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Fuzzy Observer Design for a Class of Takagi-Sugeno Descriptor Systems

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Abstract

This paper deals with the problem of observers design for descriptor systems. The approach can be used to solve this problem in this paper consists in designing an observer described by differential equations only. The idea of the proposed approach is to separate the dynamic relations of the static relations in the descriptor model. First, the method used for decomposed the differential part of the algebraic part is developed, secondly we give an observer design permitting to estimate the unknown state. Two classes of systems are considered. The first one concern the descriptor linear systems. An observer design of this class is given. The second class concern the descriptor nonlinear systems. First, the Takagi-Sugeno fuzzy model is employed to approximate the descriptor nonlinear systems. Next, a fuzzy observer design is designed to estimate the unknown state. The convergence of the state estimation error is studied using the Lyapunov theory and the stability conditions are given in terms of Linear Matrix Inequalities (LMIs). Finally, to illustrate the proposed methodology, a nonlinear heat exchanger pilot model is considered.

Keywords: linear systems, nonlinear systems, descriptor systems, Takagi-Sugeno model, Fuzzy observer, linear matrix inequality (LMI)

1 Introduction

All recently there has been a great deal of interest in using dynamic Takagi-Sugeno fuzzy models [5] to approximate nonlinear systems. This approach consists to construct nonlinear model by means of interpolating the behaviour of several linear time invariant submodels. Once the T-S fuzzy models are obtained, linear control methodology can be used to design local state feedback controllers for each linear model. Aggregation of the fuzzy rules results in a generally nonlinear model, but in a very special form, which is exactly the same as a time varying and nonlinear system described by a set of Polytopic Linear Inclusions. The Takagi-Sugeno model has been generalized to singular systems. Many physical systems are naturally modeled as systems of differential and algebraic equations (DAE)[8],[9],[10]. These systems are variously called descriptor systems, singular systems, or differential algebraic equations (DAEs). This formulation includes both dynamic and static relations. Consequently this formalism is much more general than the usual one and can model physical constraints or impulsive behavior due to an improper part of the system. Descriptor systems appear in many fields of system design and control such as constrained robots, power systems, hydraulic or electrical networks. Many control issues have been extended to the descriptor case, in particular the observer design for descriptor systems has been intensively addressed see e.g. [1],[2],[11],[12] a linear fractional transformation parameterization of linear observers is done in [3] and [4] introduces the proportional integral (PI) observer. This paper presents a new method for state-estimation of Takagi-Sugeno descriptor systems (TSDS), is organized as follows: the class of studied systems are defined in section 2 and section 3 and the method used to decompose the dynamic relations of the static relations and design observer are detailed in these sections. Finally, in section 4, we illustrate the performance of the proposed observer (given in section 3) in simulation through a nonlinear heat exchanger pilot model.

2 Observer design for a class of descriptor linear systems

In this section, the following class of descriptor linear systems is considered:

$$\left\{ \begin{array}{rcl} E\dot{x}(t) & = & Ax(t) + Bu(t) \\ y(t) & = & Cx(t) \end{array} \right. \tag{1}$$

where $x = \begin{pmatrix} X_1^T & X_2^T \end{pmatrix}^T \in R^n$ is the state variable with $X_1 \in R^r$, $X_2 \in R^{n-r}$, $u \in R^m$ is the control input, $y \in R^p$ is the measured output. $E \in R^{n\times n}$ is constant matrix with $rank(E) = r$. $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{p\times n}$ are real known constant matrices.
with $E = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$; $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$; $B = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$; $C = \begin{pmatrix} C_1 & C_2 \end{pmatrix}$.
where A_{22} constant matrix is invertible ($rank(A_{22}) = n - r$).
In order to design an observer, we will make the following assumptions:

H1) $(E,\ A)$ is regular, i.e. $det(sE - A) \neq 0\ \forall s \in C$

H2) A descriptor system is impulse observable, i.e.

$$rank(\begin{pmatrix} E & A \\ 0 & E \\ 0 & C \end{pmatrix}) \quad = \quad n + rank(E)$$

H3) A descriptor system is R detectable, i.e.

