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Delay-Dependent α-Stable Linear Systems with Multiple Time Delays

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Abstract

In this paper we investigate the delay dependent exponential stability a of class of multiple time delay systems. A new sufficient condition is derived by using the Lyapunov functional method, linear matrix inequality (LMI). For this purpose, we use a decomposition technique of the delay term matrices. Then we investigate exponential stability of system and at the end some numerical examples are given to illustrate efficiency of our method.

Mathematics Subject Classification: 37B25, 37N30

Keywords: Exponential stability, Matrix decomposition LMI, Lyapunov functional

1 Introduction

The problem of investigation of time delay systems has been exploited over many years. Time delay is very often encountered in various technical systems, such as electric, pneumatic and hydraulic networks, chemical processes, long transmission lines, etc. The existence of pure time lag, regardless if it is present in control and the state, may cause undesirable system transient response, or even instability. During the last three decades, the problem of stability analysis of time delay systems has received considerable attention and many papers dealing with this problem have appeared [1]. The developed stability criteria are classified often into two categories according to their

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dependence on the size of the delay: delay-dependent and delay-independent stability criteria. It has been shown that delay-dependent stability conditions that take into account the size of delays, are generally less conservative than delay-independent ones which do not include any information on the size of delays [2]. Further, the delay-dependent stability conditions can be classified into two classes: frequency-domain and time-domain approaches. In the second approach, we have the comparison principle based techniques [3] for functional differential equations [4] and respectively the Lyapunov stability approach with the Krasovskii and Razumikhin based methods [1]. The stability problem is thus reduced to one of finding solutions to Lyapunov [5] or Riccati equations [6] or analyzing eigenvalue distribution of appropriate finite-dimensional matrices [7].

The stability with a given convergence rate related to the exponential stability has been studied in [8, 9, 10, 11] and the references therein. There are many different methods to deal with the exponential stability problem. Among the well known Lyapunov stability methods, the Lyapunov function method is a powerful tool for studying system stability, even for linear systems. Time-delay stability conditions for time-invariant systems were formulated in both algebraic Riccati equations and LMI [12, 13, 14, 15, 16].

This paper is organized as follows. In section 2, we give some notations, definitions and auxiliary propositions. Sufficient conditions for the exponential stability with a given convergence rate are presented in section 3. Two illustrating examples are provided to show the superiority of our results in section 4. In section 5, we make concluding remarks.

2 Preliminaries

We start by introducing notations and definitions that will be employed throughout the paper.

$\langle x, y\rangle$ or $x^T y$ denotes the scalar product of two vectors $x, y \in R^n$,

$\|x\|$ denotes the Euclidean vector norm of $x \in R^n$,

$M^{n\times r}$ denotes the space of all $n \times r$-matrices,

A^T denotes the transpose of matrix A, A is symmetric and $A = A^T$,

$\lambda(A)$ denotes the eigenvalues of A, $\lambda_{max}(A) = \max\{\mathrm{Re}\lambda : \lambda \in \lambda(A)\}$,

$\|A\|$ denotes the spectral norm of the matrix defined by $\|A\| = \sqrt{\lambda_{max}(A^T A)}$,

matrix A is named non-negative definite ($A \geq 0$) if $\langle Ax, x\rangle \geq 0$, for all $x \in R^n$,

A is positive definite ($A > 0$) if $\langle Ax, x\rangle > 0$ for all $x \neq 0$, or equivalently,

$\exists c > 0$ such that $\langle Ax, x\rangle \geq c\,\|x\|^2$, for all $x \in R^n$, $C([-h, 0], R^n)$ denotes the

Banach space of all piecewise-continuous vector functions mapping $[-h, 0]$ into R^n, $*$ denotes the symmetric part.

Let us consider the following linear systems with multiple time delays:

$$\begin{aligned} \dot{x}(t) &= A_0 x(t) + \sum_{k=1}^{m} A_k x(t - h_k), \quad t \in R^+, \\ x(t) &= \phi(t), \quad t \in [-h, 0], \end{aligned} \tag{1}$$

where $0 \le h_k \le h, k = 1, 2, \ldots, m$, are positive delays and h is a positive constant, $x(t)$ is the state vector, $A_k \in R^{n\times n}$,$(k = 1, 2, \ldots, m)$ are constant system matrices and $\phi(t) \in C([-h, 0], R^n)$ with $\|\phi\|_h = Sup\,\|\phi(t)\|$ for all $t \in [-h, 0]$, where $C([-h, 0], R^n)$ is the Banach space of continuous functions which map $[-h, 0] \longrightarrow R^n$ with the topology of uniform convergence, $\|\phi(t)\|$ denotes the Euclidean vector norm of ϕ.

