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New Characterization of the Phenomenon of the Thermosolutale Natural Convection in Porous Media with Non-Uniform Porosity

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Abstract

The purpose of this work is to study numerically the natural convection and thermosolutal in a square cavity filled with a porous medium saturated by an incompressible Newtonian fluid. The Boussinesq approximation is adopted. The thermo-physical properties of the fluid are considered constants except the density which varies linearly with the temperature and concentration. The horizontal walls are subjected to temperature and concentration constant gradients and the vertical walls are adiabatic and impermeable. The flow is assumed laminar and non-permanent. The porosity is considered variable along the vertical axis and expression of the permeability is given as a function of porosity. The mathematical model used is that of Darcy – Brinkman. The equations of continuity, and momentum, and energy and mass are solved numerically using the Rung-Kutta method.
The study was focused on influence Lewis number, and Buoyancy ratio number, and Aspect ratio number on the flow behavior. The results are in perfect agreement with results obtained by other authors working in the same field.

Keywords: natural convection, square cavity, porous media, Rung Kutta, Darcy-Brinkman formulation

1. Introduction

The study of heat transfer and mass in porous media has been particularly developed in recent years, because of their potential applications in various domains. In effect, the phenomenon of convection can occur in porous media, as geothermal, oceanography, astrophysics, biology, chemical process reservoirs for the storage of natural gas and radioactive waste, etc.
The majority of experimental work, analytical and numerical addressing convective heat transfer and mass transfer in porous media have been given in the book by Nield and Bejan [1]. Goyeau and al. [2] have treated the natural and thermosolutal convection in porous media. They proposed relationships to collect heat transfer and mass. A study by Khair and Bejan [3] identified four regimes possible of convection thermosolutal according to the values of buoyancy ratio N and Lewis number Le, for a vertical plate filled by a porous medium saturated by a fluid.
Nithiarasu and al [4] have studied the natural convection in a rectangular cavity filled with a porous medium non-Darcian, taking into account the effects of viscous constraints and inertia in the equation of movement that is reduced to the Navier-Stokes equations. Porosity is assumed to vary according to an empirical law as follows an exponential function. A study is done for different values of Rayleigh numbers and Darcy.
A. Mrabti [5] studied the phenomenon of natural convection in a cylindrical cavity of aspect ratio equal to unity, a porous medium saturated by a Newtonian fluid the Prandtl number equal to 0.7. The cavity is heated from below. The vertical walls are adiabatic and impermeable and rigid. The author observed that increasing the

number of Rayleigh the structure of the flow becomes bicellular with intensification of flow with increasing porosity of the medium. This bicellular structure disappears when increases the number of Darcy.

In a study based on linear stability, Mahidjiba and al [6] illustrate the effect of boundary conditions thermal and solutales the Neumann and Dirichlet respectively, on thermosolutal convection in a horizontal rectangular cavity filled by a porous medium with uniform porosity and isotropic. The authors show that there are three critical Rayleigh numbers; the Rayleigh number supercritical, Rayleigh supercritical and Rayleigh oscillating.

Lamine Kalla [7] milking a study of heat transfer and mass transfer in the cavity subject to temperature gradients and concentration requires the determination of the intensity of convective transfer. This intensity is given in the case of thermal transfer, the Nusselt number, characterized the heat transfer at the walls, but does not serve as a condition of similarity flows. .

Recently, studies theoretical and digital of the phenomenon of the diffusive double convection are carried out by Sammouda and al. [8]. The author considered a cylindrical cavity filled by a porous environment and saturated by a Newtonian fluid. The thermo-physical properties are constant, except for the density which varies linearly with the temperature according to the approximation of Boussinesq. The side wall of the enclosure is supposed to be rigid, impermeable and adiabatic. The horizontal walls are maintained at constant temperatures and concentrations. The porosity of the medium is considered variable; this variation is described by an exponential empirical law according to the ray of the enclosure.

The object of this article is to study the effect of these parameters on the natural and thermosolutal convection in porous media. The model considered is that of Darcy-Brinkman which is adapted more to study the strictly permeable mediums. Initially, we present the description of the physical model as well as the conservation equations which govern the phenomenon. In a last part, the digital procedure is briefly pointed out and then the numerical simulations for broad ranges of parameters are presented are analyzed.