$$rank(\begin{pmatrix} sE - A \\ C \end{pmatrix}) = n \ \ \forall s \in C$$

The idea to build an observer for a class of descriptor linear systems (1) is to separate the dynamic relations of the static relations as follow:

$$\left\{ \begin{array}{rcl} \dot{X}_1(t) & = & A_{11}X_1(t) + A_{12}X_2(t) + B_1u(t) \\ 0 & = & A_{21}X_1(t) + A_{22}X_2(t) + B_2u(t) \\ y(t) & = & C_1X_1(t) + C_2X_2(t) \end{array} \right. \tag{2}$$

Using the fact that A_{22}^{-1} exist, system (2) can be rewritten as:

$$\left\{ \begin{array}{lcl} \dot{X}_1(t) & = & MX_1(t) + Nu(t) \\ X_2(t) & = & QX_1(t) + Ru(t) \\ y(t) & = & SX_1(t) + Gu(t) \end{array} \right. \tag{3}$$

where

$$\left\{ \begin{array}{lcl} M & = & A_{11} - A_{12}A_{22}^{-1}A_{21} \\ N & = & B_1 - A_{12}A_{22}^{-1}B_2 \\ Q & = & -A_{22}^{-1}A_{21} \\ R & = & -A_{22}^{-1}B_2 \\ S & = & (C_1 - C_2A_{22}^{-1}A_{21}) \\ G & = & -C_2A_{22}^{-1}B_2 \end{array} \right. \tag{4}$$

In descriptor form, system (3) takes the form:

$$\left\{ \begin{array}{lcl} E\dot{x}(t) & = & Kx(t) + Fu(t) \\ y(t) & = & Dx(t) + Gu(t) \end{array} \right. \tag{5}$$

where $K = \begin{pmatrix} M & 0 \\ Q & -I \end{pmatrix}$, $F = \begin{pmatrix} N \\ R \end{pmatrix}$, $D = \begin{pmatrix} S & 0 \end{pmatrix}$.
Our candidate descriptor observer for system (5) takes the following form:

$$\left\{ \begin{array}{lcl} E\dot{\hat{x}}(t) & = & K\hat{x}(t) + Fu(t) - J(\hat{y}(t) - y(t)) \\ \hat{y}(t) & = & D\hat{x}(t) + Gu(t) \end{array} \right. \tag{6}$$

where $\hat{x}(t)$ and $\hat{y}(t)$ denote the estimated state vector and output vector respectively.
$J = \begin{pmatrix} L \\ 0 \end{pmatrix}$ is the gain of observer.
Denoting the state estimation error by

$$e(t) = \hat{x}(t) - x(t) \tag{7}$$

It follows from (5) and (6) that the observer error dynamic is given by the differential-algebraic equation

$$E\dot{e}(t) = \begin{pmatrix} \Gamma & O \\ Q & -I \end{pmatrix} e(t) \tag{8}$$

where

$$\Gamma = M - LS \tag{9}$$

The design of the observer consists to determine the gain L to ensure the convergence to zero of the estimation error. The convergence condition of the observer (6) can be formulated by the following theorem (see [6], [7] for more detail).

Theorem 2.1 *: There exists an observer (6) for (5) if the Hypotheses H1, H2 and H3 holds and there exists a symmetric positive matrix P verifying the following inequality:*

$$\Gamma^T P + P\Gamma \quad < \quad 0 \tag{10}$$

To proof the convergence of the observer in equation (6), set $e_1(t) = \hat{X}_1(t) - X_1(t)$ and $e_2(t) = \hat{X}_2(t) - X_2(t)$ the state estimation error, we get:

$$\begin{cases} \dot{e}_1(t) &= \Gamma e_1(t) \\ e_2(t) &= Q e_1(t) \end{cases} \tag{11}$$

Then, as in [6], from (11) the condition (10) implies that $\hat{x}(t)$ exponentially converges to the unknown trajectory $x(t)$ of system (5) which are identical to those of the system (1).