Definition 1 [20] *Let $\alpha > 0$ be a given number.The system* (1) *is said to be α-stable, if there is a function $\eta : R^n \longrightarrow R^n$ such that for each $\phi(t) \in C([-h, 0], R^n)$, the solution $x(t, \phi)$ of the system satisfies $\|x(t, \phi)\| \le \eta(\|\phi\|)e^{-\alpha t}$, for all $t \in R^+$.*

The following lemmas will be used to prove the main results in section 3.

Lemma 1 [17] *The following linear matrix inequality* (LMI)

$$\begin{pmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{pmatrix} < 0$$

where $Q(x) = Q^T(x), R(x) = R^T(x)$ and $S(x)$ depends affinely on x, is equivalent to

$$\begin{aligned} &Q(x) < 0, \; R(x) - S^T(x)Q^{-1}(x)S(x) < 0, \\ &R(x) < 0, \; Q(x) - S(x)R^{-1}(x)S^T(x) < 0. \end{aligned}$$

Define an operator $D : C([-h, 0], R^n) \to R^n$ as

$$D(x(t)) = x(t) + \int_{t-h_k}^{t} G_k x(s)ds, \quad k = 1, 2, 3, \ldots, m \tag{2}$$

where $x_t = x(t + s)$, $s \in [-h, 0]$ and $G_k \in R^{n\times n}$ is a constant matrix. we have the following fact about $D(x(t))$.

Lemma 2 [18] *The operator (2)is said to be stable if there exist a scalar* $0 < \delta < 1$ *and positive symmetric matrix* M*such that*

$$\begin{pmatrix} -\delta M & hG_k^T M \\ * & -M \end{pmatrix} < 0.$$

3 Main results

Consider the linear delay system (1), where the matrices $A_k, k = 1, 2, \ldots, m$ are constant. Let us set
$B_0 = A_0 + \alpha I, B_k = e^{\alpha h_k} A_k, k = 1, 2, \ldots, m.$
We change matrices $B_k, k = 1, 2, \ldots, m$ in diagonal form and denote it as G_k. By decomposing G_k we have

$$\begin{aligned} G_k &= \begin{pmatrix} g_{11k} & 0 & \ldots & 0 \\ 0 & g_{22k} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & g_{nnk} \end{pmatrix} = \begin{pmatrix} g_{11k} - \varepsilon & 0 \ldots & 0 & \\ 0 & g_{22k} - \varepsilon & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & g_{nnk} - \varepsilon \end{pmatrix} \\ &+ \begin{pmatrix} \varepsilon & 0 & \ldots & 0 \\ 0 & \varepsilon & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \varepsilon \end{pmatrix} = G_{k\varepsilon} + B_{\varepsilon}. \end{aligned} \tag{3}$$

Considering that ε is small enough then we take
$G_k = G_{k\varepsilon}$.

Theorem 1 *System* (1) *is said to be* α*-stable if there exist positive definite symmetric matrices* P, R_2 *and* $\varepsilon > 0$ *which satisfy the following LMIs:*

$$\begin{pmatrix} -I & hG_{k\varepsilon}^T \\ hG_{k\varepsilon} & -I \end{pmatrix} < 0, \tag{4}$$

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} & \ldots & A_{1,m+1} \\ * & A_{22} & 0 & \ldots & 0 \\ * & * & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \\ * & * & * & \ldots & A_{m+1,m+1} \end{pmatrix} < 0 \tag{5}$$

