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Limitations on the Computation of Electric Field in Rectangular Waveguide Based Microwave Components Using Modal Expansion

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Abstract

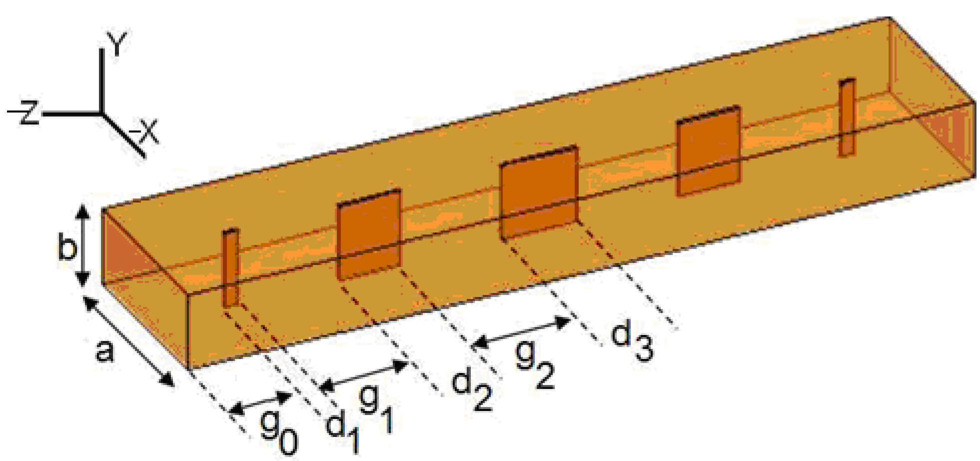
A critical examination of the modal expansion method to study discontinuities in waveguide based microwave components is made. We consider the case of E-plane metal insert filters as part of our study to assess the places where maximum electric fields occur so as to cause damage by multipaction in a high vacuum environment. Comparisons are made with results obtained using commercially available soft wares.

I. INTRODUCTION

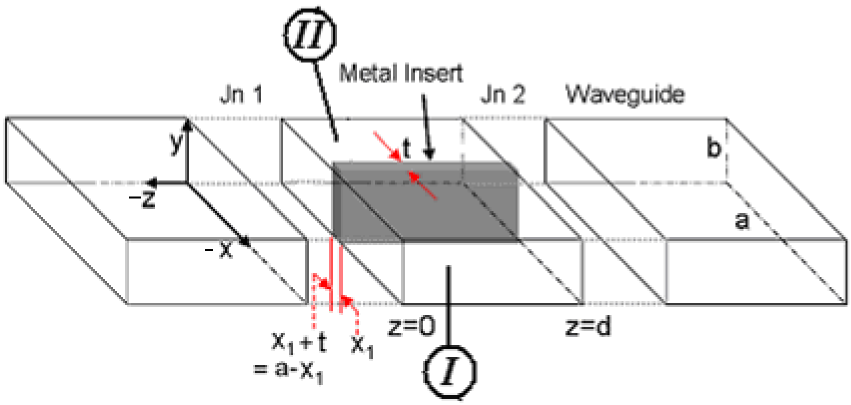
Metal inserts or septums in rectangular waveguides as shown in figure 1 are used by Indian Space Research Organization (ISRO) in their microwave filters fitted inside satellites. In the environment of space these components get damaged due to a problem called multipaction which is impact of electrons on the metallic walls of the waveguide in the high electric field existing inside. Thus it is necessary to estimate the maximum electric field and its place of occurrence in these filters for a given power level. In this short study we try to estimate the electric field distribution when a single such septum of a given thickness and length is present inside a rectangular waveguide when excited by an incident wave of unit amplitude by the method of modal expansion tried out in the past with slightly differing formulations [1], [2]. The purpose here is to compare the results obtained by this method with those of HFSS (high frequency structure simulator of Ansoft) which uses the finite element method. The fields at $z = 0$ in fig. 1 are plotted as a function of x from 0 to a for different levels of accuracy and are presented in section III along with the plots obtained from HFSS with the maximum possible accuracy. The conclusions are that the modal expansion method is of limited use.

