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Mathematical Model for Detecting Thermally Damaged Composites Using Rayleigh-Lamb Waves

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Abstract

This work presents a new method for detecting thermally damaged composites, because theses materials having now many applications especially in aerospace vehicles. Many techniques have been used to detect if the material has a thermal damage or not, but the present method has many advantages to be used. This method depending on a simple and effective idea depending on that both Raleigh and Lamb waves' velocities are varied as the composite materials properties are changed as a result of thermal damage. Wave phase velocity of both Lamb and Rayleigh waves are found to be varied as the laminates properties are changed due to thermal effects, the density of the materials is studied here only to be an investigator for applying such idea.

Keywords: Laminated Composite materials, Lamb waves, Rayleigh waves, thermally damaged composites, wave number

1 INTRODUCTION

Composites materials are a combination of more than one material that are combined together to form a new material. There are three main types of composite materials filamentary, laminated, and particulate. Each composite material consists of two main parts: fibers, and matrix, fibers are considered as the load carrying members such as, glass, boron, graphite, while matrix are considered as a supporter and connector, such metal matrix, polymers, and ceramics, polymer matrix composites are a low temperature sensitive so they are used in low temperatures applications such composites like E-glass/Epoxy, Carbon/Epoxy, Armid/Epoxy…etc. the term

Carbon/Epoxy means that carbon represents the fibers while epoxy represents the matrix. The second type of composites depending on type of matrix is metal matrix composites that are used in intermediate temperature ranges like Boron/Aluminum, Graphite/Copper. The third type is ceramic matrix composites, which is used in high temperature applications such as; Silicon/Silicon carbide, Carbide/Aluminum. The advantages of composites are they have high specific strength and stiffness because they have small specific weight; the second advantage is they have high fatigue and corrosion resistance, clean to handle, fewer parts for structural system, and cab be tailored easily. The disadvantages of composites are high cost; structural analysis is more complex, sensitivity to impact loading and for aerospace structures environmental protection is required. Composite usage is increases in last two decades in many industrials such as thermal protection, structural missiles, aerospace and aircraft industrials and vehicles. Wave propagation becomes a very important subject, especially in composite materials research because the velocity of waves propagates in the composite materials is changed with any variation in the properties of the composite materials. Tow kinds of waves are usually used in these studies that are Rayleigh waves, and lamb waves, next a brief definition will be given about each of them.[5].

- Rayleigh surfaces waves: which propagates on the free surface of homogenous, isotropic semi-infinite elastic solids. It can be propagated in unbounded isotropic solids. Where there is a bounding surface, however, elastic surface waves may also occur. These waves are similar to gravitational surface waves in liquids. Lord Rayleigh (1887) was first to investigate these waves and showed that their effect decreases rapidly with depth, and that their velocity of propagation is smaller than that of body waves.
- Lamb waves: these are horizontally traveling, non-undispersive acoustic waves. They are longitudinal and purely horizontal, and as a result, there are no buoyancy effects and propagation is not dispersive. Lamp waves offer a promising method of evaluating thermal damage in composite materials. Lamp waves are a good tool to study the thermally damaged composites.

As a material is thermally damaged, the material properties are also altered. Since the lamp wave velocity is directly related to these parameters, an effective tool can be developed to monitor the damage in composites by measuring the velocity of these waves. Lamp waves can be measured experimentally, by using some special devices,. Michael et al. (1995). [3]. Another kinds of devices were used by: Yoseph Bar et al.(1998).[2].