3 Fuzzy observer design for a class of T-S descriptor nonlinear systems

3.1 Takagi-Sugeno descriptor nonlinear systems

Consider the following form of descriptor nonlinear systems:

$$\begin{cases} E\dot{x}(t) &= f(x(t)) + g(x(t))u(t) \\ y(t) &= Cx(t) \end{cases} \tag{12}$$

where $x = \begin{pmatrix} X_1^T & X_2^T \end{pmatrix}^T \in R^n$ is the state variable with $X_1 \in R^r$, $X_2 \in R^{n-r}$, $u \in R^m$ is the control input, $y \in R^p$ is the measured output. f, g are nonlinear functions. $E \in R^{n\times n}$ with $rank(E) = r$, $C \in R^{p\times n}$ are real known constant matrices.
with $E = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$; $C = \begin{pmatrix} C_1 & C_2 \end{pmatrix}$.
In the present work, the class of T-S descriptor nonlinear systems that we consider here assumed represent descriptor nonlinear systems (12), takes the following form:

$$\begin{cases} E\dot{x}(t) &= \sum_{i=1}^{q} h_i(x(t))(A_i x(t) + B_i u(t)) \\ y(t) &= Cx(t) \end{cases} \tag{13}$$

where q is the number of submodels, $A_i \in R^{n\times n}$, $B_i \in R^{n\times m}$ are real known constant matrices. with $A_i = \begin{pmatrix} A_{11i} & A_{12i} \\ A_{21i} & A_{22i} \end{pmatrix}$; $B_i = \begin{pmatrix} B_{1i} \\ B_{2i} \end{pmatrix}$.

where A_{22i} constant matrices are invertible $(rank(A_{22i}) = n - r)$.
The $h_i(x(t))$ are the weighting functions that ensure the transition between the contribution of each sub model. They have the following properties:

$$\begin{cases} 0 \leq h_i(x(t)) \leq 1 \\ \sum_{i=1}^{q} h_i(x(t)) = 1 \end{cases} \tag{14}$$

As in section 2, in order to design an observer for each sub model of system (8) $(i = 1, \ \ldots, \ q)$, we will make the following assumptions:

H4) $(E, \ A_i)$ is regular, i.e. $det(sE - A_i) \neq 0 \ \forall s \in C$

H5) A descriptor system is impulse observable, i.e.

$$rank(\begin{pmatrix} E & A_i \\ 0 & E \\ 0 & C \end{pmatrix}) \quad = \quad n + rank(E)$$

H6) A descriptor system is R detectable, i.e.

$$rank(\begin{pmatrix} sE - A_i \\ C \end{pmatrix}) = n \quad \forall s \in C$$

3.2 Fuzzy observer design

The idea is to build an observer for every local model. In a similar way to the technique used to aggregate the local models, the observer global is obtained by aggregation of the locals observers.
Every local descriptor system of system (13) is represented as follows:

$$\begin{cases} E\dot{x}(t) &= A_i x(t) + B_i u(t) \\ y(t) &= Cx(t) \end{cases} \tag{15}$$

Based on the separate the dynamic relations of the static relations and the fact that A_{22i}^{-1} exist, and using above theory developed in section 2 (see (2), ..., (6)), our candidate fuzzy observer for system (13) takes the form:

$$\begin{cases} E\dot{\hat{x}}(t) &= \sum_{i=1}^{q} \bar{h}_i(x(t))(K_i\hat{x}(t) + F_i u(t) - J_i(\hat{y}(t) - y(t))) \\ \hat{y}(t) &= \sum_{i=1}^{q} \bar{h}_i(x(t))(D_i\hat{x}(t) + G_i u(t)) \end{cases} \tag{16}$$

where $\hat{x}(t)$ and $\hat{y}(t)$ denote the estimated state vector and output vector respectively. The activation functions $\bar{h}_i(x(t))$ (see(18))are the same than those used in the T-S model (13).

$$\left\{\begin{array}{lcl} K_i & = & \begin{pmatrix} M_i & 0 \\ Q_i & -I \end{pmatrix} \\ F_i & = & \begin{pmatrix} N_i \\ R_i \end{pmatrix} \\ J_i & = & \begin{pmatrix} L_i \\ 0 \end{pmatrix} \\ D_i & = & \begin{pmatrix} S_i & 0 \end{pmatrix} \end{array}\right. \tag{17}$$

and

$$\left\{\begin{array}{lcl} \bar{h}_i(x(t)) & = & h_i(x(t)) = h_i(X_1(t), X_2(t) = Q_i X_1(t) + R_i u(t)) = h_i(X_1(t), u(t)) \\ M_i & = & A_{11i} - A_{12i} A_{22i}^{-1} A_{21i} \\ N_i & = & B_{1i} - A_{12i} A_{22i}^{-1} B_{2i} \\ Q_i & = & -A_{22i}^{-1} A_{21i} \\ R_i & = & -A_{22i}^{-1} B_{2i} \\ S_i & = & (C_1 - C_2 A_{22i}^{-1} A_{21i}) \\ G_i & = & -C_2 A_{22i}^{-1} B_{2i} \end{array}\right. \tag{18}$$