II. Formulation of the Modal Expansion Method

The structure in fig. 1 has no variations along the $y-$axis and so it is sufficient to consider TE_{m0} modes only [3]. In the figure the $x-$axis shown might create some confusion and hence we clarify that whenever x appears in the equations below it actually stands for $x+a$ of the figure. Thus according to the text the value of x at which the septum is located is $x_1 < x < x_1 + t$ where $x_1 + t = a - x_1$. The electric field in the region of the main waveguide that is in the $z < 0$ region



(a)



(b)

Fig. 1 Waveguide metal-insert filter (a) Geometry (b) Exploded view of a section containing one metal-insert (thickness, t and length, d) bifurcating the guide into two regions I and II. The septums are all placed at the centre of the broad wall.

when an incident wave of amplitude π/a in the dominant mode approaches from $z=-\infty$ is given by

$$E_y = \frac{\pi}{a} sin\left(\frac{\pi x}{a}\right) e^{-jk_{z10}z} + \sum_{m=1}^{M} B_{m0}\left(\frac{2m-1}{a}\pi\right) sin\left(\frac{2m-1}{a}\pi x\right) e^{\gamma_{zm0}z}, \qquad \text{..... (1)}$$

where $k_{z10} = \sqrt{\omega^2\mu_0\varepsilon_0 - \left(\pi/a\right)^2}$, $\gamma_{zm0} = \sqrt{\left(\frac{2m-1}{a}\pi\right)^2 - \omega^2\mu_0\varepsilon_0}$ for $m>1$ and $\gamma_{z10} = jk_{z10}$.

Here the usual notation ω for angular frequency and μ_0, ε_0 for free space permeability and permittivity respectively have been used. $B_{m0}\left(\frac{2m-1}{a}\pi\right)$ are the amplitudes of the scattered (reflected) wave whose magnitude and phase has to be determined. The reason for selecting odd modes only is the symmetry of the problem about the centre of the broad wall. The tangential component of the magnetic field in relation to the face of the septum at $z=0$ that is a surface of dimension $t \times b$ as seen in fig. 1 is expressed as

$$j\omega\mu_0 H_x = -jk_{z10}\left(\frac{\pi}{a}\right) sin\left(\frac{\pi x}{a}\right) e^{-jk_{z10}z} + \sum_{m=1}^{M} \gamma_{zm0} B_{m0}\left(\frac{2m-1}{a}\pi\right) sin\left(\frac{2m-1}{a}\pi x\right) e^{\gamma_{zm0}z}. \qquad \text{.....(2)}$$

One must note that in both equations (1) and (2) the upper limit of summation M should be ideally infinity but for practical calculations the summation must be suitably truncated as dictated by the need for accuracy. The electric and the magnetic fields in the region of the bifurcated waveguide that is in the region $0<z<d$ in fig.1 are respectively expressed with x_1 defined in the same figure as

$$E_y = \sum_{m=1}^{M}\left[A'_{m0}\, e^{-\gamma'_{zm0}\, z} + B'_{m0}\, e^{\gamma'_{zm0}(z-d)}\right]\left(\frac{m\pi}{x_1}\right)\left[\begin{array}{l}\{\theta(x)-\theta(x-x_1)\}\sin\dfrac{m\pi x}{x_1} - (-1)^m \times \\ \times\{\theta(x-x_1-t)-\theta(x-a)\}\sin\dfrac{m\pi(x-x_1-t)}{x_1}\end{array}\right] \quad ...(3)$$

$$j\omega\mu_0 H_x = \sum_{m=1}^{M}\gamma'_{zm0}\left[-A'_{m0}\, e^{-\gamma'_{zm0} z} + B'_{m0}\, e^{\gamma'_{zm0}(z-d)}\right]\left(\frac{m\pi}{x_1}\right)\left[\begin{array}{l}\{\theta(x)-\theta(x-x_1)\}\sin\dfrac{m\pi x}{x_1} - (-1)^m \times \\ \left\{\begin{array}{l}\theta(x-x_1-t)\\ -\theta(x-a)\end{array}\right\}\sin\dfrac{m\pi(x-x_1-t)}{x_1}\end{array}\right] \quad (4)$$

where $\gamma'_{zm0} = \sqrt{\left(\frac{m}{x_1}\pi\right)^2 - \omega^2\mu_0\varepsilon_0}$, $\theta(x)$ is the unit step function defined as $\theta(x)=1$ for $x>1$ and $\theta(x)=0$ for $x<1$ and A'_{m0} and B'_{m0} multiplied by $m\pi/x_1$ are again the unknown scattered amplitudes. In addition to these the fields in the region $z>d$ are obtained as

$$E_y = \sum_{m=1}^{M} A_{m0}\left(\frac{2m-1}{a}\pi\right)\sin\left(\frac{2m-1}{a}\pi x\right)e^{-\gamma_{zm0}(z-d)} \quad .....(5)$$

$$j\omega\mu_0 H_x = \sum_{m=1}^{M} -\gamma_{zm0} A_{m0}\left(\frac{2m-1}{a}\pi\right)\sin\left(\frac{2m-1}{a}\pi x\right)e^{-\gamma_{zm0}(z-d)} \quad .....(6)$$