Leach, Briggs, and Carlie [1985] investigated a range of material properties for a thermoplastic PEEK matrix with carbon fibers. The temperature range used was 50° C to 240° C. it was found that above the glass transition temperature, strength decreased by 50% and modulus fell on the order of 25%. Below the glass transition temperature the modulus only decreased slightly while the strength fell by 20%. Flame resistance of this particular material was noted to be better than that of aluminum. The result for heating in air of AS4/3501-6 composites was examined by Street, Russell, and Bonsang [1988]. In this study, the temperature was extended to 350° C and the damage assessed by measuring interlaminar fracture toughness, interlaminar shear strength, and hardness. The data suggested that for overheating at 350° C, less than 5 minutes is sufficient to seriously degrade all the parameters measured. A combined time temperature profile of 300° C for 15 minutes led to a loss of interlaminar fracture toughness. Up to 30 minutes at temperatures of 225° C caused insignificant damage, while a temperature of 300° C for the same period resulted in a 50% loss of interlaminar shear strength. Barcol hardness tests of the 300° C damage only dropped by 20% for a 30 minute exposure. The sensitivity of the Barcol hardness test as an indicator of thermal damage seems somewhat low as illustrated by this study. Operating conditions can cause what are labeled as "thermal spikes", or temperature fluctuations in aircraft. Damage generated under these conditions was studied by Stansfield and Pritchard [1989]. For composites that undergo these thermal fluctuations in service where moisture ingress is possible, the matrix dominated mechanical properties can be severely limited. It was concluded that matrix cracking and fiber debonding were the main damage mechanisms.

Michael D. Seal and Barry T. Smith [1995] provide a method of quantifying the amount of thermal damage in composite materials, components which have nonvisible damage. The results of the experimental measurement show that the velocity of the lowest order symmetric lamb mode dropped significantly for extended thermal damage.

Theoretical calculations with reduced values of C_{11} show that a 25-30% reduction in this parameter lowered velocities of the S_0 mode by 15%. Yosef B. Cohen and Chang [1998], uses a new method leaky lamb waves as a practical quantitative tool for both inversion of the elastic properties and characterization of flows. The emphasis of the current study is on the detection and characterization of flows. A data- acquisition system have developed a rapid and user-friendly data-acquisition system as well as improved their analytical tools to automatically determine the wave speeds and elastic constants. This development simplifies the process of characterizing flaws in composites and bonded joints and the determination of the material properties degradation. Michel V. Agostini, et.al [2001], made a simulation of lamb wave's propagation in composites for evaluating the initial integrity of the mechanical structures and/or for a reliable monitoring of damage and aging.

1.1 Mathematical Model Derivation

For any laminated composite unit cell, symmetry and continuity conditions also exist and should be satisfied in the Y-Z plan. Accordingly, the relevant field equations and appropriate interface and symmetry conditions of the representative repeating cell are as follows:

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}+\frac{\partial\sigma_x z}{\partial z}=\rho\ddot{u}$$
$$\frac{\partial\sigma_y}{\partial y}+\frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yz}}{\partial z}=\rho\ddot{v} \qquad (1)$$
$$\frac{\partial\sigma_z}{\partial z}+\frac{\partial\sigma_{xz}}{\partial x}+\frac{\partial\sigma_{yz}}{\partial y}=\rho\ddot{w}$$

Also stresses in the case of composites can be given as:

$$\sigma_x=(\lambda+2\mu)\frac{\partial u}{\partial x}+\lambda(\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z})$$
$$\sigma_y=(\lambda+2\mu)\frac{\partial v}{\partial y}+\lambda(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z})$$
$$\sigma_z=(\lambda+2\mu)\frac{\partial w}{\partial z}+\lambda(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}) \qquad (2)$$
$$\sigma_{xy}=\mu(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x})$$
$$\sigma_{xz}=\mu(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x})$$
$$\sigma_{yz}=\mu(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y})$$

These equations hold for both materials 1 and 2, as in figure (1) below, but at the center line of laminate 1 and 2 the symmetry conditions at $y_1=0$ are as follows:$v_1=0$; $\sigma_{xy1}=0$,and $\sigma_{zy1}=0$; and at $y_2=0$, $v_2=0$; $\sigma_{xy2}=0$and $\sigma_{zy2}=0$.[6].

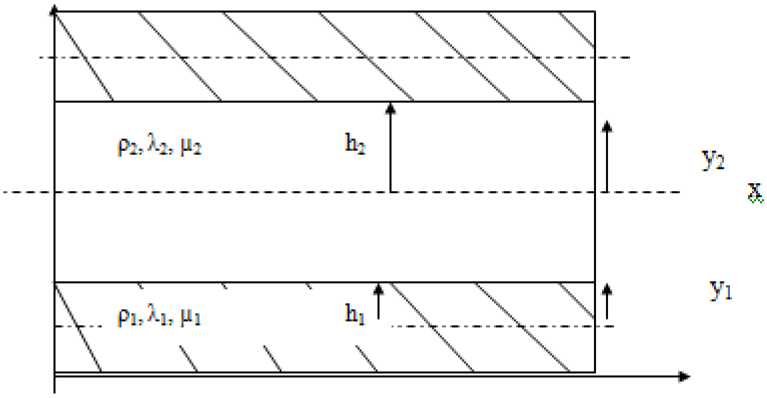

Figure (1). Laminated composite geometry

Furthermore, at the interface $y_1 = h_1$, $y_2 = -h_2$ where $h_1$and h_2 are half-thicknesses of laminates 1 and 2; respectively then the continuity relations:

$$u_1 = u_2 = u^*, v_1 = v_2 = v^*, w_1 = w_2 = w^* \quad (3)$$

$$\sigma_{y1} = \sigma_{y2} = \sigma_y^*, \sigma_{xy1} = \sigma_{xy2} = \sigma_{xy}^*, \quad \sigma_{yz1} = \sigma_{yz2} = \sigma_{yz}^* \quad (4)$$

Where the "*" denotes an interfacial value.

1.2Averaging Process

If equation (1) is averaged across its laminates thickness according to the relation

$$(\bar{\ })_i = \frac{1}{h_i}\int_0^{hi} (\ )_i dy \text{ ,I=1,2} \quad (5)$$

One gets for material 1

$$\frac{\partial}{\partial x}\bar{\sigma}_{x1} + \frac{\partial}{\partial z}\bar{\sigma}_{xz1} - \rho_1 \bar{\ddot{u}}_1 = \frac{1}{h_1}[\sigma_{xy1}(0) - \sigma_{xy1}(h_1)] \quad (6)$$

If the appropriate relevant symmetry and continuity conditions are utilized then equation (6) becomes

$$\frac{\partial}{\partial x} n_1 \bar{\sigma}_{x1} + \frac{\partial}{\partial z} n_1 \bar{\sigma}_{xz1} - \rho_1 n_1 \bar{\ddot{u}}_1 = p \quad (7)$$

where

$$n_1 = \frac{h_1}{h_1 + h_2} = \frac{h_1}{h} \quad (8)$$

where n_1: is the volume fraction of material 1, h is the total thickness of the repeating unit cell and

$$p = -\frac{\overset{*}{\sigma}_{xy}}{h} \tag{9}$$

P is called a momentum interaction term. Referring to figure (2) below which shows the shear distribution "σ_{xy}" and the displacement in the y-direction "v" as linear function of y, it can be noticed that:

$$\sigma_{xy1}(h_1) = \overset{*}{\sigma}_{xy} = -\sigma_{xy2}(h_2) \tag{10}$$

and

$$v_1(h_1) = v^* = -v_2(h_2) \tag{11}$$

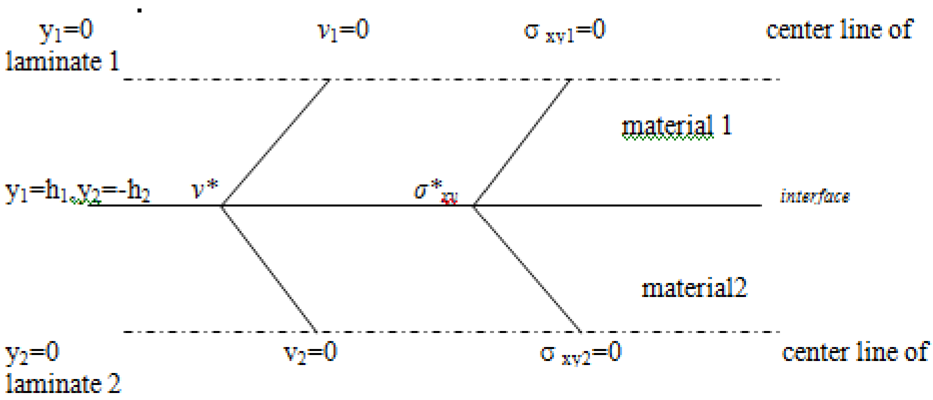

Figure (2). variation of σ_{xy} and v with y

Similarly for material 2 in view of equations (7), (10) and (37). [6].

$$\frac{\partial}{\partial x} n_2 \overline{\sigma}_{x2} + \frac{\partial}{\partial z} n_2 \overline{\sigma}_{xz2} - \rho_2 n_2 \ddot{\overline{u}}_2 = -P \tag{12}$$

Similarly for material 2:

$$n_2 = \frac{h_2}{h}$$

where n_2: is the volume fraction of material 2.

