Contemporary Engineering Sciences, Vol. 7, 2014, no. 28, 1565 - 1578
HIKARI Ltd, www.m-hikari.com
http://dx.doi.org/10.12988/ces.2014.4858

An Explicit Fuzzy Observer Design for a Class of Takagi-Sugeno Descriptor Systems

Ilham Hmaiddouch

ECPI, Departement GE, ENSEM
Univ. Hassan II Casablanca
B.P 8118, Oasis, Casablanca, Morocco

Boutayna Bentahra

ECPI, Departement GE, ENSEM
Univ. Hassan II Casablanca
B.P 8118, Oasis, Casablanca, Morocco

Abdellatif El Assoudi

ECPI, Departement GE, ENSEM
Univ. Hassan II Casablanca
B.P 8118, Oasis, Casablanca, Morocco

Jalal Soulami

ECPI, Departement GE, ENSEM
Univ. Hassan II Casablanca
B.P 8118, Oasis, Casablanca, Morocco

El Hassane El Yaagoubi

ECPI, Departement GE, ENSEM
Univ. Hassan II Casablanca
B.P 8118, Oasis, Casablanca, Morocco

Abstract

In [1] the authors proposed a study concerning the design of explicit state observer for a class of nonlinear descriptor systems described by Takagi-Sugeno (T-S) model with measurable premise variables. In this paper, the aim is to extend this result to a class of T-S descriptor systems when the premise variables are not measurables. This new approach is based on the singular value decomposition. The convergence of the state estimation error is studied using the Lyapunov theory and the stability conditions are given in terms of Linear Matrix Inequalities (LMIs). Finally, numerical simulations are given in order to highlight the performance of the proposed estimator.

Keywords: T-S descriptor system, fuzzy observer, unmeasurable premise variables, Linear Matrix Inequality (LMI), Singular Value Decomposition

1 Introduction

It is well known that many physical systems are naturally modeled as systems of differential and algebraic equations such as chemical, electrical and mechanical engineering systems [2], [3], [4]. These systems are variously called descriptor systems, singular systems, implicit, or differential algebraic equations (DAEs). This formulation includes both dynamic and static relations. Consequently this formalism is much more general than the usual one and can model physical constraints or impulsive behavior due to an improper part of the system.

Based on the T-S fuzzy model (see [5], [6]), the nonlinear observer synthesis and its application for dynamical systems has received a great deal of attention over the two last decades. More precisely, the problem of designing a fuzzy observer for differential nonlinear systems described by T-S fuzzy models with measurable or unmeasurables premise variables and its application is widely studied in the literature [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17]. For nonlinear descriptor systems described by T-S descriptor models, the problem of fuzzy observer design has been widely investigated see for instance [18], [19], [20], [21], [22], [23]. The numerical simulation of such descriptor models usually combines an ODE numerical method together with an optimization algorithm.

Based on the singular value decomposition approach, the aim of this paper is to give a fuzzy observer design to a class of T-S descriptor systems with unmeasurable premise variables permitting to estimate the unknown state without the use of an optimization algorithm. In other words, an explicit fuzzy observer design for descriptor nonlinear systems with unmeasurable premise variables is considered.

The rest of the paper is organized as follows. The class of studied systems is defined in section 2 and the main result about fuzzy observer design for T-S descriptor systems with unmeasurable premise variables is detailed in section 3. Section 4 is devoted to a numerical example to demonstrate the utility of the proposed approach.

2 Takagi-sugeno descriptor systems

Consider the following general form of descriptor nonlinear systems:

$$\begin{cases} E\dot{x}(t) &= f(x(t)) + g(x(t))u(t) \\ y(t) &= h(x(t)) \end{cases} \tag{1}$$

where $x \in \mathbf{R}^n$ is the state variable, $u \in \mathbf{R}^m$ is the control input, $y \in \mathbf{R}^p$ is the measured output. f, g and h are nonlinear functions. $E \in \mathbf{R}^{n\times n}$ is constant matrix with $rank(E) = r$.

