**Contemporary Engineering Sciences, Vol. 7, 2014, no. 31, 1699 - 1705**
**HIKARI Ltd, www.m-hikari.com**
**http://dx.doi.org/10.12988/ces.2014.411228**

# Assignment Scheme for Maximizing the Network Capacity in the Massive MIMO

**Kwihoon Kim[1], Sunhyung Kwon[1], Woongsik You[1] and Kyounghee Lee[2*]**

[1]Future Research Creative Laboratory
ETRI, Daejeon, Korea

[2]Department of Computer Engineering, Pai Chai University
Daejeon, Korea



**Abstract**

The throughput performance of Massive MIMO is limited due to pilot contamination. In the conventional research, we considered maximum network capacity when the number of user and total pilots fixed. However, we want to calculate theoretically maximum network capacity when the number of user and total pilot symbols change continuously. Then total network capacity changes according to total pilot symbols and pilot assignment. So this paper proposes to calculate the maximum network capacity considering total pilot symbols and pilot assignment to minimize the pilot contamination.

**Keywords:** OFDM, Massive MIMO, Pilot Symbols, Pilot Assignment, Pilot Contamination

## 1 Introduction

Massive MIMO technique was proposed employing (theoretically) unlimited number of antennas at each base station in [1].

---

[*] Corresponding author

In LTE-A, the base station has the eight antenna. But a new Massive MIMO antenna uses the infinite. For Massive MIMO environments, AWGN and intra-cell interference is not all, only a portion of inter-cell interference remains. Because interference is small, the capacity of cell is very large.

Though Massive MIMO has such a special feature, it has inter-cell interference because base station cannot get the perfect channel information. We have to use limited pilots; other cells will reuse necessarily the same pilot. We call this phenomenon the problem of pilot contamination. This is the fundamental limitation of the Massive MIMO environment. [2][3][4]

This paper proposes to solve the problem of pilot contamination. Traditional paper calculates theoretically the capacity of network when the number of user and pilots are fixed in the massive MIMO environments.

However the number of user is changing sustainably and we need the different total pilots according to the number of user.

At this point, the total number of pilots and pilot allocation for each user in a cell will affect the capacity of the entire network.

This paper proposes the method decreasing pilot contamination when the number of user and total pilots change. In other words, we propose to calculate the maximum network capacity through total pilot symbols and pilot assignment in the Massive MIMO.

## 2 Proposed Scheme

In the traditional paper, we suppose the $K^{max}$(The maximum pilots in a cell) and $U_j$(The total user numbers in the j-th cell) are the same and constant. However generally $U_j$ is changed continuously. If $K^{max}$ is increasing, we include many users in the cell but data rate per user is decreasing. Meanwhile if $K^{max}$ is decreasing, we include a little of users in the cell but data rate per user is increasing. For maximizing the total network capacity, we need that $K^{max}$ must be selected according to $U_j$. Pilots will be assigned to user in the random or for maximizing the total network capacity.

k-th user's capacity:

$$\begin{aligned} C^u_{j,k} &= B_W \left(\frac{1}{\alpha}\right)\left(\frac{T_{block} - T_{pilot}}{T_{block}}\right)\left(\frac{T_u}{T_s}\right) log_2\left(1 + SIR^u_{j,\varphi_k}\right) \\ &= B_W \left(\frac{1}{\alpha}\right)\left(\frac{J \cdot T_s - \tau \cdot T_s}{J T_s}\right)\left(\frac{T_u}{T_s}\right) log_2\left(1 + SIR^u_{j,\varphi_k}\right) \\ &= B_W \left(\frac{1}{\alpha}\right)\left(\frac{J-\tau}{J}\right)\left(\frac{T_u}{T_s}\right) log_2\left(1 + SIR^u_{j,\varphi_k}\right) \qquad (1) \end{aligned}$$

