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Digital Compensation of the Analog-Digital Highway Nonlinearity in a Case of Unknown Initial Phase of the Test Signal

V. A. Makovy, A. P. Ermakov

JCS Concern "Sozvezdie"
Voronezh, Russia

O. V. Chernoyarov

Department of Radio Engineering Devices
National Research University "MPEI", Moscow, Russia

B. I. Shakhtarin

Department of Self-contained Information and Control Systems
Bauman Moscow State University, Moscow, Russia

Abstract

We consider the digital compensation algorithm of analog-digital highway nonlinearity, based on the measurement of the nonlinearity using the harmonic signal. The technique is presented, allowing providing the independence of the compensation quality on an initial phase of the test signal. We demonstrate the simulation work of the synthesized algorithm, confirming its operability and efficiency.

Keywords: Measuring signal, instantaneous nonlinear element, Chebyshev polynomials, analog-digital highway, analog-digital converter, compensation nonlinear element, digital compensation

1 Introduction

Analog-digital highway (ADH), which has been introduced into modern software, defined radio aids has a nonlinear transfer function, owing to the various technological features of microchips and active elements manufacturing [1]. Availability of the analog-digital converter (ADC) in a signal passage path adds some features to the possibilities of the nonlinear interference suppression by the digital compensation of a path transfer function [2]. In Fig. 1 the block diagram of typical ADH is presented, in terms of amplitude characteristic nonlinearity. Here are the following designations: LA – linear amplifier, NE – nonlinear element, ADC NE – digitizer nonlinearity of ADC, LQ – linear quantizer, DSP – digital signal processing unit.

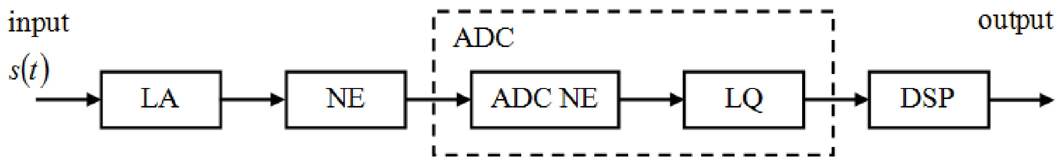

Fig. 1. Block diagram of the nonlinear analog-digital highway.

The input signal, passing through linear amplifier LA, incomes to the nonlinear element NE input, which describes amplitude characteristic nonlinearity of the analog highway. The basic contribution to nonlinearity of such ADH is inserted by the buffer amplifier, which is carrying out the matching of frequency-selective input circuits with ADC input. Such situation is characteristic for modern ADH implementing signal transformation to the digital form on a radio frequency. Further, the signal using ADC is converted to the digital format and incomes to the digital signal processing unit DSP, in which required processing algorithms are realized.

For the analysis of nonlinearity properties, we present ADC in the form of series connection of linear digitizer and nonlinear quantizer. Digitizer nonlinearity is comparatively small and is identified by ADC NE. We model nonlinear quantizer as series connection of uniform linear quantizer LQ and continuous nonlinear element NE, producing nonlinearity, which is equivalent to amplitude characteristic nonlinearity of the digitizer and quantizer, included in ADC.

ADH nonlinearity and, accordingly, nonlinear interference level at the signal reception can be reduced by digital compensation both of the quantizer nonlinearity and of the whole ADH nonlinearity. Digital nonlinearity compensation means an introduction of the additional nonlinearity into a digital part of the signal pass highway, which leads to the aggregate conversion response of nonlinear ADH and compensating nonlinear element (CNE) becoming more linear.

2 Nonlinearity Compensation of Analog-Digital Highway

In order to measure the ADH nonlinearity, the one-frequency harmonious signal is employed of a kind

$$s(t) = A\cos(\omega_s t + \varphi_0) = A\cos\Psi .$$

Here A is a test signal amplitude (without losing generality, we suppose $A = 1$ during further analysis), ω_s is any frequency from received frequency band, φ_0 and Ψ are initial and total phases of harmonic wave.