In this paper, the class of T-S descriptor systems that we consider, represents descriptor nonlinear systems (1), takes the following form:

$$\begin{cases} E\dot{x}(t) &= \sum_{i=1}^{q} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) &= Cx(t) \end{cases} \tag{2}$$

where $A_i \in \mathbf{R}^{n\times n}$, $B_i \in \mathbf{R}^{n\times m}$, $C \in \mathbf{R}^{p\times n}$ are real known constant matrices. $\xi(t)$ represent the premise variable. The $\mu_i(\xi(t))$ are the weighting functions that ensure the transition between the contribution of each sub model:

$$\begin{cases} E\dot{x}(t) &= A_i x(t) + B_i u(t) \\ y(t) &= Cx(t) \end{cases} \tag{3}$$

They depend on measurable or unmeasurable premise variables (state of the system), and have the following properties:

$$\begin{cases} 0 \leq \mu_i(\xi(t)) \leq 1 \\ \sum_{i=1}^{q} \mu_i(\xi(t)) = 1 \end{cases} \tag{4}$$

Recall that to obtain a Takagi-Sugeno model of the system (1) we use the procedure of fuzzy model construction given in [6].

In the sequel, we assume the following hypotheses (for each sub-model (3), $i = 1, \ \ldots, \ q$):

H1) $(E,\ A_i)$ is regular, i.e. $det(sE - A_i) \neq 0\ \forall s \in \mathbf{C}$

H2) All sub-models are impulse observable, i.e.

$$rank(\begin{pmatrix} E & A_i \\ 0 & E \\ 0 & C \end{pmatrix}) \;=\; n + rank(E)$$

H3) All sub-models are detectable, i.e.

$$rank(\begin{pmatrix} sE - A_i \\ C \end{pmatrix}) = n \quad \forall s \in \mathbf{C}$$

H4) $rank(\begin{pmatrix} E \\ C \end{pmatrix}) = n.$

3 Fuzzy observer design

Based on the singular value decomposition, our aim in this section is to design an explicit fuzzy observer for nonlinear descriptor system (2) with unmeasurable premise variables. For this object, noticing that under hypothesis H_4, there exist a non-singular matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that:

$$\begin{cases} aE + bC &= \mathtt{I}_n \\ cE + dC &= \mathtt{0} \end{cases} \tag{5}$$

Our candidate fuzzy observer witch is not in descriptor form is given by the following equations:

$$\begin{cases} \dot{z}(t) &= \sum_{i=1}^{q} \mu_i(\hat{\xi}(t))(N_i z(t) + L_{1i} y(t) + L_{2i} y(t) + G_i u(t)) \\ \hat{x}(t) &= z(t) + by(t) + Kdy(t) \end{cases} \tag{6}$$

where $\hat{x}(t)$ denote the estimated state vector, N_i, L_{1i}, L_{2i}, G_i and K are unknown matrices of appropriate dimensions, which must be determined such that $\hat{x}(t)$ will asymptotically converge to $x(t)$.
Denoting the state estimation error by:

$$e(t) \;=\; x(t) - \hat{x}(t) \tag{7}$$

Then by substituting (2) and (6) into (7), we obtain:

$$e(t) \;=\; (\mathtt{I}_n - bC - KdC)x(t) - z(t) \tag{8}$$

From (5), we have:

$$e(t) \;=\; (a + Kc)Ex(t) - z(t) \tag{9}$$

Then, the dynamic of this observer error is:

$$\dot{e}(t) = (a+Kc)E\dot{x}(t) - \dot{z}(t) \tag{10}$$

By substituting (2) and (6) into (10), we obtain:

$$\begin{aligned}\dot{e}(t) = & \sum_{i=1}^{q}\mu_i(\xi(t))(a+Kc)(A_i x(t)+B_i u(t)) \\ & -\sum_{i=1}^{q}\mu_i(\hat{\xi}(t))(N_i z(t)+L_{1i}y(t)+L_{2i}y(t)+G_i u(t))\end{aligned} \tag{11}$$

Using (9), equation (11) can be written as:

$$\begin{aligned}\dot{e}(t) = & \sum_{i=1}^{q}\mu_i(\xi(t))(a+Kc)(A_i x(t)+B_i u(t))+\sum_{i=1}^{q}\mu_i(\hat{\xi}(t))N_i e(t) \\ & -\sum_{i=1}^{q}\mu_i(\hat{\xi}(t))[(N_i(a+Kc)E+L_{1i}C+L_{2i}C)x(t)+G_i u(t)]\end{aligned} \tag{12}$$

