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Evaluation of Two-Person Games with Fuzzy Data

F. Hosseinzadeh Lotfi [a*], S. Sohraiee [b] and M. Anisi [a]

[a] Department of Mathematics, Science and Research Branch
Islamic Azad University, Tehran, Iran

[b] Department of Mathematics, Tehran North Branch
Islamic Azad University, Tehran, Iran

Abstract

The conventional game theory is based on known payoffs. In the real situations, usually the payoffs are not known and have to be approximated. In this paper, a method for the two-person zero-sum and non-zero-sum games that the payoffs are represented by fuzzy data, has been investigated. The procedure is based on Linear Complementarity Problem (LCP) which unifies bimatrix games.

Keywords: Game Theory, Two-person Games, LCP, Fuzzy Data

1 Introduction

A theory of rational behavior i.e. of foundations of economics and of the main mechanisms of social organization requires a through study of games [3]. One of the most important issues faced by the manager is based on the prediction of the others plans. The real competitive situations may be seen in business, military battles, sport and different cases of contest.
There are different types of games based on the number of players, the number of strategies and the structure of payoff matrix [9]. There are lots of methods developed by different authors to encounter with matrix games [1,4]. One of the methods is based on LCP [5].
Almost all of the methods are based on the precisely known payoffs matrix [2,6,7,8]. However there are lots of situations that the payoffs matrix is not absolutely known and has to be

[*] Corresponding author: Farhad Hosseinzadeh Lotfi
E-mail address: hosseinzadeh_lotfi@yahoo.com

approximated. In this paper, we investigate two-person games with imprecise data represented by fuzzy data. The structure of proposed procedure is based on LCP.
The sections of this paper are given as follows: Section 2, considers some preliminaries of LCP and fuzzy notation and two-person games. In Section 3, two-person games with fuzzy data has been considered. Section 4, presents a numerical example.

2 Preliminaries

In section 2.1 some preliminaries of fuzzy notation has been presented. In section 2.2 some preliminaries of LCP has been discussed and in section 2.3 a basic review of two-person games has been considered.

2.1 Fuzzy notation

Consider n players each have m criteria for evaluating their ability, which is approximated by positive triangular fuzzy number (We call $\tilde{x}_{ij} = (x_{ij}^l, x_{ij}^m, x_{ij}^r)$ as a positive fuzzy number if $x_{ij}^l > 0$) with membership function $\mu_{\tilde{x}_{ij}}$.
Let $S(\tilde{x}_{ij})$ denote the support of $\tilde{x}_{ij}$. The $\alpha - cuts$, also known as the $\alpha - level$ sets, of $\tilde{x}_{ij}$ is defined as
$(x_{ij})_\alpha = \{x_{ij} \in S(\tilde{x}_{ij}) | \mu_{\tilde{x}_{ij}}(x_{ij}) \geq \alpha\} i = 1,\ldots,m \ , j = 1,..,n$
It can also be expressed in another form:

$$(x_{ij})_\alpha = \left[(x_{ij}^l)_\alpha, (x_{ij}^u)_\alpha\right] = \left[\min_{x_{ij}} \{x_{ij} \in \tilde{x}_{ij} | \mu_{\tilde{x}_{ij}}(x_{ij}) \geq \alpha\}, \max_{x_{ij}} \{x_{ij} \in \tilde{x}_{ij} | \mu_{\tilde{x}_{ij}}(x_{ij}) \geq \alpha\}\right] \quad (1)$$

Also, it is obvious that $\left[(x_{ij}^l)_{\alpha_1}, (x_{ij}^u)_{\alpha_1}\right] \subseteq \left[(x_{ij}^l)_{\alpha_2}, (x_{ij}^u)_{\alpha_2}\right] if \ 0 < \alpha_2 < \alpha_1 \leq 1$.
Therefore, $\{x_{ij} \in S(\tilde{x}_{ij}) | \mu_{\tilde{x}_{ij}}(x_{ij}) \geq \alpha\}$ and $\{x_{ij} \in S(\tilde{x}_{ij}) | \mu_{\tilde{x}_{ij}}(x_{ij}) = \alpha\}$ have the same smallest and largest elements. We use the concept of $\alpha - cuts$ to propose a solution.