Denoting the state estimation error by

$$e(t) \;=\; \hat{x}(t) - x(t) \tag{19}$$

It follows from (13) and (16) that the observer error dynamic is given by the differential-algebraic equation

$$E\dot{e}(t) \;=\; \begin{pmatrix} \sum_{i=1}^{q}\sum_{j=1}^{q} \bar{h}_i(x(t))\bar{h}_j(x(t))\Gamma_{ij} & O \\ \sum_{j=1}^{q} \bar{h}_i(x(t)) Q_i & -I \end{pmatrix} e(t) \tag{20}$$

where

$$\Gamma_{ij} \;=\; M_i - L_i S_j \tag{21}$$

The design of the observer consists to determine the gain L_i to ensure the convergence to zero of the estimation error. The convergence condition of the observer (16) can be formulated by the following theorem (see [6], [7] for more detail).

Theorem 3.1 : *There exists an observer (16) for (13) if the Hypotheses H4, H5 and H6 holds and there exists a symmetric positive matrices P, Q verifying the following inequalities:*

$$\begin{cases} \mathcal{L}_e(\Gamma_{ii}, P) + (q-1)Q + 2\alpha P < 0 & i \in I_p = \{1, ..., q\} \\ 2\mathcal{L}_e(\Gamma_{ij}, P) - Q + 2\alpha P \leq 0 & \forall (i,j) \in I_p^2,\ i < j \text{ and } \bar{h}_i(x(t)).\bar{h}_j(x(t)) \neq 0 \end{cases} \tag{22}$$

where

$$\mathcal{L}_e(\Gamma_{ij}, P) = (\frac{\Gamma_{ij} + \Gamma_{ji}}{2})^T P + P(\frac{\Gamma_{ij} + \Gamma_{ji}}{2}) \tag{23}$$

To proof the convergence of the observer in equation (16), set $e_1(t) = \hat{X}_1(t) - X_1(t)$ and $e_2(t) = \hat{X}_2(t) - X_2(t)$ the state estimation error, we get:

$$\begin{cases} \dot{e}_1(t) = \sum_{i=1}^{q}\sum_{j=1}^{q} \bar{h}_i(x(t))\bar{h}_j(x(t))\Gamma_{ij} e_1(t) \\ e_2(t) = \sum_{j=1}^{q} \bar{h}_i(x(t)) Q_i e_1(t) \end{cases} \tag{24}$$

Then, as in [6], from (24) the condition (22)-(23) implies that $\hat{x}(t)$ exponentially converges to the unknown trajectory $x(t)$ of system (13) which are identical to those of the system (12).

4 Application to a heat exchange system

The aim of this section consists in applying the above fuzzy descriptor observer design (16) to a heat exchange process described by figure 1.

4.1 Physical model

The heat exchange process considered is presented in figure 1. The process is mainly built around a counter-flow tubular heat exchanger. The warm water flows in a closed circuit, the temperature in the hot water tank is fixed by an independently controlled electric heater. The cold water flows in an open circuit. The flows of either warm and cold water are controlled by two electro-pneumatic valves. T_1, T_3 are respectively the inlet temperatures of the warm and the cold water and T_2, T_4 are the correspondent outlet temperatures. The dynamics of actuators (electro-pneumatic valves) cannot be neglected. Indeed their time constants are equivalent to the residence time constants of the heat exchanger (0.5s - 1s). The correspondent state variables are the displacements and the velocities of the electro-pneumatic valves. The temperatures are assumed to be homogeneous in the tubular heat exchanger. Under the