Where

$$A_{11} = \sum_{k=1}^{m} (C_{1k} + C_{2k}^T G_{k\varepsilon} R_2^{-1} G_{k\varepsilon}^T C_{3k}),$$

$$A_{ii} = \sum_{k=1}^{m} (G_{k\varepsilon}^T P^T G_{k\varepsilon} R_2^{-1} G_{k\varepsilon}^T P G_{k\varepsilon}) + (C_{4k} G_{k\varepsilon} R_2^{-1} G_{k\varepsilon}^T C_{4k}),\ \ i = 2, \dots m+1,$$

$$A_{1i} = \sum_{k=1}^{m} (C_{2k} - 2C_{4k}),\ \ i = 2, \dots m+1.$$

Proof. We take the following change of the state variable

$$y(t) = e^{\alpha t} x(t), t \in R^+,$$

then the linear delay system (1) is transformed to the delay system

$$\dot{y}(t) = B_0 y(t) + \sum_{k=1}^{m} G_{k\varepsilon} y(t - h_k), \tag{6}$$

Define operators

$$D_k : C([-h, 0], R^n) \longrightarrow R^n,$$

as

$$D_k(y(t)) = y(t) + \int\limits_{t-h_k}^{t} G_{k\varepsilon} y(s) ds, \quad k = 1, 2, 3, \dots, m. \tag{7}$$

then

$$\dot{D}_k(y(t)) = \dot{y}(t) + G_{k\varepsilon} y(t) - G_{k\varepsilon} y(t-h_k) = B_0 y(t) + \sum_{k=1}^{m} G_{k\varepsilon} y(t - h_k) + G_{k\varepsilon} y(t) - G_{k\varepsilon} y(t-h_k)$$

$$= (B_0 + G_{k\varepsilon}) y(t) + \sum_{k=1}^{m} G_{k\varepsilon} y(t - h_k) - G_{k\varepsilon} y(t - h_k).$$

Consider the following Lyapunov-Krasovskii candidate:

$$V(y(t)) = \sum_{k=1}^{m} V_{1k}(y(t)) + V_{2k}(y(t)), \tag{8}$$

where

$$V_{1k}(y(t)) = D_k^T(y(t)) P D_k(y(t)), \tag{9}$$

$$V_{2k}(y(t)) = 3 \int\limits_{t-h_k}^{t} \int\limits_{t+s}^{t} y^T(\rho) R_2 y(\rho) d\rho ds, \tag{10}$$

Taking the time derivative of the Lyapunov functional along the trajectory of system (6) is given by

$$\begin{aligned}
\dot{V}_{1k}(y(t)) &= \dot{D}_k^T(y(t))PD_k(y(t)) + D_k^T(y(t))P\dot{D}_k(y(t)) = 2D_k^T(y(t))P\dot{D}_k(y(t)) \\
&= 2(y(t) + \int_{t-h_k}^{t} G_{k\varepsilon}y(s)ds)^T P((B_0+G_{k\varepsilon})y(t) + \sum_{k=1}^{m} G_{k\varepsilon}y(t-h_k) - G_{k\varepsilon}y(t-h_k)) \\
&= y(t)^T((P(B_0+G_{k\varepsilon}) + (B_0+G_{k\varepsilon})^T P)y(t) - 2y(t)^T PG_{k\varepsilon}y(t-h_k) \\
&+ (2y(t)^T)P\sum_{k=1}^{m} G_{k\varepsilon}y(t-h_k) + 2(\int_{t-h_k}^{t} G_{k\varepsilon}y(s)ds))^T P(B_0+G_{k\varepsilon})y(t) \\
&- 2(\int_{t-h_k}^{t} G_{k\varepsilon}y(s)ds)^T PG_{k\varepsilon}y(t-h_k) + 2(\int_{t-h_k}^{t} G_{k\varepsilon}y(s)ds)^T P\sum_{k=1}^{m} G_{k\varepsilon}y(t-h_k) \\
k &= 1,2,\ldots,m,
\end{aligned} \tag{11}$$

$$\dot{V}_{2k}(y(t)) = 3y(t)^T h_k R_2 y(t) - 3\int_{t-h_k}^{t} y(s)^T R_2 y(s)ds, k = 1,2,\ldots,m, \tag{12}$$