$$SIR^{u}_{j,\varphi_k} = \begin{cases} \frac{\beta^{2}_{j,\varphi_k j}}{\sum_{l=1,l\neq j}^{L} \beta^{2}_{j,\varphi_k l}} , & \varphi_k \in \Omega_j \\ 0 , & \varphi_k \ not \in \ \Omega_j \end{cases} \tag{2}$$

where $\Omega_j$: Used pilot set in the j-th cell,

$\varphi_k$ : Assigned pilot for the k-th user

**A. The Optimal Case**

Firstly, we propose the optimal method for the total pilots and the pilot assignment. The total network capacity:

$$\begin{aligned} C^{u}_{network}(K^{max},\boldsymbol{\Omega}) &= \sum_{j=1}^{L}\sum_{k=1}^{U_j} C^{u}_{j,k} \\ &= \sum_{j=1}^{L}\sum_{k=1}^{U_j} B_W\left(\frac{1}{\alpha}\right)\left(\frac{J-\tau}{J}\right)\left(\frac{T_u}{T_s}\right)log_2\left(1+SIR^{u}_{j,\varphi_k}\right) \\ &= \sum_{\varphi_k \in \Omega_j} B_W\left(\frac{1}{\alpha}\right)\left(\frac{J-\tau}{J}\right)\left(\frac{T_u}{T_s}\right)log_2\left(1+\frac{\beta^{2}_{j,\varphi_k j}}{\sum_{l=1,l\neq j}^{L}\beta^{2}_{j,\varphi_k l}}\right) \\ &\quad + \sum_{\varphi_k \in \Omega_j}\sum_{l=1,l\neq j}^{L} B_W\left(\frac{1}{\alpha}\right)\left(\frac{J-\tau}{J}\right)\left(\frac{T_u}{T_s}\right)log_2\left(1+\frac{\beta^{2}_{l,\varphi_k l}}{\beta^{2}_{l,\varphi_k j}+\sum_{i=1,i\neq l}^{L}\beta^{2}_{l,\varphi_k i}}\right) \\ &\quad + \sum_{\varphi_k \ not \in \Omega_j}\sum_{l=1,l\neq j}^{L} B_W\left(\frac{1}{\alpha}\right)\left(\frac{J-\tau}{J}\right)\left(\frac{T_u}{T_s}\right)log_2\left(1+\frac{\beta^{2}_{l,\varphi_k l}}{\sum_{i=1,i\neq l}^{L}\beta^{2}_{l,\varphi_k i}}\right) \end{aligned} \tag{3}$$

The optimal total pilots and pilot set according to user:

$$(K^{max,opt},\boldsymbol{\Omega}^{opt}) = \ arg \max_{(K^{max},\boldsymbol{\Omega})} C^{u}_{network}(K^{max},\boldsymbol{\Omega}) \tag{4}$$

The pilot set allocated for each cell:

$$\boldsymbol{\Omega}^{opt} = \{\Omega^{opt}_1, \Omega^{opt}_2, \ \ldots \ , \Omega^{opt}_L\} \tag{5}$$

$$\Omega^{opt}_j = \{The\ optimal\ pilots\ in\ 1 \sim K^{max,opt}\} \tag{6}$$

$$\mathbf{U} = \{U_1, U_2, \ \ldots, U_L\} \tag{7}$$

According to equation (4), when we change τ, $\boldsymbol{\Omega}$ and $\varphi_k$ for $\mathbf{U}$, we can find $K^{max,opt}$ and $\boldsymbol{\Omega}^{opt}$ with the exhaustive search method.

For each τ = {1, … , J-1}, the computational complexity is,

$$\begin{cases} \sum_{j=1}^{L}\binom{K^{max}}{U_j}U_j! = \sum_{j=1}^{L}\binom{\tau N_{smooth}}{U_j}U_j! \ , K^{max} > U_j \\ \sum_{j=1}^{L}\binom{U_j}{K^{max}}K^{max}! = \sum_{j=1}^{L}\binom{U_j}{\tau N_{smooth}}\tau N_{smooth}! \ , U_j > K^{max} \end{cases} \tag{8}$$

We need the computation for each case. Also, because we compute $C^{u}_{network}(K^{max},\boldsymbol{\Omega})$ for each case, we must totally compute $\sum_{\Omega_j}\log_2(\cdot)+\sum_{\Omega_j}\sum_{L}\log_2(\cdot)$.

