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Computational Approach to Cost and Profit Analysis of Clocked Queueing Networks

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Abstract

Cost and profit analysis of a queueing network problem has a significant value for modern-economics and informatics age. Here, we consider a clocked queueing network with renewal model and developed a computational approach to analyze cost and profit structure of the system and find its optimality with respect to arrival and service parameters of the system by applying advanced technique of optimization for nonlinear systems and in turn a scientific computing with C++ has been applied to numerically demonstrate the model.

Mathematics Subject Classification: 90Bxx, 90B22, 60K25

Keywords: Cost and Profit / Computing / Clocked queueing networks.

Introduction

Although the analysis of networks with queues has occupied a prominent place in Operational Research but its cost and profit analysis is also demanding aspect of today. Percus and Percus [6] analyze time series transformations in clock queueing networks. Authors considered that the messages involved are queued and transmitted one per cycle. In another work, Percus and Percus [7] discussed a model for queue-length in clocked queueing networks. In this paper, authors analyze a discrete network of queues. Moreover, recently, Mishra [4] has focused on the cost analysis of renewal model in clocked queueing network. Author ignored the parameter of arrival rate of the system in this optimization of total cost function.

Till now, cost analysis has taken a significant place in the queueing theory. Several researchers have contributed to this field by their respective works and gave an impetus theoretical development to this area, vide for example, Taha [2], Grass and Hariss [1], Mores [3] but profit analysis along with cost factor is also an important aspect of nowadays. Further, Wang and Ming [8] discussed the profit analysis of $M/E_K/1$ machine repair problem with a non reliable service station. Also, Mishra and Yadav [5] discussed the cost and profit analysis of $M / E_k / 1$ queueing system with removable service station.

There are the notations and their descriptions used in the analysis of this model. λ = arrival rate at the system, μ = service rate in the system, ρ = traffic intensity of the system, E(L) = expected queue length of the system, C_s = cost per service per unit time, C_h = holding cost per customer per unit time, T(C) =total expected cost (TEC) of the system, R = earned revenue for providing service per customer, T (P) = total expected profit of the system, TOC= total optimal cost of the system and TOP= total optimal profit of the system.

Description of the model

Here, it is considered the unit element of the network of a 2x2 buffered switch, which regards as a system of two queues working in parallel, each one with deterministic server of unit serving time. A message entering either of the two inputs of the switch goes with probability half to either of the two output queues of the switch. Any node in the network will see a time series, each element of which is the presence or absence of an information packet to be transmitted to the next node. Thus it is observed that the transformation of discrete time series by the basic process of passage through a queue and of mixing.

Thus, applying above preliminary ideas and assumptions authors [6] ultimately obtained the second stage mean queue length of renewal model in the clocked queueing networks as:

$$\frac{\rho^2}{4(1-\rho)}\left(1+\frac{1}{2}\frac{\rho}{2-\rho}\right)=E(L)\quad \text{(say)} \tag{1}$$

Optimal cost analysis of the system

The expected number of customers in a clocked queue network is given as:

$$E(L)=\frac{4\rho^2-\rho^3}{8\rho^2-24\rho+16}=\frac{X}{Y}, \text{ where } X=4\rho^2-\rho^3, Y=8\rho^2-24\rho+16 \tag{2}$$

Now, the desired total cost function of the system is defined as:

$$T(C)=C_s\mu+C_h\frac{X}{Y}, \tag{3}$$

Differentiating total cost function expressed by equation (3) with respect to μ and λ partially we have:

$$\frac{\partial T(C)}{\partial \mu} = C_s + C_h \frac{YX_1^{'} - XY_1^{'}}{Y^2} = U\,;\, \frac{\partial \text{T(C)}}{\partial \lambda} = C_h \frac{YX_2^{'} - XY_2^{'}}{Y^2} = V \quad \text{(say)} \tag{4}$$

For optimal values of μ and λ, we have the following conditions:

$$\frac{\partial T(C)}{\partial \mu} = C_s + C_h \frac{YX_1^{'} - XY_1^{'}}{Y^2} = 0 \text{ and } \frac{\partial \text{T(C)}}{\partial \lambda} = C_h \frac{YX_2^{'} - XY_2^{'}}{Y^2} = 0 \tag{5}$$

For the solution of above nonlinear systems we require the following expressions that can be obtained from equation (4) as: $\frac{\partial U}{\partial \mu}, \frac{\partial V}{\partial \lambda}, \frac{\partial U}{\partial \lambda}, \frac{\partial V}{\partial \mu}$.