Provided the matrices G_i, K, L_{1i}, L_{2i} and N_i satisfy:

$$\begin{cases} N_i(a+Kc)E+L_{1i}C+L_{2i}C=(a+Kc)A_i \\ G_i=(a+Kc)B_i \end{cases} \tag{13}$$

Then, from (5) and (13), we have:

$$N_i = (a+Kc)A_i - L_{2i}C + (N_i(b+Kd)-L_{1i})C \tag{14}$$

Take:

$$L_{1i} = N_i(b+Kd) \tag{15}$$

Then:

$$N_i = (a+Kc)A_i - L_{2i}C \tag{16}$$

It follows system (12) is equivalent to:

$$\begin{aligned}\dot{e}(t) = & \sum_{i=1}^{q}(\mu_i(\xi(t))-\mu_i(\hat{\xi}(t)))(a+Kc)(A_i x(t)+B_i u(t)) \\ & +\sum_{i=1}^{q}\mu_i(\hat{\xi}(t))N_i e(t)\end{aligned} \tag{17}$$

Using the fact that:

$$\begin{cases} \sum_{i=1}^{q}(\mu_i(\xi(t))-\mu_i(\hat{\xi}(t))A_i = \sum_{i,j=1}^{q}\mu_i(\xi(t))\mu_j(\hat{\xi}(t))(A_i-A_j) \\ \sum_{i=1}^{q}(\mu_i(\xi(t))-\mu_i(\hat{\xi}(t))B_i = \sum_{i,j=1}^{q}\mu_i(\xi(t))\mu_j(\hat{\xi}(t))(B_i-B_j) \end{cases} \tag{18}$$

Then, the equation (17) becomes:

$$\begin{aligned}\dot{e}(t) &= \sum_{i,j=1}^{q} \mu_i(\xi(t))\mu_j(\hat{\xi}(t))(a+Kc)(\Delta A_{ij}x(t)+\Delta B_{ij}u(t)) \\ &\quad + \sum_{i=1}^{q} \mu_i(\hat{\xi}(t))N_i e(t)\end{aligned} \tag{19}$$

where $\Delta A_{ij} = A_i - A_j$ and $\Delta B_{ij} = B_i - B_j$.
Multiplying by $\sum_{i=1}^{q} \mu_i(\xi(t))$, equation (19) can be reduced to the equation:

$$\dot{e}(t) = \sum_{i,j=1}^{q} \mu_i(\xi(t))\mu_j(\hat{\xi}(t))[N_j e(t) + \Phi_{ij}x(t) + \Gamma_{ij}u(t))] \tag{20}$$

where

$$\begin{cases} \Phi_{ij} &= (a+Kc)\Delta A_{ij} \\ \Gamma_{ij} &= (a+Kc)\Delta B_{ij} \\ i,j &\in \{1,\ldots,q\} \end{cases} \tag{21}$$

Let $\bar{e}(t) = [e^T(t)\ x^T(t)]^T$, we have:

$$\bar{E}\dot{\bar{e}}(t) = \sum_{i,j=1}^{q} \mu_i(\xi(t))\mu_j(\hat{\xi}(t))[\mathcal{M}_{ij}\bar{e}(t) + \mathcal{N}_{ij}u(t)] \tag{22}$$

where

$$\begin{cases} \bar{E} = \begin{pmatrix} \mathtt{I}_n & \mathtt{0}_n \\ \mathtt{0}_n & E \end{pmatrix} \\ \mathcal{M}_{ij} = \begin{pmatrix} N_j & \Phi_{ij} \\ \mathtt{0}_n & A_i \end{pmatrix} \\ \mathcal{N}_{ij} = \begin{pmatrix} \Gamma_{ij} \\ B_i \end{pmatrix} \end{cases} \tag{23}$$