By making averaging of equation (1) in x-direction

$$n_1\bar{\sigma}x_1 = n_1(\lambda+2\mu)_1\frac{\partial\overline{u}_1}{\partial x}+n_1\lambda_1\frac{\partial\overline{w}_1}{\partial z}-\lambda_1 s$$
$$n_2\bar{\sigma}x_2 = n_2(\lambda+2\mu)_2\frac{\partial\overline{u}_2}{\partial x}+n_2\lambda_2\frac{\partial\overline{w}_2}{\partial z}-\lambda_2 s \qquad (13)$$

and

$$s=\frac{-V^*}{h}$$

where s: is called a constitutive interaction term by expanding σ_{xy1}, σ_{xy2}, and applying the continuity conditions at the interface, p and k can be written as:

$$p=\frac{k}{h^2}\left(\overline{u}_1-\overline{u}_2\right)$$
$$k=\frac{3\mu_1\mu_2}{n_1\mu_2+n_2\mu_1} \qquad (14)$$

and then

$$S=\frac{1}{E}\left[\lambda_1\left(\frac{\partial\overline{u}_1}{\partial x}+\frac{\partial\overline{w}_1}{\partial z}\right)-\lambda_2\left(\frac{\partial\overline{u}_2}{\partial x}+\frac{\partial\overline{w}_2}{\partial z}\right)\right] \qquad (15)$$

where $E=\frac{E_1}{n_1}+\frac{E_2}{n_2}$. (16)

Similarly

$$\sigma xz=\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)$$
$$\bar{\sigma}xz=\mu_\alpha\left(\frac{\partial\overline{u}_\alpha}{\partial z}+\frac{\partial\overline{w}_\alpha}{\partial x}\right) \qquad (17)$$

For material 1:

$$C_{11}\frac{\partial^2\overline{u}_1}{\partial x^2}+C_{12}\frac{\partial^2\overline{u}_2}{\partial x^2}+h_1\mu_1\frac{\partial^2\overline{u}_1}{\partial z^2}+\left(C_{11}-\mu_1h_1\right)\frac{\partial^2\overline{w}_1}{\partial x\partial z}+C_{12}\frac{\partial^2\overline{w}_2}{\partial x\partial z}-p_1n_1\ddot{\overline{u}}_1=\frac{k}{h^2}\left(\overline{u}_1-\overline{u}_2\right)$$

(18)

Similarly for material 2

$$C_{22}\frac{\partial^2\overline{u}_2}{\partial x^2}+C_{12}\frac{\partial^2\overline{u}_1}{\partial x^2}+h_2\mu_2\frac{\partial^2\overline{u}_2}{\partial z^2}+\left(C_{22}-\mu_2h_2\right)\frac{\partial^2\overline{w}_2}{\partial x\partial z}+C_{12}\frac{\partial^2\overline{w}_1}{\partial x\partial z}-p_2n_2\ddot{\overline{u}}_2=\frac{k}{h^2}\left(\overline{u}_2-\overline{u}_1\right)$$

(19)

where

$$C_{\alpha\alpha} = n_\alpha E_\alpha - \frac{\lambda_\alpha{}^2}{E}, \alpha = 1,2$$
$$C_{\alpha\beta} = \frac{\lambda_\alpha \lambda_\beta}{E}, \alpha, \beta = 1,2, \alpha \neq \beta \qquad (20)$$

In similar way for the y-z plan, [6, 3].

