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Algorithm for Vertex Anti-magic Total Labeling on Various Classes of Graphs

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Abstract

Graph Labeling is an interesting filed having direct or indirect involvement in solving various problems in various fields. In this paper we proposed new algorithms to construct vertex antimagic edge labeling, (a,d)-vertex antimagic labeling, vertex antimagic total labeling and (a, d)-vertex antimagic total labeling and super vertex-antimagic total labeling of various classes of graphs like paths, cycles, wheels, Fan Graph and Friend Graphs. With these results many open problems in this area can be solved.

Keywords: Graph Labeling, Vertex antimagic edge labeling, Vertex antimagic total labeling, Super vertex-antimagic total labeling, Paths, Cycles, Wheels, Fan Graph and Friend Graphs

1. Introduction

Let G = (V, E) be a graph which is finite, simple and undirected .The graph G has a vertex set V = V (G) and edge set E = E(G) .We denote an m = |E| and n = |V |. A standard graph theoretic notation is followed refer [13]. Labeled graphs are becoming an increasingly useful family of Mathematical Models for a broad range of applications [4]. It has very crucial impact in network communications explained in [5]. Many recent applications exposed its usage to frequency allocation and image authentication.

The labeling of a graph is the process of mapping that maps some set of graph elements to a set of numbers (usually positive or non negative integers). The most complete recent survey of graph labeling is [3]. Sedlacek [9] introduced labelings that generalize the idea of a magic labeling. The magic labelings is defined as a bijection of graph element to set of consecutive integers starting from 1, satisfying some kind of "constant sum" property. If this Bijection involves vertices /edges /both as graph elements to a set of integers yielding a constant sum called as magic constant, it will be called as Vertex /Edge /Total Magic Labeling.

In [6] Hartsfield and Ringel introduced the concepts of an antimagic graphs. According to them an Antimagic labeling is an edge labeling of the graph with integers 1,2,….,m so that the weight at each vertex is different from the weight at each vertex. Bodendiek and Walter [7] defined the concept of an (a, d)-antimagic labeling as an edge labeling in which the vertex weights form an arithmetic progression starting from a and have common difference d. Martin Baca, Francois Bertault and MacDougall [8] introduce the notions of the Vertex Antimagic Total Labeling [VATL] and (a, d)- Vertex Antimagic Total Labeling [(a, d)- VATL], and conjuncture that all regular graphs are (a, d)- VATL.

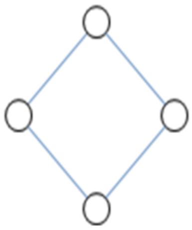
In 2004, K.A.Sugeng and others [10] introduced the notion of super vertex magic total labeling (SVMTL) and super edge magic total labeling (SEMTL). In [11] the existence of Antimagic vertex labeling of classes of hyper graphs like Cycles, Wheels and the existence and nonexistence of the Antimagic vertex labeling of Wheels have been discussed in theorems. The method to obtain Antimagic labeling for trees has been given in [12]. In [2] they proposed the algorithms to construct antimagic labeling, (a, d)-antimagic labeling, vertex antimagic total labeling and (a, d)-vertex antimagic total labeling of complete graphs, which is a generalization of several other types of labelings.

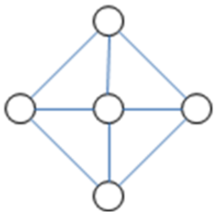
In this paper we proposed algorithms for above said Vertex Antimagic Total labeling on various classes of graphs like paths, cycles, wheels, fan Graphs and Friend graphs. These algorithms are almost similar to all classes with minor changes because of structural differences. With these we are also going to study the behavior of the graphs with specific graph size. For given Graph size we can identify the number of possible labelings, possible values of a and d to form (a,d) Vertex Anti magic Total Labelings and also the possibility of forming super vertex magic total labeling (SVMTL).

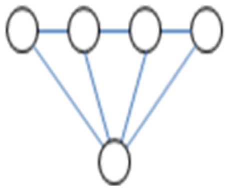
2. Preliminaries

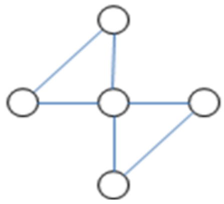
General definitions of Paths, cycles, wheels, fan and Friendship graphs are as follows. A Path P_n is a cycle without an edge from first vertex to last vertex. Cycle is a graph where there is an edge between the adjacent vertices only and the vertex is adjacent to last one (Fig1.a). Wheel is a Cycle with central hub, where all vertices of cycle are adjacent to it (Fig1.b).Fans and Friendship Graphs are sub classes of wheels. If a path is connected to central hub it is a Fan Graph (Fig1.c). A Friendship Graph consists of n triangles with one common vertex called as hub where n is size of Friendship Graph (Fig 1.d).