Theorem 3.1 *: There exists an observer (6) for (2) if the hypotheses H1, H2, H3 and H4 hold and there exists symmetric positive matrices P_1, P_2, matrices Q and W_j for $j = 1,\ldots,q$ verifying the following LMI:*

$$\Sigma_{ij} = \begin{pmatrix} m_{11} & * & * \\ m_{21} & m_{22} & * \\ m_{31} & m_{32} & m_{33} \end{pmatrix} < 0 \qquad \forall\, i,j \in \{1,\ldots,q\} \tag{24}$$

where:

$$\left\{\begin{array}{lll} m_{11} & = & (aA_j)^T P_1 + P_1(aA_j) + (cA_j)^T Q^T + Q(cA_j) - C^T W_j^T - W_j C \\ m_{22} & = & A_i^T P_2 + P_2 A_i \\ m_{33} & = & 0_n \\ m_{21} & = & (a\Delta A_{ij})^T P_1 + (c\Delta A_{ij})^T Q^T \\ m_{31} & = & (a\Delta B_{ij})^T P_1 + (c\Delta B_{ij})^T Q^T \\ m_{32} & = & B_i^T P_2 \end{array}\right. \tag{25}$$

The observer gains N_j, L_{1j}, L_{2j}, G_j and K are given by:

$$\left\{\begin{array}{lll} N_j & = & (a + P_1^{-1}Qc)A_j - P_1^{-1}W_j C \\ L_{1j} & = & ((a + P_1^{-1}Qc)A_j - P_1^{-1}W_j C)(b + P_1^{-1}Qd) \\ L_{2j} & = & P_1^{-1}W_j \\ G_j & = & (a + P_1^{-1}Qc)B_j \\ K & = & P_1^{-1}Q \end{array}\right. \tag{26}$$

where a, b, c and d are such that equation (5) is satisfied.

Proof of theorem 3.1 : To prove the convergence of the estimation error toward zero, let us consider the following quadratic Lyapunov function:

$$V(t) = \bar{e}^T(t)\bar{E}^T P \bar{e}(t) , \qquad P = P^T > 0 \tag{27}$$

with

$$\bar{E}^T P = P\bar{E} \geq 0 \tag{28}$$

and

$$P = \begin{pmatrix} P_1 & 0_n \\ 0_n & P_2 \end{pmatrix} \tag{29}$$

Estimation error convergence is ensured if the following condition is guaranteed:

$$\dot{V}(t) = \dot{\bar{e}}^T(t)\bar{E}^T P \bar{e}(t) + \bar{e}^T(t)\bar{E}^T P \dot{\bar{e}}(t) < 0 \tag{30}$$

By using (22) and (28), the condition (30) can be written as:

$$\begin{array}{lll} \dot{V}(t) & = & \sum_{i,j=1}^{q} \mu_i(\xi(t))\mu_j(\hat{\xi}(t))[\mathcal{M}_{ij}\bar{e}(t) + \mathcal{N}_{ij}u(t)]^T P\bar{e}(t) \\ & & + \sum_{i,j=1}^{q} \mu_i(\xi(t))\mu_j(\hat{\xi}(t))\bar{e}^T(t)P[\mathcal{M}_{ij}\bar{e}(t) + \mathcal{N}_{ij}u(t)] \\ & < & 0 \end{array} \tag{31}$$

Then:

$$\begin{array}{rcl} \dot{V}(t) & = & \sum_{i,j=1}^{q} \mu_i(\xi(t))\mu_j(\hat{\xi}(t))[\bar{e}^T(t)(\mathcal{M}_{ij}^T P + P\mathcal{M}_{ij})\bar{e}(t) \\ & & +u^T(t)\mathcal{N}_{ij}^T P\bar{e}(t) + \bar{e}^T(t)P\mathcal{N}_{ij}u(t)] \\ & < & 0 \end{array} \tag{32}$$

This implicates:

$$\dot{V}(t) \quad = \quad \sum_{i,j=1}^{q} \mu_i(\xi(t))\mu_j(\hat{\xi}(t)) \begin{pmatrix} \bar{e}(t) \\ u(t) \end{pmatrix}^T \Sigma_{ij} \begin{pmatrix} \bar{e}(t) \\ u(t) \end{pmatrix} \quad < \quad 0 \tag{33}$$

where

$$\Sigma_{ij} \quad = \quad \begin{pmatrix} \mathcal{M}_{ij}^T P + P\mathcal{M}_{ij} & P\mathcal{N}_{ij} \\ \mathcal{N}_{ij}^T P & 0 \end{pmatrix} \qquad \forall\, i,j \in \{1,\ldots,q\} \tag{34}$$