2.2 Linear Complementarity Problem

Let M be a given square matrix of order n and q a column vector in R^n. Throughout this paper we will use the symbols $w_1, w_2, \ldots, w_n$ and $z_1, z_2, \ldots, z_n$ to denote the variables in the problem. In an LCP there is no objective function to be optimized. The problem is: find $W = (w_1, w_2, \ldots, w_n)^T$ and $Z = (z_1, z_2, \ldots, z_n)^T$ satisfying

$$W - MZ = q$$
$$W \geq 0\,, Z \geq 0 \qquad (2)$$
$$w_i z_i = 0 \qquad \textit{for all } i$$

The only data in the problem is the column vector q and the square matrix $M_{n\times n}$. So we will denote the LCP of finding $W \in R^n$, $Z \in R^n$ satisfying (2) by the symbol (q, M) .It is said to be an LCP of order n. In an LCP of order n there are $2n$ variables.

2.3 Two-person games

Consider a game where in each play of the game, player I picks one out of a possible set of his m choices and independently player II picks one out of a possible set of his N choices. In a play, if player I has picked his choice, i, and player II has picked his choice j, then player I loses amount a'_{ij} dollars and player II loses an amount b'_{ij} dollar, where $A' = (a'_{ij})$ and $B' = (b'_{ij})$ are given loss matrix. If $a'_{ij} + b'_{ij} = 0$ for all i, j the game is known as a zero-sum game; in this case it is possible to develop the concept of an optimum strategy for playing the game using Von Neumann's Minimax theorem. Games that are not zero-sum games are called non-zero-sum games or bimatrix games. In bimatrix games it is difficult to define an optimum strategy. However, in this case, an equilibrium pair of strategies can be defined and problem of computing an equilibrium pair of strategies can be transformed in to an LCP. Suppose player I picks his choice i with probability of $x_{i.}$ The column vector $x = (x_i) \in R^m$ completely defines player I's strategy. Similarly let the probability vector $y = (y_j) \in R^N$ be player II's strategy. If player I adopts strategy x and player II adopts strategy y ,the expected loss of player I is obviously $x^T A' y$ And that of player II is $x^T B' y$.

The strategy pair $(\bar{x}, \bar{y})$ is said to be an equilibrium pair if no player benefits by unilaterally changing his own strategy while the other player keeps his strategy in the pair $(\bar{x}, \bar{y})$ unchanged, that is, if

$$\begin{cases} \bar{x}^T A' \,\bar{y} \leq x^T A' \,\bar{y} & ,\textit{For all probability vector } x \in R^m \\ \bar{x}^T B' \,\bar{y} \leq \bar{x}^T B' \,y & ,\textit{For all probability vector } x \in R^N \end{cases} \qquad (3)$$

Let $\bar{\alpha}, \bar{\beta}$ be arbitrary positive numbers such that $a_{ij} = a'_{ij} + \bar{\alpha} > 0$ and $b_{ij} = b'_{ij} + \bar{\beta} > 0$ For all i, j. Let $A = (a_{ij})$, $B = (b_{ij})$ Since $x^T A' y = x^T A y - \bar{\alpha}$ and $x^T B' y = x^T B y - \bar{\beta}$ for all probability vector $x \in R^m$ and $y \in R^N$, if $(\bar{x}, \bar{y})$ is an equilibrium pair of strategies for the game with loss matrices A', B' then $(\bar{x}, \bar{y})$ is an equilibrium pair of strategies for the game with loss matrices A, B and vice versa. So without any loss of generality, consider the game in which the loss matrices are A, B. Since x is a probability vector, the condition $\bar{x}^T A\bar{y} \le x^T A\bar{y}$ for all probability vectors $x \in R^m$ is equivalent to the system of constraints

$$\bar{x}^T A\bar{y} \le A_i \bar{y} \qquad (\textit{for all } i = 1,2,...,m) \tag{4}$$

Let e_r denote the column vector in R^r in which all elements are equal to 1. In matrix notation the above system of constraints can be written as $(\bar{x}^T A\bar{y})e_m \le A\bar{y}$. In similar way the condition $\bar{x}^T B\bar{y} \le \bar{x}^T B y$ by for all probability vectors $y \in R^N$ is equivalent to $(\bar{x}^T B\bar{y})e_N \le B^T \bar{x}$. Hence the strategy pair $(\bar{x}, \bar{y})$ is an equilibrium pair of strategies for the game with loss matrices A, B iff

$$\begin{aligned} A\bar{y} &\ge (\bar{x}^T A\bar{y})e_m \\ B^T \bar{x} &\ge (\bar{x}^T B\bar{y})e_N \end{aligned} \tag{5}$$

Since A, B are strictly positive matrices, $\bar{x}^T A\bar{y}$ and $\bar{x}^T B\bar{y}$ are strictly positive numbers. Let

$$\bar{\xi} = \frac{\bar{x}}{\bar{x}^T B\bar{y}} \quad \text{and} \quad \bar{\eta} = \frac{\bar{y}}{\bar{x}^T A\bar{y}} \tag{6}$$

