08.2008) (Fig. 2).

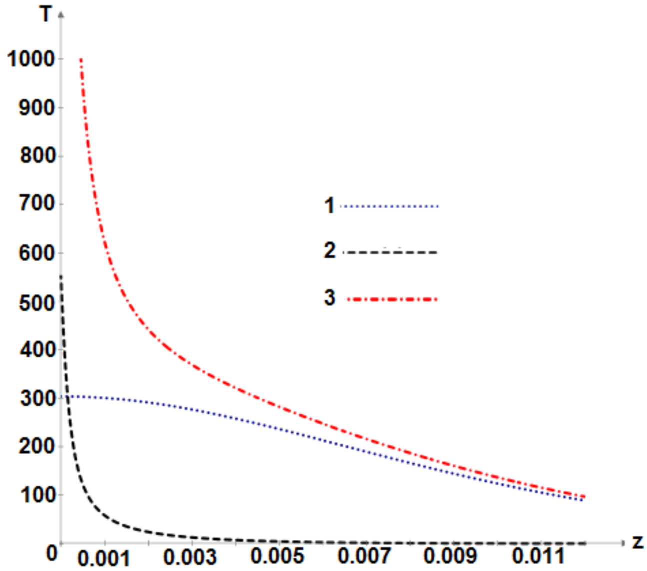

Figure 2: Temperature on the axis of the 1-plasma, 2 - laser 3 - the combined influence

The first calculation was performed for an annular source with the heat flux $q = 3000$ W, with $R_1 = 0{,}01$ m, $R_2 = 0{,}012$ m, and the influence time $\tau = 5$ sec. The second calculation was performed for the laser source $q_l = 100$ W, $R = 0{,}0005$ m, $\tau_l = 1$ sec. The third calculation was carried out for the combined influence, while plasma flow impacted with 5 sec., and only then, when the coefficient of absorption of the laser became equal to 0.99, there was a combined effect of laser plasma source and the duration of 1 ms. In the case of joint simultaneous action of a laser and plasma impact for 25 s., the temperature distribution along the axis is not very different from the third calculation.
This settlement model can be used for determination of necessary power of the combined source for material processing, in particular hardening, cutting, cladding.

References

[1] Grigoryants, A.G., I.N. Shiganov and A.I. Misyurov, Technological processes of laser processing. Moscow: Publishing House of the MSTU. N.E. Baumana. 2006.

[2] Reznikov, N.A. and oth. Metal cutting with plasma heated. Mashinostroenie. 1986 pp. 232

[3] Israphilov, I.H., D.I. Israphilov, A.T. Galiakbarov and D.A. Bashmakov, Hybrid laser-plasma technology heat treatment of metals. International Scientific Conference "Plasma technology studies, modify and retrieve materials of different physical nature": Conference Proceedings 2012, 110-111.

[4] Kozlov, V.P. and P.A. Mandrik, System of integral and differential equations with L-parameter in mathematical physics and methods of identification of thermal characteristics. Publishing of the Belarusian state university. 2000, pp. 555.

[5] Israphilov, I.H., Israphilov, D.I., Galiakbarov, A.T., Bashmakov, D.A. The development of hybrid laser-plasma technology of heat treatment and welding in the automotive industry. International scientific and practical conference "Education, science and production. New technologies as a tool for implementing the strategy of development and modernization 2020 "- Kazan, 2012, S.306-311.

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