The decrease of $V(t)$ is guaranteed if:

$$\Sigma_{ij} \quad < \quad 0 \qquad \forall\, i,j \;\in\; \{1,\ldots,q\} \tag{35}$$

Then, the use of the changes of variables:

$$\left\{ \begin{array}{rcl} Q & = & P_1 K \\ W_j & = & P_1 L_{2j} \end{array} \right. \tag{36}$$

and from (16), (21), (23) and (29) we establish the LMIs conditions (24) of theorem 3.1.

4 Numerical illustration

To demonstrate the effectiveness and applicability of the proposed approach of the observer synthesis, we consider the following Takagi-Sugeno descriptor model given in [23]:

$$\left\{ \begin{array}{rcl} E\dot{x}(t) & = & \sum_{i=1}^{4} h_i(x(t))(A_i x(t) + Bu(t)) \\ y(t) & = & Cx(t) \end{array} \right. \tag{37}$$

where

$$A_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -2 & -3 & 0 & 1 \\ 1 & 1 & -2 & 0 \\ -1 & -1 & 0 & -5 \end{pmatrix},\; A_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -2 & -3 & 0 & 1 \\ 1 & 1 & -2 & 0 \\ -1 & -1 & 0 & -1 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -2 & -3 & 0 & -3 \\ 1 & 1 & -2 & 0 \\ -1 & -1 & 0 & -5 \end{pmatrix}, \; A_4 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -2 & -3 & 0 & -3 \\ 1 & 1 & -2 & 0 \\ -1 & -1 & 0 & -1 \end{pmatrix}$$

$$E = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \; B = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \; C = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

The membership functions are given by:

$$\begin{aligned} h_1(x(t)) &= \frac{((1-x_4^2)-1)((x_4^2-5)+5)}{-16} \\ h_2(x(t)) &= \frac{((1-x_4^2)-1)(1+(x_4^2-5))}{16} \\ h_3(x(t)) &= \frac{(3+(1-x_4^2))((x_4^2-5)+5)}{16} \\ h_4(x(t)) &= \frac{(3+(1-x_4^2))(1+(x_4^2-5))}{-16} \end{aligned}$$

Noting that, in the present example, we assume that $y_1(t) = x_1(t) + x_3(t)$ and $y_2(t) = x_2(t) + x_4(t)$ can be measured. In other words, $x_1(t)$, $x_2(t)$, $x_3(t)$ and $x_4(t)$ are estimated using the proposed fuzzy observer.
The fuzzy observer for system (37) is given by:

$$\left\{ \begin{aligned} \dot{z}(t) &= \sum_{i=1}^{4} \mu_i(\hat{\xi}(t))(N_i z(t) + L_{1i} y(t) + L_{2i} y(t) + G_i u(t)) \\ \hat{x}(t) &= z(t) + by(t) + Kdy(t) \end{aligned} \right. \tag{38}$$

where b and d satisfying the equation (5) are as follows:

$$b = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \; d = 10^{-15} \begin{pmatrix} 0 & 0.1110 \\ 0 & 0.0555 \end{pmatrix}$$

We use the Matlab Control Toolbox to solve the LMI in (24), we obtain the parameters as follows:

$$N_1 = 10^5 \begin{pmatrix} -0.0000 & 0.0000 & -0.0000 & -0.0000 \\ -8.4444 & -4.1141 & 6.0071 & 0.7031 \\ 0.0000 & 0.0000 & -0.0000 & 0.0000 \\ 8.4444 & 4.1141 & -6.0071 & -0.7031 \end{pmatrix}$$

$$N_2 = 10^5 \begin{pmatrix} -0.0000 & 0.0000 & -0.0000 & -0.0000 \\ -8.4444 & -4.1141 & 6.0071 & 0.7031 \\ 0.0000 & 0.0000 & -0.0000 & 0.0000 \\ 8.4444 & 4.1141 & -6.0071 & -0.7032 \end{pmatrix}$$