2. Geometry and mathematical model

The geometric configuration of the flow is considered square of height H, width L illustrated in Figure 1. This cavity is filled with a porous medium isotropic saturated by a Newtonian and incompressible fluid. The basics lower and upper of the cavity are maintained at temperatures and concentrations constant and uniform, respectively (T_C,T_F) and (C_{inf} , C_{sup}). The side walls are rigid and impermeable. The effects Crusaders Soret and Dufour on the diffusion of heat and mass are neglected. The thermo-physical properties of the fluid are assumed to be constant except the density which is taken in account in the volume forces term. This density is considered variable linearly with the temperature and concentration.

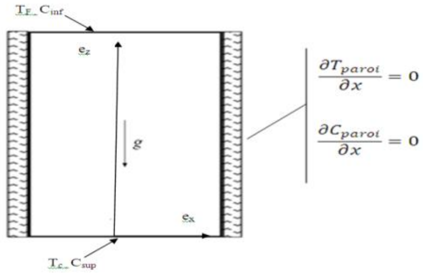


Figure 1: The geometric configuration of the flow

Using the model of Darcy-Brinkman equations governing this problem are:

$$div\vec{V} = 0 \tag{1}$$

$$\frac{\partial T}{\partial t} + \vec{V}.\vec{\nabla}T = k_e \vec{\nabla}.\left(\vec{\nabla}T\right) \tag{2}$$

$$\varepsilon(z)\frac{\partial C}{\partial t} + \vec{V}\vec{\nabla}C = \vec{\nabla}(\varepsilon(z)D\vec{\nabla}C) \tag{3}$$

The movement equation written in vector form:

$$\left(\frac{1}{\varepsilon}\frac{\partial \vec{V}}{\partial t} + \left(\frac{\vec{V}}{\varepsilon}.\vec{\nabla}\right)\frac{\vec{V}}{\varepsilon}\right) = -\frac{\mu}{\rho_0 K}\vec{V} + (1-\beta_T(T-T_0)-\beta_C(C-C_0))\vec{g} - \frac{1}{\rho_0}\vec{\nabla}p + \frac{\mu}{\rho_0\varepsilon}\Delta\vec{V} \tag{4}$$

The density of the fluid depends on the variations of temperature and concentration developed by first order Taylor [9] as:

$$\rho = \rho_0\left(1-\beta_T\left(T-T_0\right)-\beta_C\left(C-C_0\right)\right) \tag{5}$$

The variation of the porosity is represented using an exponential law and can be written as follows

$$\varepsilon(z) = \varepsilon_\infty\left(1 + C_1\left(1 - e^{-C_2 z}\right)\right) \tag{6}$$

Permeability is given as:

$$K(z) = \frac{\varepsilon^3 d^2}{180\left(1-\varepsilon\right)^2} = K_\infty\ f\left(z\right) \tag{7}$$

with

$$f(z)=\frac{\chi(z)^3}{\left[1+C_{por1}(1-\chi(z))(1-C_{por2}\chi(z))\right]} \tag{8}$$

$$\chi(z)=1-C_{por}e^{-C_2 z} \tag{9}$$

$$C_{por}=\frac{\varepsilon_\infty-1}{\varepsilon_\infty}\quad C_{por1}=\frac{\varepsilon_\infty}{(1-\varepsilon_\infty)^2(2-\varepsilon_\infty)}\quad C_{por2}=\frac{\varepsilon_\infty}{(2-\varepsilon_\infty)} \tag{10}$$

In two-dimensional flows, it is more common to use the formulation vorticity - stream function therefore the momentum equation can be written in the form:

$$\begin{aligned}&\frac{1}{\varepsilon(z)}\frac{\partial\Omega}{\partial t}+\frac{\partial U}{\partial t}(\frac{\partial}{\partial z}(\frac{1}{\varepsilon(z)}))+(\frac{2\Omega}{\varepsilon(z)}+\frac{\partial}{\partial z}(\frac{1}{\varepsilon(z)})U))(W\frac{\partial}{\partial z}(\frac{1}{\varepsilon(z)}))+\frac{U}{\varepsilon^2(z)}\frac{\partial\Omega}{\partial x}+\frac{W}{\varepsilon^2(z)}\frac{\partial\Omega}{\partial z}\\&+\frac{U}{\varepsilon(z)}\frac{\partial}{\partial x}(\frac{\partial}{\partial z}(\frac{1}{\varepsilon(z)})U)+\frac{W}{\varepsilon(z)}\frac{\partial}{\partial z}(\frac{\partial}{\partial z}(\frac{1}{\varepsilon(z)})U)=-\frac{\upsilon}{K(z)}\Omega-\upsilon(\frac{\partial}{\partial z}(\frac{1}{K(z)})U-(\beta_T g\frac{\partial T}{\partial x}+\beta_S g\frac{\partial C}{\partial x})\\&+\frac{\upsilon}{\varepsilon(z)}(\frac{\partial^2\Omega}{\partial x^2}+\frac{\partial^2\Omega}{\partial z^2})+\upsilon(\frac{\partial}{\partial z}(\frac{1}{\varepsilon(z)}))(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial z^2})\end{aligned} \tag{11}$$

