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Optimality Analysis of Machine Interference Model with Spares less than Servers

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Abstract

The paper focuses on the optimality analysis of machine interference with spares less than number of servers. The N-R method has been used to optimize the total cost function. Numerical demonstration has also been presented to exhibit the use of the model.

Keywords: Arrival rate, Optimality analysis, Service rate, Spares

1. Introduction

Optimality analysis of any queuing system plays very important role in the study of the efficiency and economic level of the queuing system. A queuing model is of great application if the associated cost is optimum and if associated cost is high it seriously affects the applicability of the queuing model. It is the cost factor which enables us to judge the efficiency of any queuing system.

Shawky (2000) made an attempt on the machine interference model *M/M/C/K/N* with balking, reneging, and spares without computing the cost of the system. Mishra and Mishra (2004), Mishra and Yadav (2008), and Mishra and Shukla (2009) have already developed cost models for various queueing systems.

In the present paper, we attempt the optimality analysis of the machine interference model without balking, reneging, and with number of spares less than number of servers by controlling both the parameters of arrival and service. Previously, no attempt has been made on the optimality analysis of this model.

2. Optimality Analysis of the Model

When $Y < C$ i.e. the number of spares is less than the number of servers then the expected number of customers in the system, as given by Shawky (2000), is

$$L = P_0\left[\sum_{n=1}^{Y-1}\frac{K^n}{(n-1)!}\left(\frac{\lambda}{\mu}\right)^n + \frac{K^Y}{(Y-1)!}\left(\frac{\lambda}{\mu}\right)^Y {}_2F_1\left(1, -K; Y+1; -\frac{\lambda}{\mu}\right) + \frac{K^{Y+1}}{(Y+1)!}\left(\frac{\lambda}{\mu}\right)^{Y+1}\right.$$

$$\times {}_2F_1\left(2, 1-K; Y+2; -\frac{\lambda}{\mu}\right) + \frac{K^Y K_{(C-Y)}}{(C-1)!}\left(\frac{\lambda}{\mu}\right)^C {}_2F_0\left(1, C-K-Y; -; -\frac{\lambda}{C\mu}\right)$$

$$\left. + \frac{K^Y K_{(C-Y+1)}}{C.C!}\left(\frac{\lambda}{\mu}\right)^{C+1} {}_2F_0\left(2, C-K-Y+1; -; -\frac{\lambda}{C\mu}\right)\right] = y P_0,$$ say, where ${}_2F_1$ and ${}_2F_0$ are hyper geometric functions, K is the capacity of the system, L is the expected number of customers in the system, and P_0 is probability of the empty system. We define the total cost function of the system as, $TC = C_1(m+C)\mu + C_2 L$; $m < C$, where, C_1 is the service cost per customer per unit time, C_2 is the waiting cost per customer per unit time, m is the number of customers in the system, and TC is the total cost of the system and optimize it for optimum service and arrival rates by applying N-R method. The following optimum parameters are given in the following table with its graphic representation.

Table-1

$C = 6$, $Y = 3$, $K = 30$, $m = 4$, $C_2 = 2.40$

C_1	λ_0	μ_0	TC_0
3.50	4.39	6.13	269.11
3.80	4.35	6.07	285.25
4.00	4.33	6.05	296.53
4.50	4.27	5.98	323.52
5.00	4.22	5.91	349.92

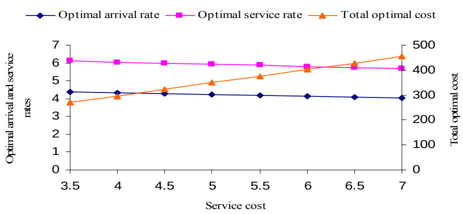

Fig. 1 Service Cost vs. Total Optimal Cost

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