$$C_{11}\frac{\partial^2 \overline{w}_1}{\partial z^2} + C_{12}\frac{\partial^2 \overline{w}_2}{\partial z^2} + h_1\mu_1\frac{\partial^2 \overline{w}_1}{\partial x^2} + \left(C_{11} - \mu_1 h_1\right)\frac{\partial^2 \overline{u}_1}{\partial x \partial z} + C_{12}\frac{\partial^2 \overline{u}_2}{\partial x \partial z} - p_1 n_1 \ddot{\overline{w}}_1 = \frac{k}{h^2}\left(\overline{w}_1 - \overline{w}_2\right) \qquad (21)$$

and

$$C_{22}\frac{\partial^2 \overline{w}_2}{\partial z^2} + C_{12}\frac{\partial^2 \overline{w}_1}{\partial z^2} + h_2\mu_2\frac{\partial^2 \overline{w}_2}{\partial x^2} + \left(C_{22} - \mu_2 h_2\right)\frac{\partial^2 \overline{u}_2}{\partial x \partial z} + C_{12}\frac{\partial^2 \overline{u}_1}{\partial x \partial z} - p_2 n_2 \ddot{\overline{w}}_2 = \frac{k}{h^2}\left(\overline{w}_2 - \overline{w}_1\right) \qquad (22)$$

For low frequency ranges the displacements in both laminates become very closed to each others (i.e. as $h \to 0, \overline{u}_1 \to \overline{u}_2, \overline{w}_1 \to \overline{w}_2$) so, the last equation becomes

$$\frac{\partial^2 u}{\partial x^2} + C_{20}{}^2\frac{\partial^2 u}{\partial z^2} + \left(1 - C_{20}{}^2\right)\frac{\partial^2 w}{\partial x \partial z} + k_\circ{}^2 u = 0$$
$$\frac{\partial^2 w}{\partial z^2} + C_{20}{}^2\frac{\partial^2 w}{\partial x^2} + \left(1 - C_{20}{}^2\right)\frac{\partial^2 u}{\partial x \partial z} + k_\circ{}^2 w = 0 \qquad (23)$$

where

$$C_{20}{}^2 = \frac{c_2{}^2}{C_\circ{}^2}; \ddot{u} = -w^2 u; \ddot{w} = -u^2 w$$
$$k_\circ{}^2 = \frac{w^2}{C_\circ{}^2}$$
$$C_\circ{}^2 = \left(C_{11} + C_{22} + 2C_{12}\right)/\rho_c \qquad (24)$$
$$C_2{}^2 = \left(n_1\mu_1 + n_2\mu_2\right)/\rho_c$$
$$\rho_c = n_1\rho_1 + n_2\rho_2$$

By expanding the equation:

$$\overline{\sigma} xz = \mu_\alpha\left(\frac{\partial \overline{u}_\alpha}{\partial z} + \frac{\partial \overline{w}_\alpha}{\partial x}\right) \qquad (25a)$$

It yields

$$\bar{\sigma}xz_c = \mu_c\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \tag{25b}$$

$$\bar{\sigma}xz_c = n_1\bar{\sigma}xz_1 + n_2\bar{\sigma}xz_2$$

and

$$\mu_c = n_1\mu_1 + n_2\mu_2 \tag{26}$$

where $\bar{\sigma}xz_c$ is the average shear stress in x-z plane for the composite. μ_c : is the mixture shear modulus.

Finally

$$\begin{aligned} \bar{\sigma}z_c &= C_\circ^{\,2}\rho_c\frac{\partial w}{\partial z} + \lambda_c\frac{\partial u}{\partial x} \\ \lambda_c &= n_1\lambda_1 + n_2\lambda_2 + \frac{1}{E}(\lambda_1 - \lambda_2)^2 \end{aligned} \tag{27}$$

Which called the Lame's constant (mixture).[1].

Whereas

$$\bar{\sigma}z_c = n_1\bar{\sigma}z_1 + n_2\bar{\sigma}z_2$$

Is the composite average normal stress in the Z-direction.

If the material is homogenous material then

$$\begin{aligned} \frac{\partial\sigma x}{\partial x} + \frac{\partial\sigma xz}{\partial z} &= \rho\ddot{u} \\ \frac{\partial\sigma z}{\partial z} + \frac{\partial\sigma xz}{\partial x} &= \rho\ddot{w} \end{aligned} \tag{28}$$

$$\begin{aligned} \sigma x &= (\lambda + 2\mu)\frac{\partial u}{\partial x} + \lambda\frac{\partial w}{\partial z} \\ \sigma z &= (\lambda + 2\mu)\frac{\partial w}{\partial z} + \lambda\frac{\partial u}{\partial x} \\ \sigma xz &= \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \end{aligned} \tag{29}$$