Fig1.a Path(P_4)









Fib1.b Cycle(C_4) Fib1.c Wheel(W_4) Fib1.d Fan Graph(F_4) Fib1.e Friend Graph(T_2)

Labeling, the process of assigning integers to elements of graph can be termed as a mapping function from integers to Graph elements. Magic Labeling is defined as a bijection from {1,2,3,…n} to n-graph elements such that the sum of each element is a magic constant k. If the component is vertex/edge/both it is called as Vertex/Edge/Total magic Total Labeling.

For a Graph G with v vertices and e edges if there is a one to one function from set of integers {1,2,…e} to edges of Graph and vertices will be assigned label as sum of edges incident to it. If the vertices are pair wise different then it will be Vertex Antimagic Labeling (VAL). If the vertices are assigned with no duplicates it turned as Strong-VAL otherwise weak-VAL. For any two fixed integers a, d if the vertices are assigned with labels is { a , a+d , …. , a+(v-1)d } , where a>0 and d≥0 is called (a, d)-VAL. If the vertices are assigned with consecutive numbers then, it will be Super Vertex Antimagic Labeling (SVAL).

Similarly for a Graph G with v vertices and e edges if there is a one to one function from set of integers {1,2,…v+e} to vertices and edges of Graph and weight of vertices is sum of labels assigned to vertex and labels assigned to edges incident to it. If the vertices are pair wise different then it will be Vertex

Antimagic Total Labeling (VATL). If the vertices are assigned with no duplicates it turned as Strong VATL otherwise weak VATL. For any two fixed integers a, d if the vertices are assigned with labels is { a , a+ d , …. , a+(v-1)d } , where $a>0$ and $d\geq 0$ is called (a, d) VATL. If the vertices are assigned with consecutive numbers then, it will be Super Vertex Antimagic Total Labeling (SVATL).

Next section provides algorithms to identify above all for different topologies of Graphs. Cumulative number of all such possibilities can be calculated. And open problems are answered and properties of graphs are observed.

3. Proposed Work

Here in this section we discuss algorithm to identify various features of vertex antimagic labeling. We give generalized algorithm in common to all kinds of Graphs mentioned in this paper. Because of topological differences each graph structure requires some modifications to algorithm. First we discuss algorithm then required changes for each kind of graph are given in this section. The following are the functions used in designing algorithm.

np_x : Generates variation of size x from set of n set of numbers which are available and unused labels.
isPwd (array, size) : A Boolean function return true if given array consist different adjacent values otherwise returns false.
isDuplicate(array, size) : A Boolean function return true if given array consist duplicate values otherwise returns false.
isRegDiff(array, size) : A function return rate of change if given array consist adjacent values with a regular difference given by arithmetic progression otherwise returns -1.
isSVATL (array) : A Boolean function return true if isRegDiff (array, size) returns the value 1 otherwise returns false.
Input : Graph size n
Output : possible number of Strong VAEL, weak VAEL, total VAEL, (a, d) VAEL with a, d values, strong VATL, weak VATL, total VATL, (a, d) VATL with a, d values and checks existence of SVATL

Algorithm for VAEL

1. Read Graph size n and set labels range {1,2,….,r}.
2. for i: 1 to r
   If there is a rP_n and is not an isomorphic then set them as labels of hub and spokes.
3. for j:1 to r
   if there is a rP_n, go to step 4.

otherwise display "checked all possible assignments".
4. set them as labels of edges.
5. For all vertices calculate weight.
   Weight[v] = Sum of labels of all edges incident to it.
   If(isPwd(weight, n)) VAELcnt++;
      If(isDuplicate(weight, n)) StrongVAELcnt++;
      Otherwise weakVAELcnt++;
      Set d= isRegDiff(weight, n)
      If(d>=1) adVAELcnt++;
      Id(d==1) SVAELcnt++;
6. Stop.

Algorithm for VATL

1. Read Graph size n and set labels range {1,2,….r}.
2. for i: 1 to r
   if there is a rP_n and is not an isomorphic then set them as labels of hub and spokes.
3. for j:1 to r
   if there is ar rP_3, go to step 4.
   Otherwise display "checked all possible assignments".
4. Set them as labels of last edge, first vertex and first edge.
   previous vertex=1 current vertex=2
   for i:1 to r
   if there is a rp_2 go to step5.
   Otherwise go to step4.
5. Set them as labels of current vertex and current edge.
   Previous vertex=current vertex Current vertex= current_vertex+1
   If current vertex=n go to step 5.
   Otherwise reset labels assigned to previous vertex as available.
   Previous vertex=previous_vertex-1 Current vertex= current_vertex-1
6. For all vertices calculate weight.
   Weight[v] = Sum of labels assigned to it and all edges incident to it.
   If(isPwd(weight, n)) VATLcnt++;
      If(isDuplicate(weight, n)) StrongVATLcnt++;
      Otherwise weakVATLcnt++;
      Set d= isRegDiff(weight, n)
      If(d>=1) adVATLcnt++;
      If(d==1) SVATLcnt++;
7. Stop.