Vorticity and velocity component are expressed in terms of the stream function, such as:

$$U=\frac{\partial\Psi}{\partial z}\ ;\ W=-\frac{\partial\Psi}{\partial x}\ ;\ \Omega=\Delta\Psi \tag{12}$$

We introduce the non-dimensional following variables:

$$\Omega^*=\frac{\Omega H^2}{k_e}\qquad W^*=\frac{WH}{k_e}\qquad t^*=\frac{tk_e}{H^2}$$

$$U^*=\frac{UH}{k_e}\quad x^*=\frac{x}{H}\quad z^*=\frac{z}{H}\quad \psi^*=\frac{\psi}{k_e}$$

$$T^*=\frac{T-T_F}{\Delta T}\qquad C^*=\frac{C-C_F}{\Delta C}$$

The obtained equations after introduction of these variables while omitting the index * are written:

$$\frac{\partial T}{\partial t}=(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2})-\frac{\partial(UT)}{\partial x}-\frac{\partial(WT)}{\partial z} \tag{13}$$

$$\frac{\partial C}{\partial t}=\frac{1}{Le}\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial z^2}\right)+\frac{1}{Le\varepsilon(z)}\frac{\partial \varepsilon(z)}{\partial z}\frac{\partial C}{\partial z}-\frac{1}{\varepsilon(z)}\left(\frac{\partial UC}{\partial x}+\frac{\partial WC}{\partial z}\right) \quad (14)$$

$$\frac{\partial \Omega}{\partial t}=-\Pr\frac{\varepsilon(z)}{Da}\frac{\Omega}{f(z)}+\Pr\frac{\varepsilon(z)}{Da}\frac{\partial}{\partial z}\left(\frac{1}{k(z)}\right)U-\Pr Ra\,\varepsilon(z)\left(\frac{\partial T}{\partial x}+N\frac{\partial C}{\partial x}\right)$$
$$+\Pr\left(\frac{\partial^2 \Omega}{\partial x^2}+\frac{\partial^2 \Omega}{\partial z^2}\right)+\Pr\varepsilon(z)\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon(z)}\right)\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial z^2}\right)-\varepsilon(z)\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon(z)}\right)\frac{\partial U}{\partial t}$$
$$-2\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon(z)}\right)\Omega W-\left[\varepsilon(z)\left[\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon(z)}\right)\right]^2+\frac{\partial^2}{\partial z^2}\left(\frac{1}{\varepsilon(z)}\right)\right]UW-\frac{U}{\varepsilon(z)}\frac{\partial \Omega}{\partial x}$$
$$-\frac{W}{\varepsilon(z)}\frac{\partial \Omega}{\partial z}-U\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon(z)}\right)\frac{\partial U}{\partial x}-\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon(z)}\right)W\frac{\partial U}{\partial z} \quad (15)$$

The components of the velocity and vorticity are written as:

$$U=\frac{\partial \Psi}{\partial z}\ ;\ W=-\frac{\partial \Psi}{\partial x}\ ;\ \Omega=\Delta\Psi \quad (16)$$

The boundary conditions can be written in dimensionless form as follows:

$$T=1 \text{ at } z=0, T=0 \text{ at } z=1 \quad \frac{\partial T_{axe}}{\partial x}=0 \quad \frac{\partial T_{paroi}}{\partial x}=0 \quad (17)$$

$$C=1 \text{ at } z=0,\quad C=0 \text{ at } z=1 \quad \frac{\partial C_{axe}}{\partial x}=0 \quad \frac{\partial C_{paroi}}{\partial x}=0 \quad (18)$$

$$\Psi=0 \text{ at } x=0\ ; \Psi=\frac{\partial \Psi}{\partial x}=\frac{\partial \Psi}{\partial z}=0 \text{ at } x=1\ ;\ \Psi=\frac{\partial \Psi}{\partial r}=\frac{\partial \Psi}{\partial z}=0 \text{ at } z=1 \quad (19)$$

$$\Omega\big|_{x=0}=0\ ;\ \Omega\big|_{x=1}=\frac{2}{\Delta x^2}\Psi\Big|_{x=1-\Delta x}\ ;\ \Omega\big|_{\tilde{z}=0}=\frac{2}{\Delta z^2}\Psi\Big|_{z=\Delta z}\quad \Omega\big|_{z=1}=\frac{2}{\Delta z^2}\Psi\Big|_{z=1-\Delta z} \quad (20)$$

3. Numerical method

The basic equations that govern the problem of convection thermosolutal are nonlinear and coupled. Generally, they do not admit analytical solutions. Accordingly, recourse to numerical methods proves mandatory.