$$N_3 = 10^5 \begin{pmatrix} -0.0000 & 0.0000 & -0.0000 & -0.0000 \\ -8.4444 & -4.1140 & 6.0071 & 0.7031 \\ 0.0000 & 0.0000 & -0.0000 & 0.0000 \\ 8.4444 & 4.1140 & -6.0071 & -0.7031 \end{pmatrix}$$

$$N_4 = 10^5 \begin{pmatrix} -0.0000 & 0.0000 & -0.0000 & -0.0000 \\ -8.4444 & -4.1140 & 6.0071 & 0.7031 \\ 0.0000 & 0.0000 & -0.0000 & 0.0000 \\ 8.4444 & 4.1140 & -6.0071 & -0.7031 \end{pmatrix}$$

$$L_{11} = 10^5 \begin{pmatrix} -0.0000 & -0.0000 \\ 6.0071 & 0.7031 \\ -0.0000 & 0.0000 \\ -6.0071 & -0.7031 \end{pmatrix}, \ L_{12} = 10^5 \begin{pmatrix} -0.0000 & -0.0000 \\ 6.0071 & 0.7031 \\ -0.0000 & 0.0000 \\ -6.0071 & -0.7032 \end{pmatrix}$$

$$L_{13} = 10^5 \begin{pmatrix} -0.0000 & -0.0000 \\ 6.0071 & 0.7031 \\ -0.0000 & 0.0000 \\ -6.0071 & -0.7031 \end{pmatrix}, \ L_{14} = 10^5 \begin{pmatrix} -0.0000 & -0.0000 \\ 6.0071 & 0.7031 \\ -0.0000 & 0.0000 \\ -6.0071 & -0.7031 \end{pmatrix}$$

$$L_{21} = 10^5 \begin{pmatrix} 0.0000 & 0.0000 \\ 3.6272 & -0.7031 \\ -0.0000 & -0.0000 \\ -3.6272 & 0.7031 \end{pmatrix}, \ L_{22} = 10^5 \begin{pmatrix} 0.0000 & 0.0000 \\ 3.6272 & -0.7031 \\ -0.0000 & -0.0000 \\ -3.6272 & 0.7031 \end{pmatrix}$$

$$L_{23} = 10^5 \begin{pmatrix} 0.0000 & 0.0000 \\ 3.6272 & -0.7032 \\ -0.0000 & -0.0000 \\ -3.6272 & 0.7032 \end{pmatrix}, \ L_{24} = 10^5 \begin{pmatrix} 0.0000 & 0.0000 \\ 3.6272 & -0.7032 \\ -0.0000 & -0.0000 \\ -3.6272 & 0.7032 \end{pmatrix}$$

$$G_1 = \begin{pmatrix} -0.0000 \\ 1.0000 \\ -0.0000 \\ -1.0000 \end{pmatrix}, \ G_2 = \begin{pmatrix} -0.0000 \\ 1.0000 \\ -0.0000 \\ -1.0000 \end{pmatrix}, \ G_3 = \begin{pmatrix} -0.0000 \\ 1.0000 \\ -0.0000 \\ -1.0000 \end{pmatrix}$$

$$G_4 = \begin{pmatrix} -0.0000 \\ 1.0000 \\ -0.0000 \\ -1.0000 \end{pmatrix}, \ K = 10^5 \begin{pmatrix} 0.0000 & -0.0000 \\ -3.4063 & 3.4063 \\ 0.0000 & -0.0000 \\ 3.4063 & -3.4063 \end{pmatrix}$$

The initial conditions of the T-S model (37) are: $x_0 = \begin{bmatrix} 1 & 1 & 1 & -1 \end{bmatrix}^T$.
The initial conditions of the fuzzy observer (32) are: $\hat{x}_0 = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 1 \end{bmatrix}^T$.

The simulation results are given in figure 1 where the dotted lines denote the state variables estimated by the fuzzy observer (37). This simulation shows that the estimation states converge to their corresponding state variables.