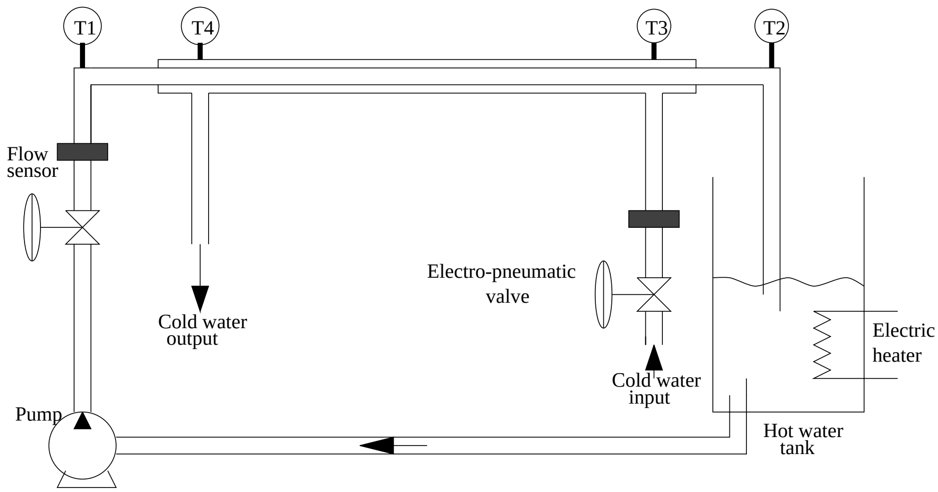

Figure 1: heat exchanger plant

hypotheses that the circuit of the thermal exchange is a closed system which contains a constant mass of water, the inertia of the fluid is negligible and the flow is turbulent.

The controlled variables of our problem are the temperatures T_2 and T_4, which are manipulated with the flows which are a function of electro-pneumatic valves current I_{vw} and I_{vc}. The current on electro-pneumatic valve is actual manipulated variable of the process. Furthermore, the heat-exchanger is just one part of the plant, so the actuators should also be modeled. The electro-pneumatic valve is a system that exhibits inherent second-order dynamics. For the heat-exchanger, we perform the energy balance for the characterization of the temperature. A descriptor model of the process takes the form:

$$\left\{ \begin{array}{rcl} E\dot{x}(t) & = & f(x(t)) + g(x(t))u(t) \\ y(t) & = & h(x(t)) \end{array} \right. \tag{25}$$

where $x = [x_1, \cdots, x_8]^T$ is the state vector, $u = (u_1, u_2)$ is the control vector, $(x_1, x_4) = (T_2, T_4)$ is the output measurements, x_2, x_5 are respectively the displacement of the warm water valve and the cold water valve, x_3, x_6 are respectively the velocity of the warm water valve and the cold water valve and finally, x_7, x_8 are respectively the acceleration of the warm water valve and the cold water valve.

$$f(x(t)) = \begin{pmatrix} e_1x_2 - a_1x_1x_2 - b_1x_1 + b_1x_4 \\ x_3 \\ x_7 \\ e_2x_5 - a_2x_4x_5 + b_2x_1 - b_2x_4 \\ x_6 \\ x_8 \\ -x_7 - \omega_0^2x_2 - 2\eta\omega_0x_3 \\ -x_8 - \omega_0^2x_5 - 2\eta\omega_0x_6 \end{pmatrix}, \; g(x(t)) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ k_0\omega_0^2 & 0 \\ 0 & k_0\omega_0^2 \end{pmatrix}$$

$$E = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad h(x(t)) = \begin{pmatrix} x_1(t) \\ x_4(t) \end{pmatrix}$$

a_1, a_2, b_1, b_2, e_1, e_2, are physical constants which derive from the energy balance transfer.
k_0 is the static gain of the valve, ω_0 is the undamped natural frequency and finally, η is the damping factor.