As part of our study, we opted for the Rung Kutta method [10] which turns out the most appropriate and easiest to use for problems with simple geometry.

We use the method S.O.R (***S**imultaneous* ***O**ver* ***R**elaxation*) to solve the equation of the stream function [11]. The iterations are repeated until the difference between two successive values of the variables Ω, Ψ, U,W,*T* and *C between two increments of time* t and t+ Δt respectively are negligible, such as:

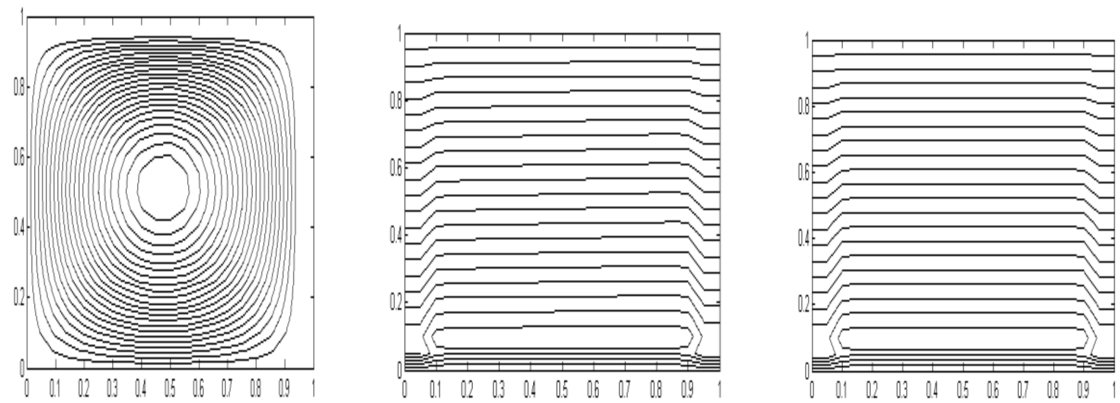
$$\frac{\sum_i \sum_j \left| f_{i,j}^{n+1} - f_{i,j}^{n} \right|}{\sum_i \sum_j \left| f_{i,j}^{n+1} \right|} \leq 10^{-4} \qquad (21)$$

4. Results and interpretations

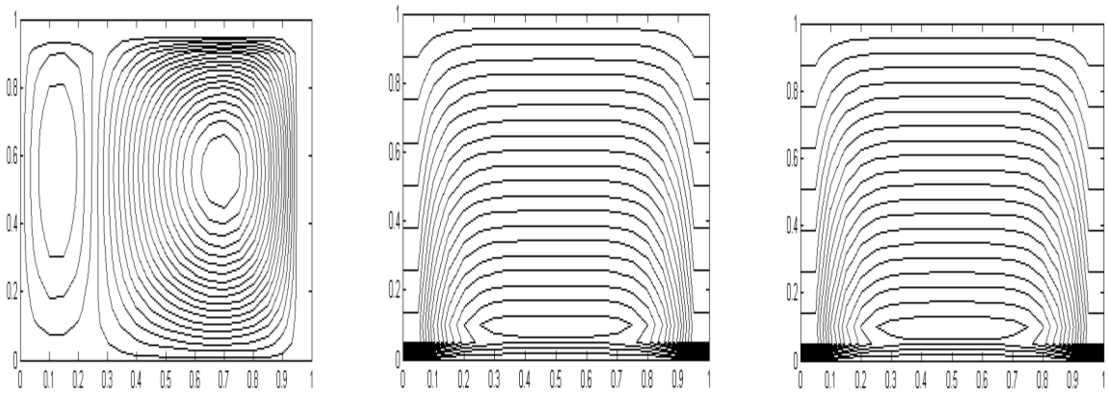
4.1 Effect of Aspect ratio A

One considers a square cavity filled by a porous matrix with variable porosity saturated by a Newtonian fluid convectant with number of Rayleigh equal to 10^4.The number of Lewis taken is 1 the number Buoyancy ratio N is taken equal to -1. The number of Darcy taken is 0.1; and number of Prandtl equal to 3.The enclosure is heated by bottom, for different values of the aspect ratio A. For an aspect ratio A = 1 there is the appearance of a single roller which has filled the entire cavity and the same deformation for isothermal and isoconcentration lines.