Now in order to determine the unknowns B_{m0}, A_{m0}, A'_{m0} and B'_{m0} by this technique of modal expansion one needs to equate the E_y component of the electric field on either side of both the junctions at $z=0$ and $z=d$. For the magnetic field component H_x however in addition to equating it on either sides of these junctions one needs to take into account the surface current densities (if the conductivity of metals are taken to be ideally infinite) on the face of the septum that is as already explained surfaces of dimensions $t\times b$ at $z=0$ and $z=d$. To make the point clear we write the equation at the $z=0$ surface only first for the electric field

$$\frac{\pi}{a}\sin\left(\frac{\pi x}{a}\right) + \sum_{m=1}^{M} B_{m0}\left(\frac{2m-1}{a}\pi\right)\sin\left(\frac{2m-1}{a}\pi x\right)$$
$$= \sum_{m=1}^{M}\left[A'_{m0} + B'_{m0}\, e^{-\gamma'_{zm0} d}\right]\left(\frac{m\pi}{x_1}\right)\left[\begin{array}{l}\{\theta(x)-\theta(x-x_1)\}\sin\dfrac{m\pi x}{x_1} - (-1)^m \times \\ \{\theta(x-x_1-t)-\theta(x-a)\}\sin\dfrac{m\pi(x-x_1-t)}{x_1}\end{array}\right] \quad .....(7)$$

and then for the magnetic field

$$- jk_{z10}\frac{\pi}{a}sin\left(\frac{\pi x}{a}\right)+\sum_{m=1}^{M}\gamma_{zm0}B_{m0}\left(\frac{2m-1}{a}\pi\right)sin\left(\frac{2m-1}{a}\pi x\right)$$

$$=\sum_{m=1}^{M}\gamma'_{zm0}\left[-A'_{m0}+B'_{m0}\,e^{-\gamma'_{zm0}d}\right]\left(\frac{m\pi}{x_1}\right)\left[\begin{array}{l}\{\theta(x)-\theta(x-x_1)\}sin\dfrac{m\pi x}{x_1}-(-1)^m\times\\ \{\theta(x-x_1-t)-\theta(x-a)\}sin\dfrac{m\pi(x-x_1-t)}{x_1}\end{array}\right]-j\omega\mu_0 J_y$$

.(8)

where J_y is a y-directed surface current density on the face of the septum at $z=0$.

III. METHOD OF SOLUTION AND RESULTS

The difficulty with this modal expansion technique is to obtain an estimate of the unknown quantity J_y in any meaningful way. It is easy to determine it by the integral equation formulation of the problem or by the finite element method but after applying these methods the modal expansion becomes trivial. So in order to calculate the fields by this (modal expansion) method we have to assume $J_y=0$ and it is a belief of the authors that earlier workers had done the same thing.

The first step in the solution of the problem is to write two more equations of the types of Eqs. (7) and (8) but now at the $z=d$ plane. These four equations can be converted into a $4M$ set of algebraic equations having $4M$ unknowns B_{m0}, A_{m0}, A'_{m0} and B'_{m0}. This is easily accomplished by multiplying each of the Eqs. (7), (8) and the two others at $z=d$ by $sin\left(\frac{2q-1}{a}\pi x\right)$ and then integrating from 0 to a. The electric field can now be calculated at any place using Eqs.(1), (3) and (5) once all the B_{m0}, A_{m0}, A'_{m0} and B'_{m0}s are determined by solving the $4M$ set of equations. We present the results for $M=300$, 400 and 500 the absolute value of the electric field amplitudes at $z=0$ as a function of x from 0 to $a=28.5$ mm in figures 2-4 respectively using Eq. (1). The value of t is 0.6 mm and the frequency of operation is 8.15 GHz. We took d in fig. 1(b) to be 2 mm. The incident electric field amplitude is normalized to unity. In figure 5 the corresponding calculation made using HFSS is presented. If one tries to increase the value of M to 600 for example in the modal expansion technique then the entire method fails if we start from without knowing J_y in Eq. (8). The expected value of E_y on the face of the septum is zero and this is most accurately predicted by HFSS as the black square shaped dot is closest to zero in fig. 5. In the method of modal expansion it cannot be brought down below a value of 0.5.