4.2 Takagi-Sugeno model

To express the model of the heat exchange system as a Takagi-Sugeno model with the measurable parameters (temperatures T_2 and T_4) as decision variables, we rewrite the equation (25) in the following equivalent state space form:

$$\left\{ \begin{array}{rcl} E\dot{x}(t) & = & A(x(t))x(t) + Bu(t) \\ y(t) & = & Cx(t) \end{array} \right. \tag{26}$$

where

$$A(x(t)) = \begin{pmatrix} -b_1 & e_1 - a_1x_1 & 0 & b_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ b_2 & 0 & 0 & -b_2 & e_2 - a_2x_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -w_0^2 & -2\eta w_0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -w_0^2 & -2\eta w_0 & 0 & -1 \end{pmatrix} \tag{27}$$

$$B = g(x(t)),\ C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{28}$$

To constructing T-S model of heat exchange system, considering the sector of the nonlinearities of the terms $z_j \in [z_{jmin},\ z_{jmax})]$ of the matrix $A(x(t))$ with $j = 1, 2$:

$$\left\{ \begin{array}{lcl} z_1(t) & = & e_1 - a_1 x_1(t) \\ z_2(t) & = & e_2 - a_2 x_4(t) \end{array} \right. \tag{29}$$

Thus, we can transform the nonlinear terms under the following shape:

$$z_j(t) \quad = \quad M_{1j}(t) z_{jmax} + M_{2j}(t) z_{jmin}; \quad j = \{1, 2\} \tag{30}$$

where

$$\left\{ \begin{array}{lcl} M_{1j}(t) & = & \dfrac{z_j(t) - z_{jmin}}{z_{jmax} - z_{jmin}} \\ M_{2j}(t) & = & \dfrac{z_{jmax} - z_j(t)}{z_{jmax} - z_{jmin}} \end{array} \right. \tag{31}$$

Then, the global fuzzy model is inferred as:

$$\left\{ \begin{array}{lcl} E\dot{x}(t) & = & \sum_{i=1}^{4} h_i(x(t))(A_i x(t) + Bu(t)) \\ y(t) & = & Cx(t) \end{array} \right. \tag{32}$$

where

$$h_i(x(t)) = \frac{w_i(x(t))}{\sum_{i=1}^{4} w_i(x(t))} \tag{33}$$

with

$$\left\{ \begin{array}{lcl} w_1(x(t)) & = & M_{21}(x(t)) M_{22}(x(t)) \\ w_2(x(t)) & = & M_{21}(x(t)) M_{12}(x(t)) \\ w_3(x(t)) & = & M_{11}(x(t)) M_{22}(x(t)) \\ w_4(x(t)) & = & M_{11}(x(t)) M_{12}(x(t)) \end{array} \right. \tag{34}$$

for all $t \geq 0$, $h_i(x(t)) \geq O$ and $\sum_{i=1}^{4} h_i(x(t)) = 1$.

$$A_1 = \begin{pmatrix} -b_1 & z_{1min} & 0 & b_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ b_2 & 0 & 0 & -b_2 & z_{2min} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -w_0^2 & -2\eta w_0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -w_0^2 & -2\eta w_0 & 0 & -1 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} -b_1 & z_{1min} & 0 & b_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ b_2 & 0 & 0 & -b_2 & z_{2max} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -w_0^2 & -2\eta w_0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -w_0^2 & -2\eta w_0 & 0 & -1 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} -b_1 & z_{1max} & 0 & b_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ b_2 & 0 & 0 & -b_2 & z_{2min} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -w_0^2 & -2\eta w_0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -w_0^2 & -2\eta w_0 & 0 & -1 \end{pmatrix}$$

$$A_4 = \begin{pmatrix} -b_1 & z_{1max} & 0 & b_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ b_2 & 0 & 0 & -b_2 & z_{2max} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -w_0^2 & -2\eta w_0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -w_0^2 & -2\eta w_0 & 0 & -1 \end{pmatrix}$$

4.3 Descriptor observer design

Based on the on-line measurements of the temperature of the warm water x_1 and the temperature of the cold water x_4, we shall show the previous result (16) can be used to estimate the displacement, the velocity and the acceleration of the warm water valve x_2, x_3, x_7 and the displacement, the velocity and the acceleration of the cold water valve x_5, x_6, x_8. Using subsection 3.2, the construction of the Fuzzy descriptor observer algorithm for heat exchange system requires that the above system (32) takes the form (13). In our case: $X_1 = [x_1 \ \ x_2 \ \ x_3 \ \ x_4 \ \ x_5 \ \ x_6]^T$, $X_2 = [x_7 \ \ x_8]^T$.

$E = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$ with $rank(E) = 6$.