**B. The Suboptimal Case Ⅰ**

We propose that the suboptimal method reduces the computational complexity because the optimal method has the problem of computational complexity. The other cell uses the value which is assigned and the current cell computes the new value. Because the pilot of the other cell fixed, the second and the third term of equation (3) disappear. Therefore we assign the total pilot symbol and the pilot of the maximum capacity in the j-th cell. However, if the total pilot symbols are differ from the previous pilot symbols, we recalculate the optimal method for the entire cell.

j-th cell capacity:

$$C_j^u(K_j^{max},\Omega_j)=\sum_{k=1}^{U_j}C_{j,k}^u$$
$$=\sum_{k=1}^{U_j}B_W\left(\frac{1}{\alpha}\right)\left(\frac{J-\tau}{J}\right)\left(\frac{T_u}{T_s}\right)log_2\left(1+SIR^u_{j,\varphi_k}\right)$$
$$=\sum_{\varphi_k\in\Omega_j}B_W\left(\frac{1}{\alpha}\right)\left(\frac{J-\tau}{J}\right)\left(\frac{T_u}{T_s}\right)log_2\left(1+\frac{\beta^2_{j,\varphi_k,j}}{\sum_{l=1,l\neq j}^{L}\beta^2_{j,\varphi_k,l}}\right) \quad (9)$$

Optimal pilot symbols and pilot set:

$$\left(K_j^{max,subopt\_I},\Omega_j^{opt}\right)=arg\max_{(K_j^{max},\Omega_j)}C_j^u(K_j^{max},\Omega_j) \quad (10)$$

Pilot set which is allocated in the j-th cell:

$$\Omega_j^{opt}=\{optimal\ pilots\ in\ 1\sim K_j^{max,opt}\} \quad (11)$$

According to equation (10), we can find $K_j^{max,subopt\_I}$ with exhaustive search method to change τ, $\Omega_j$ and $\varphi_k$ for $U_j$.

For each τ = {1, … , J-1}, the computational complexity is,

$$\begin{cases}\binom{K_j^{max}}{U_j}U_j!=\binom{\tau N_{smooth}}{U_j}U_j!\ ,\ K_j^{max}>U_j\\ \binom{U_j}{K_j^{max}}K_j^{max}!=\binom{U_j}{\tau N_{smooth}}K_j^{max}!\ ,\ U_j>K_j^{max}\end{cases} \quad (12)$$

Because we need computation for each case, this case is L times lower than the optimal case. Also because we calculate $C_j^u(K_j^{max}, \Omega_j)$ for the each case, we calculate $\sum_{\Omega_j} \log_2(\cdot)$ totally.

### C. The Suboptimal Case II

We propose that the suboptimal method II reduces lower than suboptimal method I for the computational complexity. We allocate the pilot at random, and find only the optimal total pilot symbols.

Total network capacity:

$$C_{network}^u(K^{max}, \Omega_j) = \sum_{j=1}^{L} \sum_{k=1}^{U_j} C_{j,k}^u$$
$$= \sum_{j=1}^{L} \sum_{k=1}^{U_j} B_W \left(\frac{1}{\alpha}\right) \left(\frac{J-\tau}{J}\right) \left(\frac{T_u}{T_s}\right) log_2\left(1 + SIR_{j,\varphi_k}^u\right) \quad (13)$$

Optimal pilot symbols and pilot set:

$$(K^{max,subopt\_II}) = arg \max_{(K^{max})} C_{network}^u(K^{max}) \quad (14)$$

According to equation (13), we can find $K_j^{max,subopt\_II}$ with exhaustive search method to change τ.