Modifications to be done For Path:

For VAEL Range R is {1,2,.....n-1}.
For VATL Range R is {1,2,.....2n-1}.
we can avoid Step 2 for Paths and set label of first edge as zero.

Modifications to be done For Cycle:
For VAEL Range R is {1,2,.....n}.
For VATL Range R is {1,2,.....2n}.
we can avoid Step 2 for Cycle.
Modifications to be done For Wheel:
For VAEL Range R is {1,2,.....2n+1}.
For VATL Range R is {1,2,.....3n+1}.
Modifications to be done For FanGraph:
For VAEL Range R is {1,2,.....2n}.
For VATL Range R is {1,2,.....3n}.
set label of first edge as zero.
Modifications to be done For FriendshipGraph:
For VAEL Range R is {1,2,.....(n+1)/2}.
For VATL Range R is {1,2,.....(3n+1)/2}.
set labels of even edges as zero.

4. Results

Here we designed algorithms to produce cumulative number of strong and weak anti magic edge/total labelings. We also identified possible number of (a,d)VAEL/VATLs for different values of a and d. Also we calculated number of super(a,d) antimagic labelings if exists. Several authors analyzed behavior of a particular graph structure for some a and d values. But here these algorithms produce all such sequence of arrangement of vertices and edges labels. So, we can easily and visually understand behavior of any structure. Some of observations are as follows.

Paths: All paths of size n>=3 is Antimagic. All possible combinations are antimagic. But only some of them are strong. For an instance if the path size is 8 there are 5040 VAEL's are possible among 352 are strong 4688 are weak and it has no (a, d) VAEL. Bu t the path of size 7 has 92 strong and 628 weak labelings. Thus it gives total 720 VAELs. It has 18 (a,d) VAELs. Some such possible sequences are given in table 1. Paths with any length consists some VATLs. For example the path with length 5 is having 2610 strong 888 weak and total 3498 VATLs. It has 37 (a,d)VATLs among 14 are super VATLs. This kind of analysis can be done on any path. For n>=3, the path Pn, has neither super (a; 0)-VAT labeling nor super (a; 1)-VAT labeling. Every path Pn; n >=3, has a super (a; 3)-VAT labeling.

Cycles: In case of cycles, all with size >=3 is antimagic. These algorithms are examined for several values of n. For n=5 it resulted 30 strong and 90 weak that is total 120 VAELs. It also has 10 (a,d) VAELs for some values of a and d. Some such possible sequences are given in table 1.

For a cycle of size 4 we observed 23632 strong and 2192 weak that is 25824 total VATLs. It also has 1144 (a ,d) VATLs among 456 are super VATLs. Follow table 1 for some such possibilities. Cycle Cn has neither super (a; 0)-VAT labeling nor super (a; 2)-VAT labeling when n is even. Every cycle Cn; n odd, has a super (a; 2)-VAT labeling. Cycle Cn has a super (a; 1)-VAT labeling for all n >=2.

Wheels: Wheels are also belonging to the family of antimagic. For instance wheel of size 3 produces 240 string and 128 weak that is total 368 VAELs. There are no (a,d)VAELs for wheels. But all wheel are very powerful for VATLs. For the same wheel we got 2766336 strong 279120 weak VATLS. It implied there are 3045456 possible VATLs. There are 47088 (a,d) VATLs out of which 17904 are super VATLs.

Fan Graphs: Fan Graphs are also antimagic. For instance Fan Graph of size 3 produces 40 string and 32 weak that is total 72 VAELs. We got no (a, d) VAEL. For the same Fan Graph we got 276952 strong 29648 weak VATLS. It implied there are 306600 possible VATLs. There are 4508 (a,d) VATLs out of which 1652 are super VATLs.

Friend Graphs: Friend Graphs also admits anti magic labeling. For instance Friend Graph of size 4 produces 296 strong and 294 weak that is total 590 VAELs. We got no (a, d) VAEL. For the same Friend Graph we got 26718080 strong, 5136764weak VATLS. It implied there are 31854844 possible VATLs. There are 31728 (a, d) VATLs out of which 11184 are super VATLs.

5. Conclusion & Future scope

In this paper, we give algorithms to enumerate all Anti magic labelings on cycle graphs, wheels, Fan Graphs and Friendship graphs. The idea of the algorithms can be applied to other classes of graphs or adopted to develop algorithms for other type of labeling. In the mean time, we are still working on algorithm for other type of labeling such as edge anti magic and total, harmonious, graceful etc. We also present the number of non-isomorphic different anti magic labelings on each graph for some small size graphs. The number of non-isomorphic labeling on larger size of the remaining graphs is still an open problem.

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