In [4] it is shown that function $f(s)$ of equivalent ADC NE into ADH (Fig. 1) can be expressed in the form of a power polynomial as

$$f(s) = b_0 + \sum_{n=1}^{N-2} b_n s^n ,$$

where s is the input signal to instantaneous element, b_n, $n = \overline{0, N}$ are some coefficients, $N = 2^M$, and M is number of bits of output code of the sampling (digitization and quantization) unit SU under monotonicity condition of its transfer characteristic. However, in some cases representation of the nonlinear ADH amplitude characteristic in the form of a final number of Chebyshev polynomials is more preferable [4]

$$f(s) = \sum_{n=0}^{N-2} a_n T_n(s).$$

Here $T_n(s) = \cos(n\arccos(s))$, $s[-1{,}1]$, $n = 0{,}1{,}2{,}\dots$ is Chebyshev polynomial of n-th order, $a_n = \int_0^{2\pi} f[\cos(n\omega t)]\cos(n\omega t)\,dt/\sqrt{2\pi}$ are Fourier coefficients of the test signal spectrum on the ADH output.

Compensating nonlinear element CNE can be configured in a serial form as it is shown in dashed rectangle, Fig. 2. In modern software-defined radio facilities the ADH linearity is such, that levels of highway output nonlinear interferences (i.e., test signal harmonics a_n, $n \geq 2$) are considerably less (on 80…100 dB) than level of the first harmonic $a_1 \approx 1$. Then ADH linearization can be carried out, with the following relations taken into account:

$$a_1 a_n \approx a_n >> a_m a_n , \quad m \geq 2,\ n \geq 2 .$$

The block diagram of such highway is presented in Fig. 2.

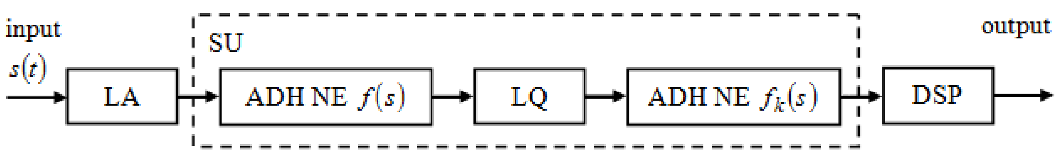

Fig. 2. ADH block diagram with series CNE connection.

The basic equation of digital nonlinearity compensation for series CNE connection has the appearance

$$f_k(f(s)) = a_1 T_1(s),$$

where $f_k(\cdot)$ is a transfer CNE function. Important advantage of the series compensation circuit shown in Fig. 2 presents itself in the possibility of the synthesizing of CNE without iterative calculation procedures of coefficients a_n. It hastens the compensation process essentially and reduces requirements to DSP block power. In this case the CNE characteristics should be following [2, 4]

$$f_k(s) = -a_0 T_0(s) + a_1 T_1(s) - \sum_{n=2}^{N-2} a_n T_n(s). \qquad (1)$$

Then, as a result of series connection of ADH NE and CNE, we receive

$$f_k(f(s)) \approx a_1^2 T_1(s).$$

The considered CNE synthetic procedure has high sensitivity to the deviation of an initial test signal phase from 0. In Fig. 3 we show the results of digital compensation received using the simulation model of the seven-bit quantizer with initial integral nonlinearity in 1 LSB. Compensation quality was determined according to relative interference level, as difference of dynamic range numbers under correct and CNE compensation. As follows from behavior of curve "a", the compensation efficiency decreases considerably under $\varphi_0 > 0.3^\circ$ already. And adequate accuracy of an initial phase definition increases with compensated ADH width.