Because we need computation for τ = {1, … , J-1}, this case need J-1 times. Also because we calculate $C_{network}^u(K^{max}, \Omega)$ for the each case, we calculate $\sum_{\Omega_j} \log_2(\cdot) + \sum_{\Omega_j} \sum_L \log_2(\cdot)$ totally.

## 3 Numerical Results

In the traditional paper, we suppose the $K^{max}$(The maximum pilots in a cell) and $U_j$(The total user numbers in the j-th cell) are the same and constant. However generally $U_j$ is changed continuously. If $K^{max}$ is increasing, we include many users in the cell but data rate per user is decreasing. Meanwhile if $K^{max}$ is decreasing, we include a little of users in the cell but data rate per user is increasing.

We simulates in the experimental environment such as <table 1>.

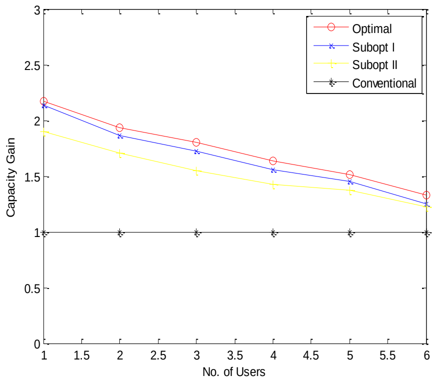
<Table 1> Experimental parameters

| Parameters | value |
|---|---|
| J | 7 |
| $N_{smooth}$ | 3 |
| $\tau$ | 1,2,3 |
| No. of available pilots | 3,6,9 |
| No. of iterations | 10,000 |

According to figure 1, we can see the capacity increasing effect comparing to traditional thesis.

For the optimal case, we can see the improvement of 133~217%. For the suboptimal case I, we can see the improvement of 125~213%. For the suboptimal case II, we can see the improvement of 123~190%.

If we check the impact for the total pilot symbols, there is the tradeoff relation between the interference and bandwidth per user. For the example, if the total pilot symbols are greater, the total users are greater and bandwidth per user is smaller.



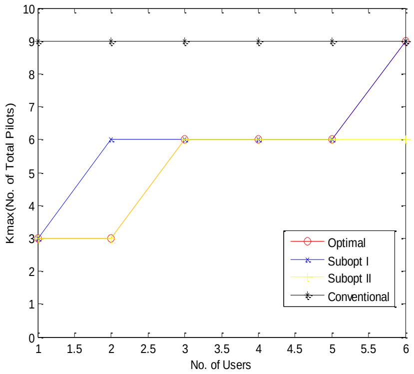




Figure 1. Capacity Gain for number of users  Figure 2. Total pilot symbols for number of users

According to figure 2, the optimal pilot symbols for number of user can be changed for selecting pilot in $\Omega_J$.

For the conventional case, we need the fixed pilots. The other hands, the optimal, the suboptimal case I and the suboptimal case II need fewer pilots than the conventional case.

## 4 Conclusion

In this paper, we propose to allocate optimally pilot for the total pilot symbols and pilot assignment. We can see the improvement of the total capacity.

Though the infinite antennas are impossible, it values we calculate theoretically the maximum capacity in the massive MIMO when user is changed.

## References

[1] T. L. Marzetta et al., “Beyond LTE: Hundreds of Base Station Antennas!”, 2010 IEEE Communication Theory Workshop.

[2] T. L. Marzetta, “Noncooperative wireless with unlimited numbers of base station antennas,” IEEE Trans. Wireless Commun., vol. 9, no. 11, pp. 3590 - 3600, Nov. 2010. http://dx.doi.org/10.1109/twc.2010.092810.091092

[3] H. Q. Ngo et al., “Uplink power efficiency of multiuser MIMO with very large antenna array,” in Proc. Of Allerton Conference on Communication, Control, and Computing, Sept. 2011. http://dx.doi.org/10.1109/allerton.2011.6120314

[4] F. Rusek et al., “Scailling up MIMO: Opportunities and challenges with very large arrays,” IEEE Signal Proces. Mag., to appear, 2012.

**Received: October 1, 2014; Published: December 2, 2014**