Introducing slack variables corresponding to the inequality constraints, (5) is equivalent to

$$\begin{aligned} &\begin{bmatrix} \bar{u} \\ \bar{v} \end{bmatrix} - \begin{pmatrix} 0 & A \\ B^T & 0 \end{pmatrix} \begin{bmatrix} \bar{\xi} \\ \bar{\eta} \end{bmatrix} = \begin{bmatrix} -e_m \\ -e_N \end{bmatrix} \\ &\begin{bmatrix} \bar{u} \\ \bar{v} \end{bmatrix} \ge 0, \begin{bmatrix} \bar{\xi} \\ \bar{\eta} \end{bmatrix} \ge 0 \\ &\begin{bmatrix} \bar{u} \\ \bar{v} \end{bmatrix}^T \begin{bmatrix} \bar{\xi} \\ \bar{\eta} \end{bmatrix} = 0 \end{aligned} \tag{7}$$

Conversely, if $(\bar{u}, \bar{v}, \bar{\xi}, \bar{\eta})$ is a solution of LCP (7) then the equilibrium pair of strategies for the original game is $(\bar{x}, \bar{y})$ where $\bar{x} = \dfrac{\bar{\xi}}{\sum \bar{\xi}_i}$ and $\bar{y} = \dfrac{\bar{\eta}}{\sum \bar{\eta}_i}$ (8)

Therefore

$$\begin{pmatrix} 0 & A \\ B^T & 0 \end{pmatrix} \begin{pmatrix} \bar{\xi} \\ \bar{\eta} \end{pmatrix} \ge \begin{pmatrix} e_m \\ e_N \end{pmatrix} \Rightarrow \begin{cases} A\bar{\eta} \ge e_m & \Rightarrow A(\bar{y}\sum \bar{\eta}_j) \ge e_m \\ B^T \bar{\xi} \ge e_N & \Rightarrow B^T(\bar{x}\sum \bar{\xi}_i) \ge e_N \end{cases} \tag{9}$$

Then we have

$$\begin{cases} A\bar{y} \ge \dfrac{1}{\sum \bar{\eta}_j} e_m \\ B^T \bar{x} \ge \dfrac{1}{\sum \bar{\xi}_i} e_N \end{cases} \tag{10}$$

By defining:

$$\overline{x}^{\,T} A \overline{y} = \frac{1}{\sum \overline{\eta}_j} \quad , \quad \overline{x}^{\,T} B \overline{y} = \frac{1}{\sum \overline{\xi}_i} \tag{11}$$

We have

$$\begin{cases} A\overline{y} \geq (\overline{x}^{\,T} A \overline{y})\, e_m \\ B^T \overline{x} \geq (\overline{x}^{\,T} B \overline{y})\, e_N \end{cases} \tag{12}$$

Therefore the conversely part has been established.
In the next section we develop the above mentioned procedure with fuzzy data and we extent the procedure when data are not precisely known and are fuzzy.

3 Two-person games with fuzzy data

In this section, we consider the problem of two-person games with fuzzy data. Since almost all the time data are not absolutely known and are approximated by manager the procedure of two-person games with fuzzy data has an important role in Game theory.
Based on section 2.3 consider a game where in each play of the game, player I picks one out of a possible set of his m choices and independently player II picks one out of a N choices. In a play if player I has picked his choice, i, and player II has picked his choice, j, then player I loses an amount $a_{ij} \in \left[a_{ij}^l, a_{ij}^u\right]$ dollar that is a α-cut (0 ≤ α ≤1) of fuzzy data and player II loses an amount $b_{ij} \in \left[b_{ij}^l, b_{ij}^u\right]$ dollar that is a α-cut (0 ≤ α ≤1) of fuzzy data, where

$$B = \begin{pmatrix} B^l & 0 \\ 0 & B^u \end{pmatrix}_{2m\times 2N} \quad \text{and} \quad A = \begin{pmatrix} A^l & 0 \\ 0 & A^u \end{pmatrix}_{2m\times 2N} \tag{13}$$

are given fuzzy loss matrices, $a_{ij}^l > 0$ and $b_{ij}^l > 0$.
Suppose player I picks his choice i with probability of $x_i + x_{m+i}$.
The column vector

$$x = \begin{pmatrix} x_1 + x_{m+1} \\ \vdots \\ x_m + x_{2m} \end{pmatrix}_{m\times 1} \tag{14}$$

completely defines player I's strategy. Particularly, in this game the lower bound and upper bound of fuzzy $\left[a_{ij}^l, a_{ij}^u\right]$ have their own contribution to be selected by probability x_i and x_{m+i}, respectively. Similarly let the probability vector