$A_i = \begin{pmatrix} A_{11i} & A_{12i} \\ A_{21i} & A_{22i} \end{pmatrix} = \begin{pmatrix} A_i(1:6,1:6) & A_i(1:6,7:8) \\ A_i(7:8,1:6) & A_i(7:8,7:8) \end{pmatrix}$ for $i = 1, 2, 3, 4$

$B = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} B(1:6,1:2 \\ B(7:8,1:2 \end{pmatrix}$; $C = \begin{pmatrix} C_1 & C_2 \end{pmatrix} = \begin{pmatrix} C(1:2,1:6) & C(1:2,7:8) \end{pmatrix}$.

Noticing that in this application $A_{22i} = A_i(7:8, 7:8) = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ are invertible.
Consequently, from theorem 2.1 a fuzzy descriptor observer for system (32) permitting to estimate x_2, x_3, x_5, x_6, x_7, x_8 takes the following form:

$$\begin{cases} E\dot{\hat{x}}(t) &= \sum_{i=1}^{4} \bar{h}_i(x(t))(K_i\hat{x}(t) + F_iu(t) - J_i(\hat{y}(t) - y(t))) \\ \hat{y}(t) &= \sum_{i=1}^{4} \bar{h}_i(x(t))(D_i\hat{x}(t) + G_iu(t)) \end{cases} \quad (35)$$

where $\bar{h}_i$, K_i, F_i, D_i, G_i, J_i are given in the above equations (17), (18).

4.4 Simulation results

In order to illustrate the performances of the above observer (35), we use the parameter values summarized in table 1. To simulate descriptor model (32), we use a Runge-Kutta method combined with the Newton-Raphson algorithm.

Parameters	Values	Parameters	Values
a_1	552.5871	k_0	0.93
a_2	92.0978	w_0	6.2832
b_1	0.2856	η	0.7
b_2	0.0952	u_1	0.006
e_1	$4.1444*10^4$	u_2	0.012
e_2	$1.4736*10^3$		

Table 1: Liste of parameters

The initial conditions of the nonlinear system (25) and T-S model (32) are:
$x_1(0) = 73\ ^oC$, $x_2(0) = 0\ m$, $x_3(0) = 0\ m/s$, $x_4(0) = 18\ ^oC$, $x_5(0) = 0\ m$, $x_6(0) = 0\ m/s$, $x_7(0) = 0.2203\ m/s^2$, $x_8(0) = 0.4406\ m/s^2$.
The initial conditions of the fuzzy observer (35) are:
$\hat{x}_1(0) = 73\ ^oC$, $\hat{x}_2(0) = 0.002\ m$, $\hat{x}_3(0) = 0.001\ m/s$, $\hat{x}_4(0) = 18\ ^oC$, $\hat{x}_5(0) = 0.001\ m$, $\hat{x}_6(0) = 0.001\ m/s$, $\hat{x}_7(0) = 0.1325\ m/s^2$, $\hat{x}_8(0) = 0.3923\ m/s^2$.
From inequalities (22) given in theorem 3.1 and with definition (21) we obtain the following observer gains L_i, $i = 1, 2, 3, 4$:

$$L_1 = \begin{pmatrix} 1.4265*10^9 & -9.9148*10^6 \\ 1.3532*10^8 & -5.6172*10^3 \\ -5.1140*10^8 & 1.8338*10^4 \\ 7.9336*10^6 & 1.0507*10^9 \\ 4.3851*10^4 & -6.2144*10^6 \\ -1.5111*10^5 & 2.6809*10^7 \end{pmatrix}, L_2 = \begin{pmatrix} 1.1715*10^9 & 3.3021*10^6 \\ 1.2598*10^8 & 1.8708*10^3 \\ -4.7891*10^8 & -6.1073*10^3 \\ -2.6423*10^6 & 9.4847*10^8 \\ -1.46051*10^4 & 4.5871*10^7 \\ 5.0330*10^4 & -1.6853*10^8 \end{pmatrix}.$$

$$L_3 = \begin{pmatrix} 1.1803*10^9 & -9.0712*10^6 \\ -4.2261*10^7 & -5.1391*10^3 \\ 1.6123*10^8 & 1.6777*10^4 \\ 7.2585*10^6 & 9.6514*10^8 \\ 4.0121*10^4 & 7.2239*10^5 \\ -1.3826*10^5 & -1.0847*10^6 \end{pmatrix}, L_4 = \begin{pmatrix} 1.3533*10^9 & 3.8001*10^7 \\ -3.9739*10^7 & 2.1529*10^4 \\ 1.5199*10^8 & -7.0283*10^4 \\ -3.0407*10^7 & 1.1311*10^9 \\ -1.6807*10^5 & 2.8185*10^7 \\ 5.7918*10^5 & -1.0198*10^8 \end{pmatrix}$$

Figure 2 shows a simulation result of the nonlinear model of the process (25) and the T-S model (32). Note that the T-S model exactly represents the nonlinear model.