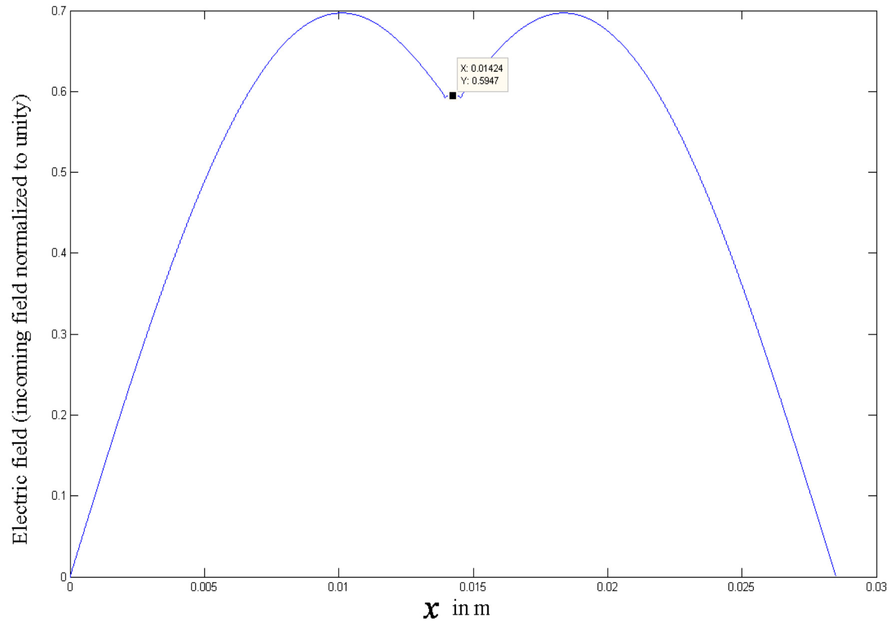


Fig. 2. Electric field as a function of x at the $z = 0$ plane for $M = 300$ by the present method

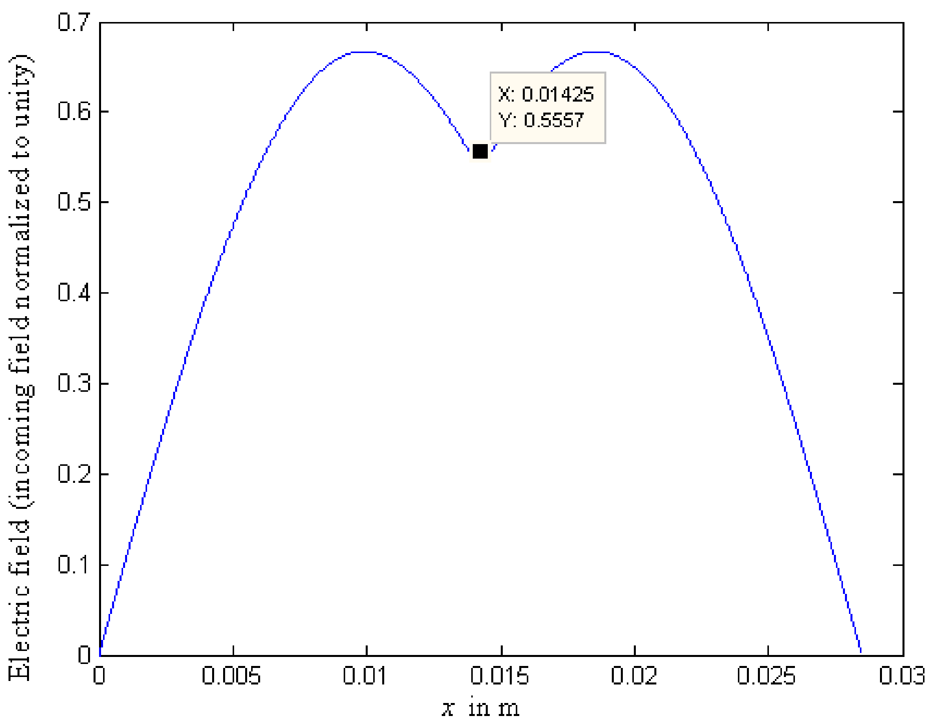


Fig. 3. Electric field as a function of x at the $z = 0$ plane for $M = 400$ by the present method