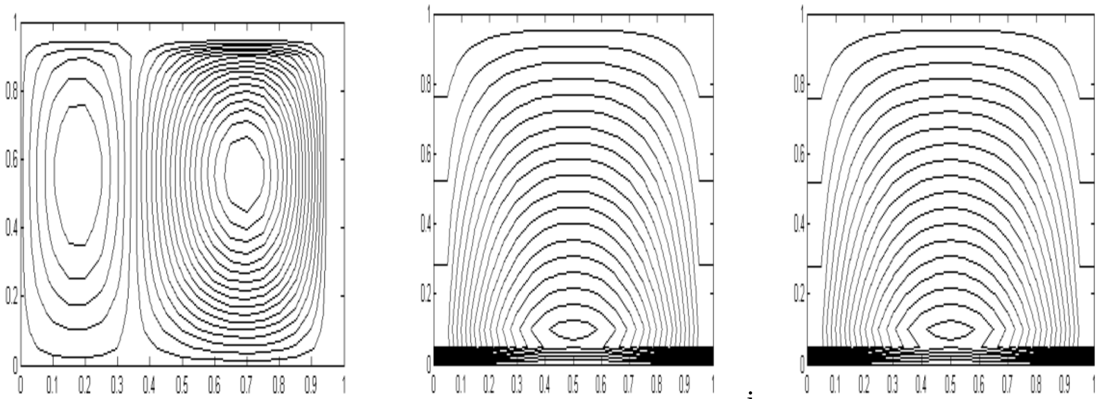
When one augment A = 5, we see the appearance of other roll, but small. The isothermal lines and the isoconcentration lines manifest more convex than previous state A=1. We augment again Aspect ratio of A=10, the second roller enlarges and lines isotherms and isoconcentration lines become more convex. In general, the axial velocity increases when A increases, but the horizontal velocity decreases when A increases.



(a) Stream function Temperature Concentration



(b)



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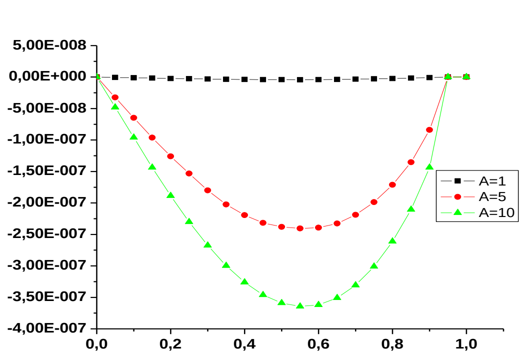
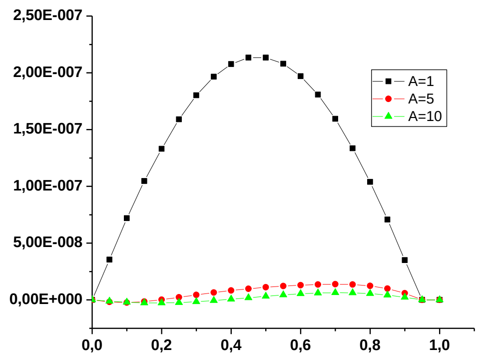
(c)

Figure 2 : Stream function, temperature and concentration with
Da=0.01 ; Ra=10000. ;Pr=3.;Le=1. ; N=-1.
(a) A=1 ; (b) A=5 ;(c) A=10