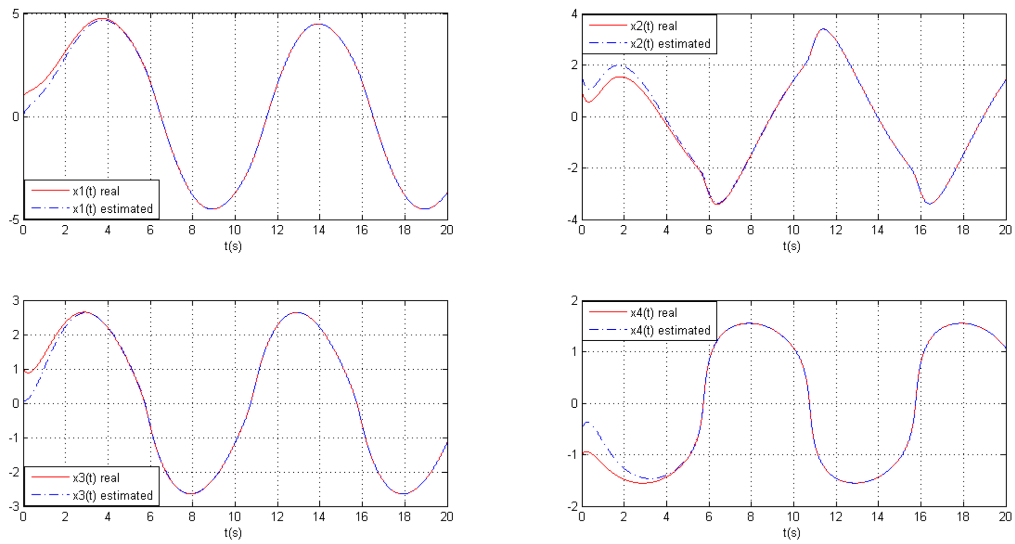

Figure 1: States and the estimation performance

5 Conclusion

Based on the singular value decomposition method and solving a system of LMIs for the determination of the observer parameters, an explicit fuzzy observer design for a class of Takagi-Sugeno descriptor systems with unmeasurable premise variables is proposed in this paper. This work is an extension of the result developed in [1] which extend the method of the observer developed for linear systems in [18] to nonlinear Takagi-Sugeno systems. To illustrate the proposed methodology, a numerical example of a Takagi-Sugeno fuzzy model is considered. The effectiveness of the proposed fuzzy observer used for the on-line estimation of unknown states in proposed model is verified by numerical simulation.

References

[1] M. Essabre , J. Soulami, and E. Elyaagoubi, Design of State Observer for a Class of Non linear Singular Systems Described by Takagi-Sugeno Model, Contemporary Engineering Sciences, vol. 6, 2013, no. 3, 99-109, HIKARI Ltd.

[2] S. L. Campbell. Singular Systems of Differential Equations,London, UK: Pitman, 1980.

[3] E. L. Lewis. A survey of linear singular systems. Circuits,Systems and Signal Processing, vol. 5, no. 1, pp. 3-36, 1986. http://dx.doi.org/10.1007/bf01600184

[4] L. Dai. Singular Control Systems, Berlin, Germany: Springer, 1989. http://dx.doi.org/10.1007/bfb0002475

[5] T. Takagi, M. Sugeno, Fuzzy identification of systems and its application to modeling and control. IEEE Trans. Syst., Man and Cyber, Vol.SMC-15, pp. 116-132, 1985. http://dx.doi.org/10.1109/tsmc.1985.6313399

[6] T. Taniguchi, K. Tanaka, H. Ohtake and H. Wang, Model construction, rule reduction, and robust compensation for generalized form of Takagi-Sugeno fuzzy systems, IEEE Transactions on Fuzzy Systems, vol. 9, No.4, pp. 525-538, August 2001. http://dx.doi.org/10.1109/91.940966

[7] K. Tanaka and M. Sano, On the Concept of Fuzzy Regulators and Fuzzy Observers, Proceedings of Third IEEE International Conference on Fuzzy Systems, Vol. 2, June 1994, pp. 767-772.