and

$$\begin{aligned} C_1^{\,2}\frac{\partial^2 u}{\partial x^2} + C_2^{\,2}\frac{\partial^2 u}{\partial z^2} + \left(C_1^{\,2} - C_2^{\,2}\right)\frac{\partial^2 w}{\partial x\partial z} &= \ddot{u} \\ C_1^{\,2}\frac{\partial^2 w}{\partial z^2} + C_2^{\,2}\frac{\partial^2 w}{\partial x^2} + \left(C_1^{\,2} - C_2^{\,2}\right)\frac{\partial^2 u}{\partial x\partial z} &= \ddot{w} \end{aligned} \tag{30}$$

where

$$\begin{aligned} C_1^{\,2} &= (\lambda + 2\mu)/\rho \\ C_2^{\,2} &= \mu/\rho \end{aligned} \tag{31}$$

The solutions of equation (30) can be written as follows

$$u = Ae^{-\alpha z} e^{i\varepsilon_1}(x - ct)$$
$$w = Be^{-\alpha z} e^{i\varepsilon_1}(x - ct) \qquad (32)$$

Where
A, B: are complex amplitudes
α: is a real positive constant
ε_1: is the wave number
c: phase velocity of Rraielgh or Lamb waves, and
$c = w/\varepsilon_1$, *w*: is the angular frequency.
If we substitute equation (32) at the following equations

$$\frac{\partial^2 u}{\partial x^2} + C_{20}^2 \frac{\partial^2 u}{\partial z^2} + (1 - C_{20}^2)\frac{\partial^2 w}{\partial x \partial z} + k_\circ^2 u = 0$$
$$\frac{\partial^2 w}{\partial z^2} + C_{20}^2 \frac{\partial^2 w}{\partial x^2} + (1 - C_{20}^2)\frac{\partial^2 u}{\partial x \partial z} + k_\circ^2 w = 0 \qquad (33)$$

It yields

$$[C_{20}^2 \alpha^2 + (k_\circ^2 - \varepsilon_1^2)]A + i\varepsilon_1 (C_{20}^2 - 1)\alpha B = 0$$
$$i\varepsilon_1 (C_{20}^2 - 1)\alpha A + [\alpha^2 + (k_\circ^2 - C_{20}^2 \varepsilon_1^2)]B = 0 \qquad (34)$$

The condition for nontrivial solution is

$$\alpha^4 + \xi\alpha^2 + \Delta = 0 \qquad (35)$$

where

$$\xi = k_\circ^2 \left(1 + \frac{1}{C_{20}^2}\right) - 2\varepsilon_1^2$$
$$\Delta = \frac{1}{C_{20}^2}(k_\circ^2 - \varepsilon_1^2)(k_\circ^2 - C_{20}^2 \varepsilon_1^2) \qquad (36)$$

Solving for α^2 yields

$$\alpha_{1,2}^2 = \frac{1}{2}\left[\xi \pm (\xi^2 - 4\Delta)^{\frac{1}{2}}\right] \qquad (37)$$

The formal solutions would be as follows:

$$u = (A_1 e^{-\alpha_1 z} + A_2 e^{-\alpha_2 z})e^{i\xi x}$$
$$w = (A_1 \delta_1 e^{-\alpha_1 z} + A_2 \delta_2 e^{-\alpha_2 z})e^{i\xi x} \qquad (38)$$

where

$$\delta_1 = \frac{k_\circ^{\ 2} - \xi^2 + C_{20}^{\ 2}\alpha_1^{\ 2}}{i\alpha_1\xi\left(1 - C_{20}^{\ 2}\right)}$$
$$\delta_2 = \frac{k_\circ^{\ 2} - \xi^2 + C_{20}^{\ 2}\alpha_2^{\ 2}}{i\alpha_2\xi\left(1 - C_{20}^{\ 2}\right)} \qquad (39)$$

Constants A_1, A_2, ζ, C can be chosen so that the B.C on the free surface Z = 0 can be satisfied.