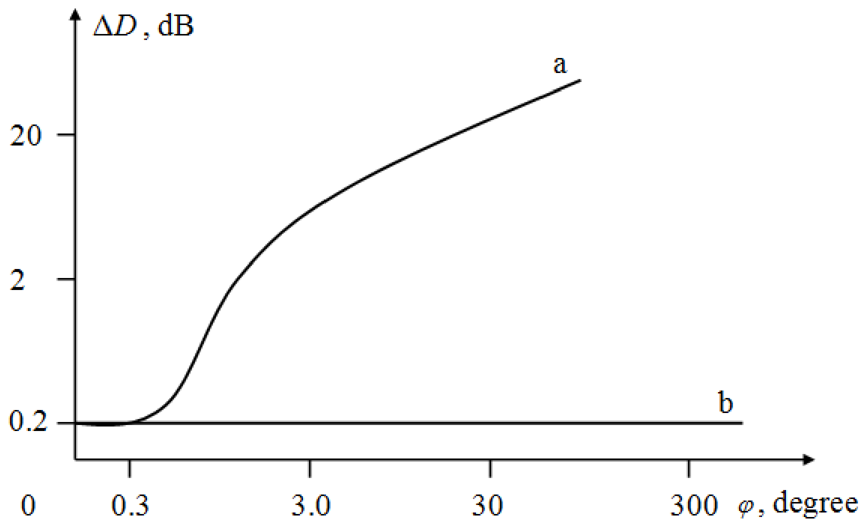

Fig. 3. Dynamic range contraction of the compensated highway depending on an initial phase of the test signal: a – initial algorithm (under arbitrary initial phase value), b – algorithm with reduction of an initial phase to zero.

There are known various methods of maintenance of a zero initial phase [5]. But all of them are not free from defects. For example, in the wideband signal correlation processing device with digital nonlinearity compensation [6] the digital sine wave generator synchronized on a phase of formed signal with reference time of signal sampling for calculation of spectral coefficients is used as test waveform shaper. Reconstruction filter at the digital generator output and narrow-band filter reducing the level of test signal harmonics have nonuniform group-delay dependence on frequency. With increasing analog filter discrimination selectivity, we have the group-delay increases and long-term and temperature stability of phase characteristics worsening. Analog ADH part also contains sensibly complex filters, used for the ensuring of the discrimination under great tunings out. In a combination with the test signal changed on frequency within ADH operating range, it all complicates the practical application of the digital nonlinearity compensation.

3 Nonlinearity Compensation of Analog-Digital Highway Using Test Signal with Arbitrary Initial Phase

In order to exclude the dependence of digital compensation quality on an initial test signal phase, we have developed the original algorithm allowing the application of the signal with any (a priori unknown) initial phase for the nonlinearity measurement of the conversion response. This algorithm is based on successive reduction of phases of test signal harmonics to a zero initial phase of the first harmonic. Reduction of spectral components on a phase is carried out by reduction of a full phase of each spectral component by the measured initial test signal phase φ_0, which increases according to number of a harmonic

$$a_n = \mathrm{Re}\{A_n \exp\left[j\left(n\omega t + n\varphi + \varphi_n - n\varphi_0\right)\right]\}. \qquad (2)$$

Here A_n is n-th harmonic amplitude of test signal on the compensated highway output, φ_0 is first harmonic phase of test signal modulo 2π, φ_n is n-th harmonic phase under $\varphi_0 = 0$.

As the signal passes through instantaneous nonlinear element, after reduction operation depending on a sign of a_n the harmonic phase can accept either value: $\varphi_n = 0$ or $\varphi_n = \pi$. Full phase of a harmonic component can be measured accurate to 2π because of periodicity of trigonometrical functions. As a result, after fast Fourier transformation (FFT), it is possible to determine the full phase for harmonic with number *n* modulo 2π:

$$\psi_n = \mathrm{mod}_{2\pi}\left(n\varphi_0 + \varphi_n\right).$$

According to Eq. (2), calculation of a_n value requires a finding of a full initial phase of *n*-th harmonic.

As nonlinear interferences in a test signal spectrum are generated by instantanous nonlinear element, reduction of frequency spectrum components to a zero phase can be realized without full phase calculation. For the proof of this statement we prove a parity

$$\mathrm{mod}_k(a-b) = \mathrm{mod}_k(\mathrm{mod}_k\, a - b), \tag{3}$$

where k is any nonnegative integer number, a, b are real positive numbers. We present values a, b as follows

$$a = n_1 k + \alpha, \qquad b = n_2 k + \beta. \tag{4}$$

Here $n_1 = a \,\mathrm{div}\, k$, $\alpha = a \bmod k$, $n_2 = b \,\mathrm{div}\, k$, $\beta = b \bmod k$. Then, substituting Eq. (4) in Eq. (3), we come to expression

$$\mathrm{mod}_k(n_1 k + \alpha - n_2 k - \beta) = \mathrm{mod}_k(\mathrm{mod}_k(n_1 k + \alpha) - n_2 k - \beta). \tag{5}$$

Let us consider the first member of Eq. (5). As $(n_1 - n_2)k + (\alpha + \beta)$ is number comparable modulo k with number $(\alpha + \beta)$ [7], the following equality is valid

$$\mathrm{mod}_k(n_1 k + \alpha - n_2 k - \beta) = \mathrm{mod}_k(\alpha - \beta). \tag{6}$$

For second member of Eq. (5) we have

$$\mathrm{mod}_k(\mathrm{mod}_k(n_1 k + \alpha) - n_2 k - \beta) = \mathrm{mod}_k(\alpha - n_2 k - \beta) = \mathrm{mod}_k(\alpha - \beta). \tag{7}$$

So, from comparison Eqs. (6) and (7) we receive identity (3).