Now, according to the method of optimization of nonlinear system, using above expressions we can get as:

$$(\mu)_{k+1} = (\mu)_k - \frac{U\frac{\partial V}{\partial \lambda} - V\frac{\partial V}{\partial \mu}}{\frac{\partial U}{\partial \mu}\frac{\partial V}{\partial \lambda} - \frac{\partial V}{\partial \mu}\frac{\partial U}{\partial \lambda}}\,;\,(\lambda)_{k+1} = (\lambda)_k - \frac{V\frac{\partial U}{\partial \mu} - U\frac{\partial U}{\partial \lambda}}{\frac{\partial U}{\partial \mu}\frac{\partial V}{\partial \lambda} - \frac{\partial V}{\partial \mu}\frac{\partial U}{\partial \lambda}} \tag{6}$$

Here, terms on the right-hand side are evaluated at $(\mu, \lambda) = ((\mu)_k, (\lambda)_k)$.

Now, using computer programming we compute the optimal values of service rate and arrival rate as μ^* and λ^* along with corresponding Total Optimal Cost (TOC) of the system. The resulting computing table is demonstrated as in table-I.

Optimal profit analysis of the system

In this section, we discuss total optimal profit (TOP) of the system on the basis of the total revenue earned by the system in rendering its service to the customers. Let R be the earned revenue for providing service to each customer then total earned revenue T(R) is given as: TER= $R\,E(L)$.

Total profit = Total earned revenue – Total cost

$$T(P) = (R - C_h)\frac{X}{Y} - C_s\mu \text{, where, } X = 4\rho^2 - \rho^3,\ Y = 16 - 24\rho + 8\rho^2 \tag{7}$$

Differentiating total profit function expressed by equation (7) with respect to μ and λ partially, we have:

$$\frac{\partial T(P)}{\partial \mu} = (R - C_h)\frac{YX_1^{'} - XY_1^{'}}{Y^2} - C_s = S\,;\, \frac{\partial \text{T(P)}}{\partial \lambda} = (R - C_h)\frac{YX_2^{'} - XY_2^{'}}{Y^2} = W \text{ (say)} \tag{8}$$

For optimal values of μ and λ, we have the following conditions:

$$\frac{\partial T(P)}{\partial \mu} = (R - C_h)\frac{YX_1^{'} - XY_1^{'}}{Y^2} - C_s = 0 \text{ and } \frac{\partial \text{T(P)}}{\partial \lambda} = (R - C_h)\frac{YX_2^{'} - XY_2^{'}}{Y^2} = 0 \tag{9}$$

Now, applying non linear technique of optimization as in earlier section we can get the expressions as:

$$(\mu)_{k+1} = (\mu)_k - \frac{S\dfrac{\partial W}{\partial \lambda} - W\dfrac{\partial W}{\partial \mu}}{\dfrac{\partial S}{\partial \mu}\dfrac{\partial W}{\partial \lambda} - \dfrac{\partial W}{\partial \mu}\dfrac{\partial S}{\partial \lambda}};(\lambda)_{k+1} = (\lambda)_k - \frac{W\dfrac{\partial S}{\partial \mu} - S\dfrac{\partial S}{\partial \lambda}}{\dfrac{\partial S}{\partial \mu}\dfrac{\partial W}{\partial \lambda} - \dfrac{\partial W}{\partial \mu}\dfrac{\partial S}{\partial \lambda}} \quad (10)$$

Here, terms on the right- hand side are evaluated at $(\mu,\lambda)= ((\mu)_k , (\lambda)_k)$.