Also from last equations it can be written that

$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0.$$
$$C_\circ^{\ 2}\rho_c \frac{\partial w}{\partial z} + \lambda_c \frac{\partial u}{\partial x} = 0 \qquad (40)$$

By setting Z=0 and substitute u, w in equation (40) it yields

$$(i\xi\delta_1 - \alpha_1)A_1 + (i\xi\delta_2 - \alpha_2)A_2 = 0$$
$$\left(i\xi\lambda_c - C_\circ^{\ 2}\rho_c\alpha_1\delta_1\right)A_1 + \left(i\xi\lambda_c - C_\circ^{\ 2}\rho_c\alpha_2\delta_2\right)A_2 = 0 \qquad (41)$$

For nontrivial solution the determinate of the coefficient A_1 and A_2 must vanishes. Then Rayleigh" equation is produced such that:

$$(i\xi\delta_1 - \alpha_1)\left(i\xi\lambda_c - C_\circ^{\ 2}\rho_c\alpha_2\delta_2\right) - (i\xi\delta_2 - \alpha_2)\left(i\xi\lambda_c - C_\circ^{\ 2}\rho_c\alpha_1\delta_1\right) = 0 \qquad (42)$$

Substituting forδ$_1$, δ$_2$ in equation (42) yields

$$\left[\frac{k_\circ^2 - \xi^2 + C_{20}^2\alpha_1^2}{\alpha_1\left(1 - C_{20}^2\right)} - \alpha_1\right]\left[\xi\lambda_c + C_0^2\rho_c\frac{k_\circ^2 - \xi^2 + C_{20}^2\alpha_2^2}{\xi\left(1 - C_{20}^2\right)}\right] - \left[\frac{k_\circ^2 - \xi^2 + C_{20}^2\alpha_2^2}{\alpha_2\left(1 - C_{20}^2\right)} - \alpha_2\right]\left[\xi\lambda_c + C_0^2\rho_c\frac{k_\circ^2 - \xi^2 + C_{20}^2\alpha_1^2}{\xi\left(1 - C_{20}^2\right)}\right] = 0$$

(43)

Now, C_{20} represents some composites properties like the modulus of elasticity "E" of the composites studied is changed with temperature and depending on [6] and [7] and after making some linearization:

$$C_{20} \quad or \quad E = 92.2 - 0.0343\,T \qquad (44)$$

After substituting equation (44) in (43) the resulted equation relates the wave phase velocity to the effective anisotropic properties of a composite consisting of two periodically stacked materials. The surface wave is propagating along the direction of lamination; this equation can be analyzed numerically for different composites, for thornel laminate/ Carbon phenolic laminate.

2 RESULT AND DISCUSSION

From the resulted equation it can be noticed that the wave velocity increases as the volume fraction increases for the composite material, and also wave velocity increases as the reciprocal of the wave number is also increases. Fig. 1 shows the relation between modulus of elasticity and temperature the value of E decreases with temperature i.e. as the composite material becomes thermal defected its modulus of elasticity becomes smaller.

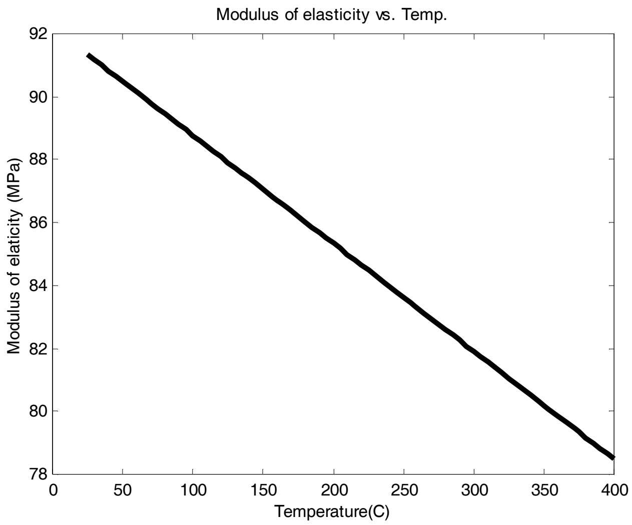

Fig.1 modulus of elasticity vs. temperature

Some of results of thermally damaged composites which investigated by wave propagation techniques specially lamb waves, shows that velocity of these waves is decreasing as the exposed temperature increases. Also it can be noticed that as the exposed temperature effected the composite material increases there will be a reducing in stiffness constant of the composite materials. Michael et al, (1993).

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