Function $\exp(jx)$ has the period 2π that is equivalent to argument reduction modulo 2π:

$$\exp(jx) = \exp[j(\mathrm{mod}_{2\pi}(x))].$$

Then, using the proved equality (3) we write down the expression (2), describing spectral coefficient computation process of the test signal with a zero initial phase, as

$$a_n = \mathrm{Re}\{A_n \exp[j(n\omega t + \psi_n - n\varphi_0)]\}. \tag{8}$$

Here ψ_n is n-th harmonic phase of the test signal modulo 2π, φ_0 is first harmonic phase of the test signal modulo 2π, n is number of test signal harmonic.

The offered reduction method of spectral components of nonlinear interferences to a zero phase of the first harmonic allows to use a signal with unknown initial phase during digital compensation. Test signal frequency ω_t and sampling frequency ω_d for input FFT array are connected by a relation

$$\omega_t = \omega_d / 4N.$$

where N is quantizing level quantity in ADH. In the output FFT array the complex number corresponding to spectral coefficient of the first test signal harmonic contains the information about an initial phase φ modulo 2π

$$\varphi = \mathrm{arctg}[\mathrm{Im}(c_1)/\mathrm{Re}(c_1)]$$

where c_1 the complex Fourier coefficient corresponding to the first test signal harmonic. Similarly initial phases of all harmonics which are necessary for CNE (1) synthesis can be found. In Fig. 3 (curve “b”) the dependence of the digital compensation error on initial test signal phase is shown received with using simulation model under reduction of a spectrum of nonlinear interferences to a zero phase. From rate of this curve it can be seen that the offered algorithm has almost completely removed influence φ_0 on digital compensation quality.

Modern ADH compensation requires the account of considerable number of harmonics in whole. For example, if 16-bit quantizer is applied, then it is necessary to conduct the analysis of nonlinear hindrances up to 65534 harmonic of a test signal. As a rule, calculations are done using special processors directed on execution of digital processing algorithms. In small-sized radio facilities the arithmetic with the fixed point and corresponding processors, for example TMS320 series, are applied to reduce energy use.

Under great *n*, the value $n\varphi_0$ can exceed the bounds of a range of allowed number values. Thereupon, in fixed equipment processors with hardware support of floating point calculations are often employed. However, for great *n* the miscalculation increases and, accordingly, reduction accuracy on a phase degrades with increasing harmonic number in case of number representation in a format with a floating point because of bounded mantissa length. This disadvantage can be overcome, having modified expression (3) and having passed from absolute value $n\varphi_0$ to its remainder of reduction modulo 2π. Then Eq. (3) has the appearance

$$\mathrm{mod}_k(a-b)=\mathrm{mod}_k(\mathrm{mod}_k\, a-\mathrm{mod}_k\, b). \tag{9}$$

The validity of Eq. (9) is proved the same as Eq. (3).

Using Eq. (9) similarly to Eq. (8) we have

$$a_n=\mathrm{Re}\{A_n \exp\left[j(n\omega t+\psi_n-\mathrm{mod}_{2\pi}(n\varphi_0))\right]\}. \tag{10}$$

In Eq. (10) the value of operands does not exceed 4π for any *n* and φ_0. In Fig. 3 (curve "b") error dependence under using the reduction operation of a test signal spectrum to a zero phase is shown. From rate of this curve follows that application of a spectrum reduction provides independence of digital compensation quality on an initial test signal phase.

5 Conclusion

We have synthesized the algorithms of the presented digital compensation methods that makes it possible to exclude almost completely the influence of an initial phase and amplitude deviations of a test signal on the outcome of digital compensation. Thus, we can successfully apply the digital nonlinearity compensation, in order to improve the linearity of ADH, for which dimensions, consumption and high linearity are important parameters.

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