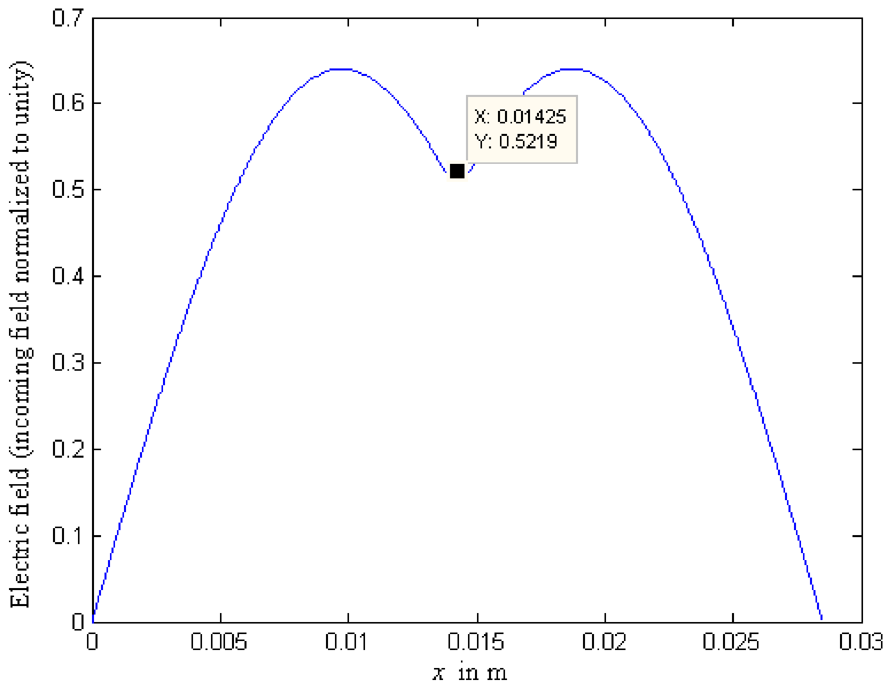


Fig. 4. Electric field as a function of x at the $z=0$ plane for $M=500$ by the present method

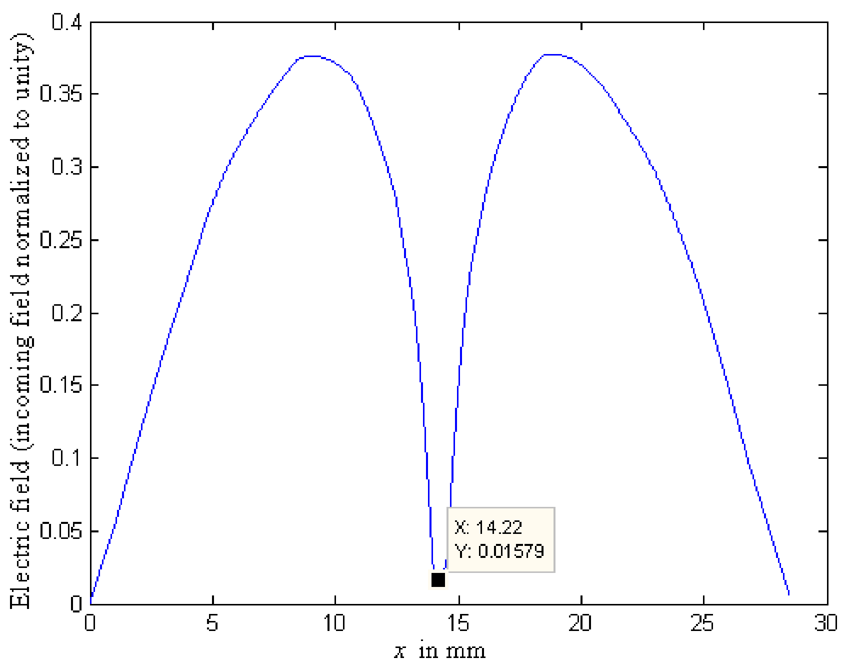


Fig. 5. Electric field as a function of x at the $z=0$ plane as computed by HFSS.

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