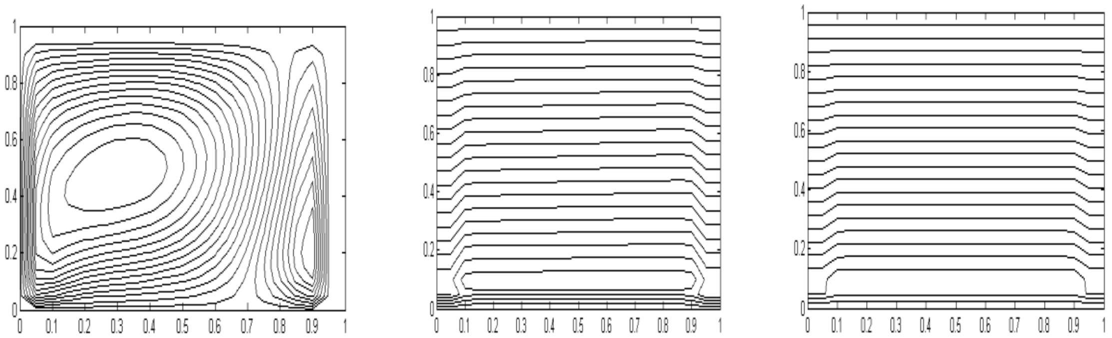
Axial velocity



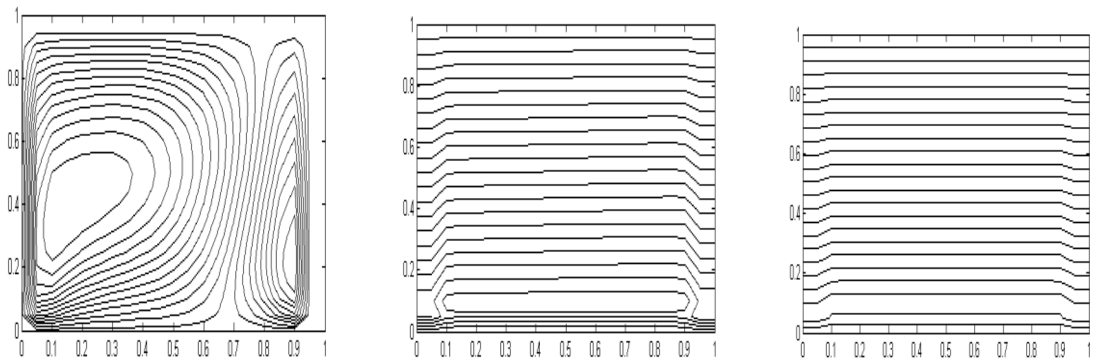
Horizontal velocity

4.2 Effect of Lewis number

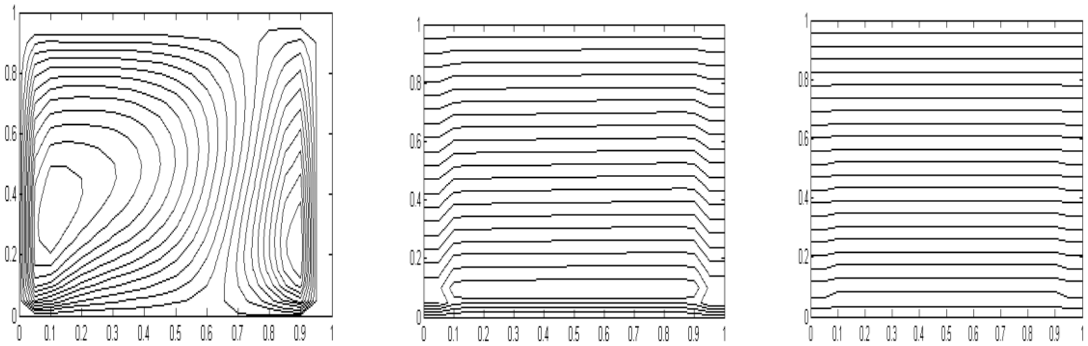
The effect Lewis is illustrated in the following figures 2 which have the function of current and lines isotherms and isoconcentration lines and speeds axial and horizontal with Pr=3; Ra=10^4; Da=0.01; N=-1.
It notes that globally, the vertical and horizontal speed increase with the increase in number the Lewis.
We note that for a Lewis number Le = 2, the appearance of two rollers, that existing in the left side is the greatest and isoconcentration lines manifest more serene than the isothermal lines. With an increase in the number of Lewis Le=3 the second roll is reduced and isoconcentration line becomes tighter than in the case Le = 2.
With a further increase in the number of Lewis, the second roll continues its reduction and lines of isoconcentration become tighter.
In fact, increasing the number of Lewis implies a similar thermal diffusivity regarding the solutal   diffusivity.