Now, using computer programming, we compute the optimal values of service and arrival rates as μ^* and λ^* along with corresponding Total Optimal Profit (TOP) of the system. The resulting computing table is demonstrated as table-II.

Sensitivity Analysis with Conclusive Observations

This section focuses on the sensitivity analysis of one parameter relative to other parameters for determining the direction of future data-input. We draw the following noteworthy observations on the basis of the optimum values computed for the performance measures of the system:

i. When service and holding cost per unit customer increases, total optimal cost (TOC) of the system also increases (vide for graph-1 & graph-2).

ii Graph-3.indicates that when service cost per unit customer increases, total optimal profit (TOP) of the system decreases.

iii Graph-4 evidences that when revenue per unit customer increases, total optimal profit (TOP) of the system also increases.

The problem of clocked queueing network has wide range of applications in the communication networking which are designed on the basis of various networks of queueing system. Moreover, cost and profit optimization strategies and its applications is a global association of experts in management, business, marketing, market analysis, pricing as well as in supply chain and inventory control.

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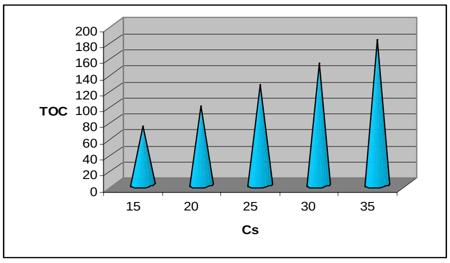
Computing Tables and Graphics

Table- I.
Computation of Total Optimal Cost

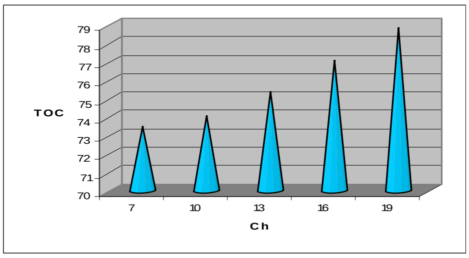
C_s	C_h	(μ^*)	(λ^*)	TOC
15	7	4.554	3.479	73.438
20	7	4.673	3.677	98.581
25	7	4.791	3.874	124.90
30	7	4.910	4.072	152.41
35	7	5.028	4.270	181.11
15	10	4.480	3.301	74.033
15	13	4.390	3.206	75.366
15	16	4.354	3.146	77.021
15	19	4.330	3.105	78.847

Table- II.
Computation of Total Optimal Profit

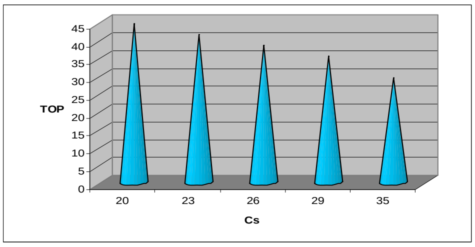
C_s	C_h	R	(μ^*)	(λ^*)	TOP
20	12	125	7.15	4.88	44.45
23	12	125	7.24	4.56	41.45
26	12	125	7.22	4.53	38.45
29	12	125	6.82	5.15	35.45
35	12	125	6.83	5.16	29.45
20	15	125	6.78	5.20	42.74
20	19	125	7.15	4.88	40.46
20	23	125	7.24	4.57	38.18
20	28	125	7.15	4.87	35.33



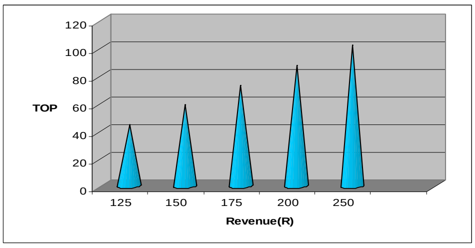
Graph-1. Service Cost Vs. TOC.



Graph-2. Holding Cost Vs. TOC



Graph-3. Service Cost Vs. TOP



Graph-4. Revenue Vs. TOP

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