From (11)-(12), we have

$$\begin{aligned}
\sum_{k=1}^{m}\dot{V}_{1k}(y(t)) &= y(t)^T(\sum_{k=1}^{m} P(B_0+G_{k\varepsilon}) + (B_0+G_{k\varepsilon})^T P)y(t) \\
&+ \sum_{k=1}^{m} 2y(t)^T(mPG_{k\varepsilon} - PG_k)y(t-h_k) \\
&+ 2\sum_{k=1}^{m}((\int_{t-h_k}^{t} G_{k\varepsilon}y(s)ds)^T P(B_0+G_{k\varepsilon})y(t)) \\
&- 2\sum_{k=1}^{m}((\int_{t-h_k}^{t} G_{k\varepsilon}y(s)ds)^T PG_{k\varepsilon}y(t-h_k)) \\
&+ 2\sum_{k=1}^{m}((\int_{t-h_k}^{t} G_{k\varepsilon}y(s)ds)^T P(B_0+G_{k\varepsilon})y(t)) \\
&- 2\sum_{k=1}^{m}((\int_{t-h_k}^{t} G_{k\varepsilon}y(s)ds)^T PG_{k\varepsilon}y(t-h_k))
\end{aligned}$$

$$+ \quad 2\sum_{k=1}^{m}((\int_{t-h_k}^{t} G_{k\varepsilon}y(s)ds)^T P\sum_{k=1}^{m} G_{k\varepsilon}y(t-h_k)), \tag{13}$$

$$\sum_{k=1}^{m}\dot{V}_{2k}(y(t)) = 3\sum_{k=1}^{m} y(t)^T h_k R_2 y(t) - 3\sum_{k=1}^{m}\int_{t-h_k}^{t} y(s)^T R_2 y(s)ds,$$

we transform $\dot{V_{1k}}$ and $\dot{V_{2k}}$ to form Inner Product, then by using Eq (6) and Eqs (11)-(12) we get

$$\begin{aligned}
\dot{V}(y(t)) &= \sum_{k=1}^{m}\langle C_{1k}y(t), y(t)\rangle + 2\sum_{k=1}^{m}\langle C_{2k}y(t-h_k), y(t)\rangle \\
&+ 2\sum_{k=1}^{m}\left\langle C_{3k}y(t), \int_{t-h_k}^{t} G_{k\varepsilon}y(s)ds\right\rangle - 2\sum_{k=1}^{m}\left\langle C_{4k}y(t-h_k), \int_{t-h_k}^{t} G_{k\varepsilon}y(s)ds\right\rangle \\
&+ 2\sum_{k=1}^{m}\left\langle PG_{k\varepsilon}y(t-h_k), \int_{t-h_k}^{t} G_{k\varepsilon}y(s)ds\right\rangle + \sum_{k=1}^{m} y(t)^T h_k R_2 y(t) \\
&- 3\sum_{k=1}^{m}\int_{t-h_k}^{t} y(t)^T R_2 y(s)ds - 2\sum_{k=1}^{m} y(t)^T C_{4k} y(t-h_k),
\end{aligned} \tag{14}$$

Where

$C_{1k} = P(B_0 + G_{k\varepsilon}) + (B_0 + G_{k\varepsilon})^T P,$

$C_{2k} = mPG_{k\varepsilon} - PG_{k\varepsilon},$

$C_{3k} = P(B_0 + G_{k\varepsilon}),$

$C_{4k} = PG_{k\varepsilon}.$

By using $2ab \leq a^T Q^{-1} a + b^T Q b$,
we get

$$\begin{aligned}
\dot{V}(y(t)) &\leq \sum_{k=1}^{m}\langle C_{1k}y(t), y(t)\rangle + 2\sum_{k=1}^{m}\langle C_{2k}y(t-h_k), y(t)\rangle - 2\sum_{k=1}^{m} y(t)^T C_{4k} y(t-h_k) \\
&+ \sum_{k=1}^{m} y(t)^T C_{3k}^T G_{k\varepsilon} R_2^{-1} G_{k\varepsilon}^T C_{3k} y(t) + \int_{t-h_k}^{t} y(s)^T R_2 y(s)ds
\end{aligned}$$