(a) Stream function Temperature Concentration

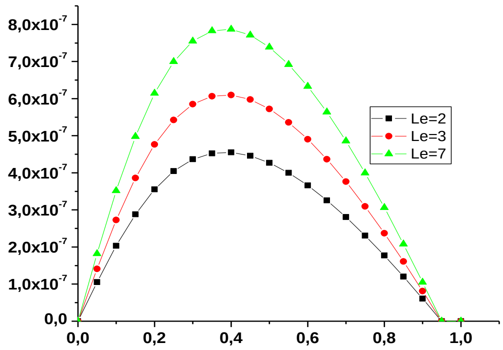


(b)

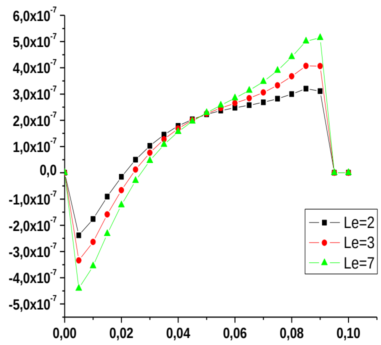


(c)

Figure 3: Stream function and temperature and concentration with
Da=0.01 ; Ra=10000. ;Pr=3.; ; N=-1 ;A=1
(a) Le=2 ; (b) Le=3 ;(c) Le=7



Axial velocity



Horizontal velocity

4.3 Effect of Buoyancy ratio N

To demonstrate the effect of Buoyancy ratio N on natural convection, we consider a square cavity. The numbers of Darcy, Prandtl, Lewis and Rayleigh are respectively Da = 0.01; Pr = 3 ; Le = 1 ; Ra=10^4 ,for different values of N number. For Buoyancy ratio N=-1, the roller is observed stream function occupies the entire cavity with a center roller located in the middle of the enclosure, when N is given the value 1, there is the appearance of another roll moving towards the right side of the cavity. Isoconcentration and isothermal lines are clamped in the upper side of the cavity.
When we taken buoyancy ratio N=-2, we observe the appearance of a roll moving toward the left side of the cavity. Isotherms and isoconcentration become very tight in the upper side of the cavity.

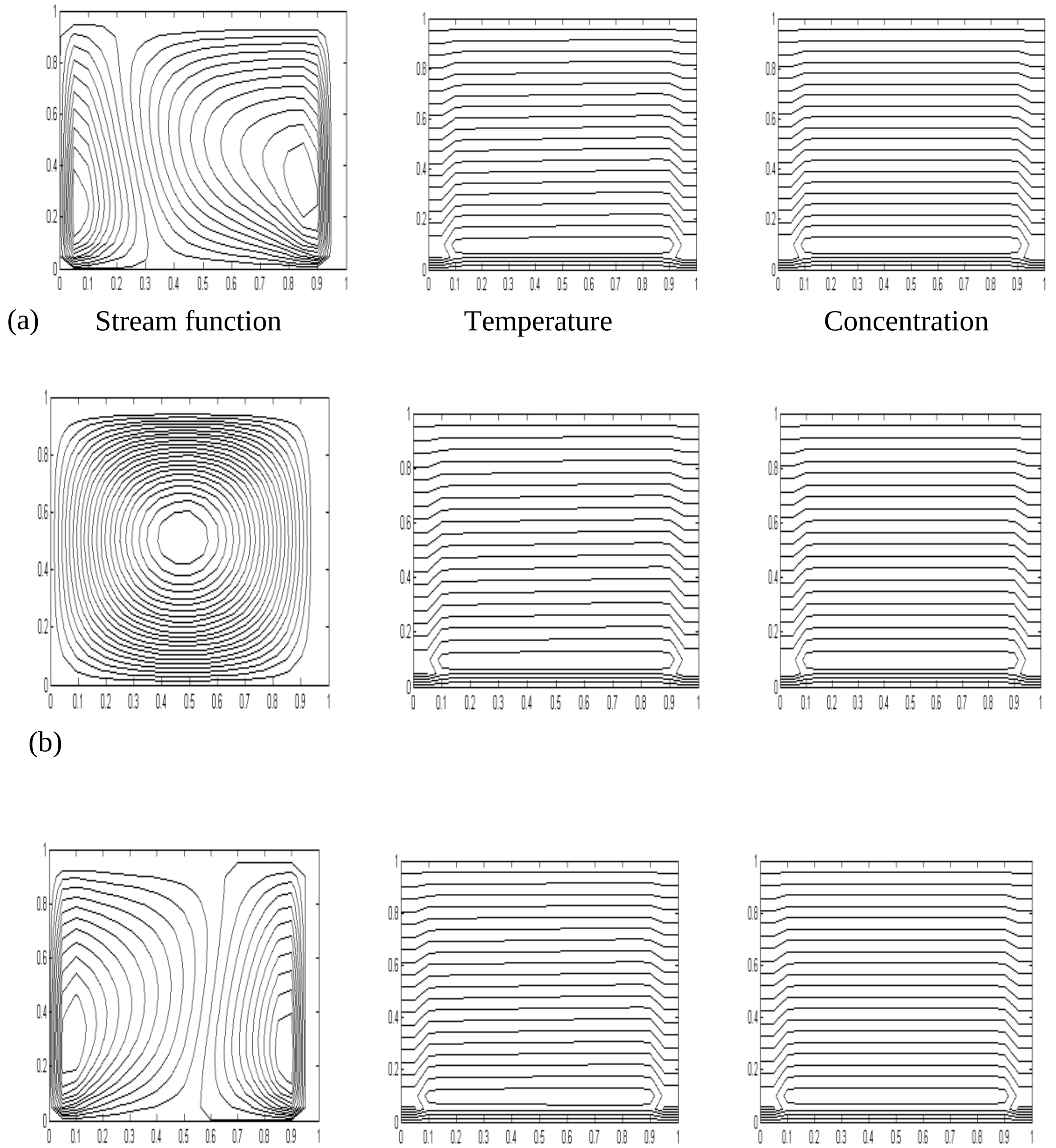
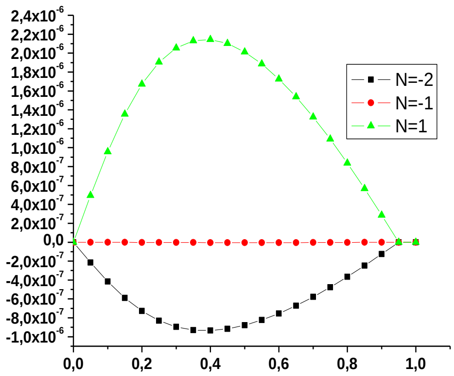
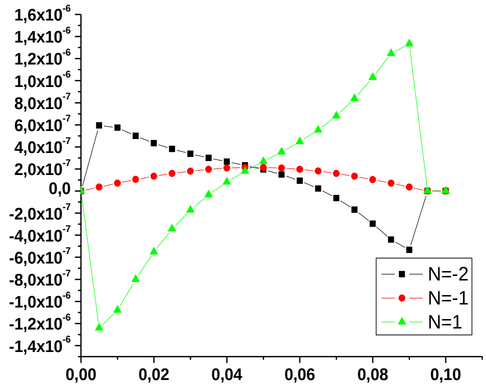