Figure 3 shows a simulation result, where the dotted lines denote the state variables estimated by the fuzzy observer (35). This simulation shows that the estimation states converge to their corresponding state variables.

5 Conclusion

Based on the separate the dynamic relations of the static relations and solving a system of LMI, a new fuzzy descriptor observer design is proposed for a class of MIMO systems. This observer synthesis can be used to solve the observer problem for a class of T-S descriptor systems. Two fuzzy observers are proposed. The first one is given for a class of descriptor linear systems. The second one which is an extension to the first class permitting to estimate the unknown state of a class of T-S nonlinear systems described by a set of differential-algebraic equations. This fuzzy observer is proposed for the on-line estimation of an unknown state in a heat exchange system. Simulation results have been given and they demonstrated the good performances of the estimator.

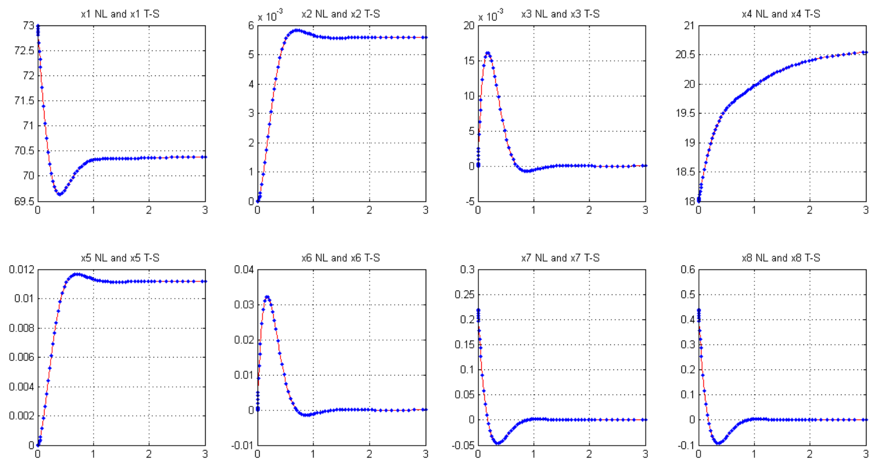

Figure 2: —— : Nonlinear model, ... : T-S model

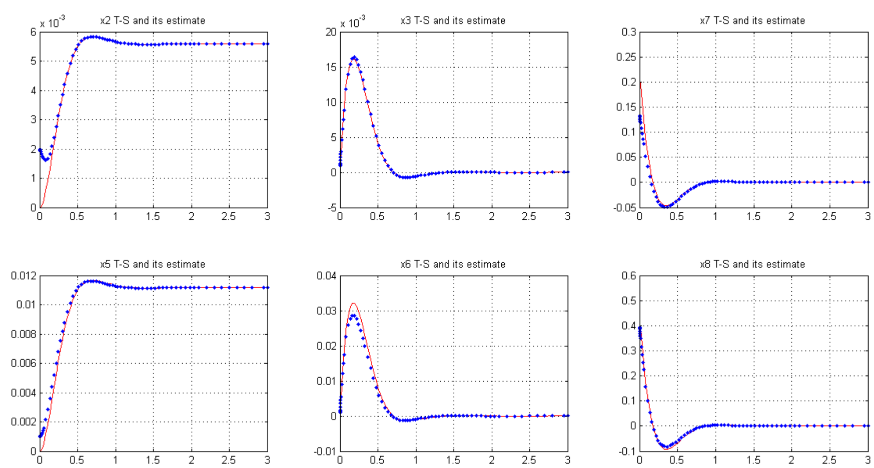

Figure 3: —— : T-S model, ... : Fuzzy observer

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