$$
\begin{aligned}
&+ \sum_{k=1}^{m}(y(t-h_k)^T G_{k\varepsilon}^T P^T G_{k\varepsilon} R_2^{-1} G_{k\varepsilon}^T P G_{k\varepsilon} y(t-h_k)) + \int_{t-h_k}^{t} y(s)^T R_2 y(s) ds \\
&+ \sum_{k=1}^{m} y(t-h_k)^T C_{4k}^T G_{k\varepsilon} R_2^{-1} G_{k\varepsilon}^T C_{4k} y(t-h_k) + \int_{t-h_k}^{t} y(s)^T R_2 y(s) ds \\
&- 2\sum_{k=1}^{m} y(t)^T C_{4k} y(t-h_k) \leq \sum_{k=1}^{m} y(t)^T (C_{1k} + C_{3k}^T G_{k\varepsilon} R_2^{-1} G_{k\varepsilon}^T C_{3k}) y(t) \\
&+ 2\sum_{k=1}^{m} y(t)^T (C_{2k} - 2C_{4k}) y(t-h_k) \\
&+ 2\sum_{k=1}^{m} y(t-h_k)^T (G_{k\varepsilon} P^T G_{k\varepsilon} R_2^{-1} G_{k\varepsilon}^T P G_{k\varepsilon})(C_{4k}^T G_{k\varepsilon} R_2^{-1} G_{k\varepsilon}^T C_{4k}) y(t-h_k),
\end{aligned}
\tag{15}
$$

$$
= z(t)^T \begin{pmatrix} A_{11} & A_{12} & A_{13} & \dots & A_{1,m+1} \\ * & A_{22} & 0 & \dots & 0 \\ * & * & 0 & \dots & 0 \\ * & * & * & 0\dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \\ * & * & * & \dots & A_{m+1,m+1} \end{pmatrix} z(t),
$$

where $z(t) := [y(t), y(t-h_k), \dots, y(t-h_m)]$. therefore, by condition (5), there is a number $\delta > 0$ such that
$\dot{V}(t, y(t)) \leq -\delta \|z(t)\|^2, \ t \in R^+,$
since
$\|z(t)\|^2 \geq \|y(t)\|^2,$
therefore
$\dot{V}(t, y(t)) \leq -\delta \|y(t)\|^2, \ \ t \in R^+$. and the proof is then completed by the same way as above.

4 Illustrative Examples

Example 1. Consider the linear delay system

$$
\dot{y}(t) = B_0(t)y(t) + \sum_{k=1}^{m} B_k y(t-h_k), t \in R^+,
$$

with
$m = 2, \alpha = 0.9, h_1 = 0.5, h_2 = 1, h = 1, \varepsilon = 0.01,$

$$A_0 = \begin{pmatrix} -2 & 1 \\ -2 & -4 \end{pmatrix}, \qquad A_1 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} 0.05 & 0 \\ 0 & 0.05 \end{pmatrix}.$$

From (3) we have

$$G_{1\varepsilon} = \begin{pmatrix} 0.141148 & 0 \\ 0 & 0.141148 \end{pmatrix}, \qquad G_{2\varepsilon} = \begin{pmatrix} 0.0983841 & 0 \\ 0 & 0.0983841 \end{pmatrix}.$$

Applying condition (5) of Theorem (3.0.4) with $R_2 = I$, we have a positive definite solution

$$P = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.25 \end{pmatrix}.$$

According to the Theorem (3.0.4), the system is 0.9-stable. This system is 0.5-stable in [19].

Example 2. Consider the linear delay system

$$\dot{y}(t) = B_0(t)y(t) + \sum_{k=1}^{m} B_k y(t - h_k), t \in R^+,$$

with
$m = 1, \alpha = 0.1, h = 1, \varepsilon = 0.001.$

$$A_0 = \begin{pmatrix} 0.02 & 0.01 \\ 0.02 & 0.03 \end{pmatrix}, A_1 = \begin{pmatrix} -0.05 & 0.01 \\ 0.02 & -0.2 \end{pmatrix},$$

From (3) we have

$$G_{1\varepsilon} = \begin{pmatrix} -0.0206 & 0 \\ 0 & -0.197 \end{pmatrix}.$$

Applying condition(5) of Theorem (3.0.4) with $R_2 = I$, we obtian

$$P = \begin{pmatrix} 0.5 & 0 \\ 0 & 2 \end{pmatrix},$$

hence by Theorem (3.0.4),the system is 0.1-stable. This system is 0.02-stable in [20].

5 Conclusion

In this paper proposed a new criterion is proposed for α stability linear systems with multiple time delays, By combining the method of decomposing delay term matrices and linear matrix inequality. linear matrix inequality based approach to the α stability analysis was derived through the stability of the operator in [18] and the selection of an appropriated Lyapunov functional. We improved and generalized some previous results. Some numerical examples are also given to show the superiority of our results.

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