Figure 4: Stream function and temperature and concentration with
Da=0.01 ; Ra=10000. ;Pr=3.; ; Le=1 ;A=1
(a) N=-2 ; (b) N=-1 ; (c) N=1



Axial velocity



Horizontal velocity

5. Conclusion

This paper deals with thermosolutal convection in porous media, once the configuration defined and the mathematical model established, an analysis of orders of magnitude is presented. The influence of different parameters characterizing this problem has been explored numerically by varying de Lewis number Le, Aspect ratio A, buoyancy ratio N. We note that for a great number of Lewis, the axial and horizontal velocities increase. We also observed that the solutal transfer promotes itself by increasing the number of number the Lewis. The increase in A leads to the appearance of other rollers and the axial velocity becomes higher. One can also observed the increase of heat transfer and solutal transfer when number A increases.
The heat transfer rate and solutal increases with the buoyancy ratio. In fact, a buoyancy ratio of positive the thermal forces and solutal are added, and consequently the intensity of the flow increases, increasing the intensity of the flow improve the heat transfer and solutal.

NOMENCLATURE

T : *Temperature*
C : *Concentration*
U : *Dimensionless radial velocity*
W : *Dimensionless axial velocity*
μ : *Fluid viscosity*
K : *Permeability of the porous matrix*
ε : *Porosity*
ε_∞ : *Infinite porosity*
K_∞ : *Infinite Permeability*
ρ : *Fluid density*
g : acceleration of gravity

k_e *: Thermal diffusivity of the porous media*
λ *: Thermal conductivity of the porous*
ν *: Fluid viscosity*
Ω *: Vorticity*
Ψ *: Stream function*
Da *: Darcy number*
Ra *: Rayleigh number*
$\Pr$ *: Prandtl number*
Le *: Lewis number*
N *: Buoyancy ratio*
A *: Aspect ratio*
D *: mass diffusivity*
t: dimensionless time
β_S : Coefficient of solutal expansion
β_T : Coefficient of thermal expansion

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