[8] K. Tanaka and H. O. Wang, Fuzzy Regulators and Fuzzy Observers: A Linear Matrix Inequality Approach, 36th IEEE Conference on Decision and Control, Vol. 2, San Diego, 1997, pp. 1315-1320. http://dx.doi.org/10.1109/cdc.1997.657640

[9] K. Tanaka, T. Ikeda, and H. O. Wang, Fuzzy Regulators and Fuzzy Observers: Relaxed Stability Conditions and LMI-Based Designs, IEEE Transactions on Fuzzy Systems, vol. 6, No.2, pp. 250-265, May 1998. http://dx.doi.org/10.1109/91.669023

[10] T. Johansen, R. Shorten, R. Murray-Smith. On the interpretation and identification of dynamic Takagi-Sugeno fuzzy models. In IEEE Transaction on Fuzzy Systems, Vol. 8, 297-313, 2000. http://dx.doi.org/10.1109/91.855918

[11] P. Bergsten and R. Palm, Thau-Luenberger observers for TS fuzzy systems, in 9th IEEE International Conference on Fuzzy Systems, FUZZ IEEE 2000, San Antonio, TX, USA, 2000. http://dx.doi.org/10.1109/fuzzy.2000.839073

[12] P. Bergsten, R. Palm, and D. Driankov, Fuzzy observers, in IEEE International Fuzzy Systems Conference, Melbourne Australia, 2001. http://dx.doi.org/10.1109/fuzz.2001.1009051

[13] K. Tanaka, and H. O. Wang, Fuzzy control systems design and analysis: A Linear Matrix Inequality Approach. John Wiley & Sons; 2001.

[14] P. Bergsten, R. Palm, and D. Driankov, Observers for Takagi-Sugeno fuzzy systems, IEEE Transactions on Systems, Man, and Cybernetics - Part B: Cybernetics, vol. 32, no. 1, pp. 114-121, 2002. http://dx.doi.org/10.1109/3477.979966

[15] D. Ichalal, B. Marx, J. Ragot, and D. Maquin, Design of observers for Takagi-Sugeno systems with immeasurable premise variables: an $\mathcal{L}_2$ approach. Proceedings of the 17th world congress, the international federation of automatic control. Seoul, Korea, July 6-11, 2008.

[16] D. Ichalal, B. Marx, J. Ragot, and D. Maquin, State and unknown input estimation for nonlinear systems described by Takagi-Sugeno models with unmeasurable premise variables. 17th Mediterranean conference on control and automation, 2009. Med'2009. June 24-26, 2009. http://dx.doi.org/10.1109/med.2009.5164542

[17] A.M. Nagy, G. Mourot, , J. Ragot, G. Schutz, S. Gill. Model structure simplification of a biological reactor. In 15^{th} IFAC Symposium on System Identification, SYSID 2009, Saint-Malo, France, 6-8 July 2009.

[18] M. Darouach and M. Boutayeb, Design of observers for descriptor systems, IEEE Transactions on Automatic Control, vol. 40, pp. 1323-1327, 1995. http://dx.doi.org/10.1109/9.400467

[19] T. Taniguchi, K. Tanaka, and H. 0. Wang, Fuzzy Descriptor Systems and Nonlinear Model Following Control, IEEE Transactions on Fuzzy Systems, vol. 8, No.4, pp. 442-452, August 2000. http://dx.doi.org/10.1109/91.868950

[20] D Koenig and S. Mammar, Design of a proportional integral observer for unknown input descriptor systems, IEEE Transactions on Automatic Control, vol. 47, pp. 2057-2063, 2002. http://dx.doi.org/10.1109/tac.2002.805675

[21] C. Lin, Q. Wang, and T. Lee, Stability and Stabilization of a Class of Fuzzy Time-Delay Descriptor Systems, IEEE Transactions on Fuzzy Systems, vol. 14, No.4, pp. 542-551, August 2006. http://dx.doi.org/10.1109/tfuzz.2006.876738

[22] Chunyu Yang, Qingling Zhang and Tianyou Chai , Observer design for a class of nonlinear descriptor systems, Joint 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference Shanghai, P.R. China, December 16-18, 2009. http://dx.doi.org/10.1109/cdc.2009.5400212

[23] H. Hamdi, M. Rodrigues, Ch. Mechmech and N. Benhadj Braiek, Observer based Fault Tolerant Control for Takagi-Sugeno Nonlinear Descriptor systems, International Conference on Control, Engineering & Information Technology (CEIT'13). Proceedings Engineering & Technology, vol. 1, pp 52-57, 2013.

Received: August 17, 2014; Published: November 24, 2014