$$y = \begin{pmatrix} y_1 + y_{N+1} \\ \vdots \\ y_N + y_{2N} \end{pmatrix}_{N\times 1} \tag{15}$$

be player II's strategy. In order to tackle the problem and based on some technical difficulties to transform the game theory with fuzzy data to LCP, we use the following pair of primary strategies, without loss the generality of structure of probability column vector (x, y) defined in (14) and (15), in the following manner:

$$x_p = \begin{pmatrix} x_1 \\ \vdots \\ x_m \\ x_{m+1} \\ \vdots \\ x_{2m} \end{pmatrix}_{2m\times 1} = \begin{pmatrix} x_p^l \\ x_p^u \end{pmatrix} \text{ and } y_p = \begin{pmatrix} y_1 \\ \vdots \\ y_N \\ y_{N+1} \\ \vdots \\ y_{2N} \end{pmatrix}_{2N\times 1} = \begin{pmatrix} y_p^l \\ y_p^u \end{pmatrix} \tag{16}$$

The pair of primary strategy $(\overline{x}_p, \overline{y}_p)$ is an equilibrium pair of primary strategies for the game with loss matrices A, B iff

$$\begin{cases} \overline{x}_p^T A\, \overline{y}_p \le x^T A\, \overline{y}_p & ,\forall x \in R^{2m} \\ \overline{x}_p^T B\, \overline{y}_p \le \overline{x}_p^T B\, y & ,\forall y \in R^{2N} \end{cases} \tag{17}$$

It means that for all probability column vector $x \in R^{2m}, y \in R^{2N}$, we have

$$\begin{cases} (\overline{x}_p^{l^T}\ \overline{x}_p^{u^T}) \begin{pmatrix} A^l & 0 \\ 0 & A^u \end{pmatrix} \begin{pmatrix} \overline{y}_p^l \\ \overline{y}_p^u \end{pmatrix} \le (x^{l^T}\ x^{u^T}) \begin{pmatrix} A^l & 0 \\ 0 & A^u \end{pmatrix} \begin{pmatrix} \overline{y}_p^l \\ \overline{y}_p^u \end{pmatrix} & ,\forall x \in R^{2m} \\ (\overline{x}_p^{l^T}\ \overline{x}_p^{u^T}) \begin{pmatrix} B^l & 0 \\ 0 & B^u \end{pmatrix} \begin{pmatrix} \overline{y}_p^l \\ \overline{y}_p^u \end{pmatrix} \le (\overline{x}_p^{l^T}\ \overline{x}_p^{u^T}) \begin{pmatrix} B^l & 0 \\ 0 & B^u \end{pmatrix} \begin{pmatrix} y^l \\ y^u \end{pmatrix} & ,\forall y \in R^{2N} \end{cases} \tag{18}$$

Hence

$$\begin{cases} \overline{x}_p^{l^T} A^l\, \overline{y}_p^l + \overline{x}_p^{u^T} A^u\, \overline{y}_p^u \le x^{l^T} A^l\, \overline{y}_p^l + x^{u^T} A^u\, \overline{y}_p^u & ,\forall x \in R^{2m} \\ \overline{x}_p^{l^T} B^l\, \overline{y}_p^l + \overline{x}_p^{u^T} B^u\, \overline{y}_p^u \le \overline{x}_p^{l^T} B^l y^l + \overline{x}_p^{u^T} B^u y^u & ,\forall y \in R^{2N} \end{cases} \tag{19}$$

Or

$$\begin{cases} \begin{pmatrix} A^l & 0 \\ 0 & A^u \end{pmatrix}\begin{pmatrix} \bar{y}_p^l \\ \bar{y}_p^u \end{pmatrix} \geq \left[({\bar{x}_p^l}^T \ {\bar{x}_p^u}^T) \begin{pmatrix} A^l & 0 \\ 0 & A^u \end{pmatrix}\begin{pmatrix} \bar{y}_p^l \\ \bar{y}_p^u \end{pmatrix} \right] e_{2m} \\ \begin{pmatrix} {B^l}^T & 0^T \\ 0^T & {B^u}^T \end{pmatrix}\begin{pmatrix} \bar{x}_p^l \\ \bar{x}_p^u \end{pmatrix} \geq \left[({\bar{x}_p^l}^T \ {\bar{x}_p^u}^T) \begin{pmatrix} B^l & 0 \\ 0 & B^u \end{pmatrix}\begin{pmatrix} \bar{y}_p^l \\ \bar{y}_p^u \end{pmatrix} \right] e_{2N} \end{cases} \tag{20}$$

Therefore we have

$$\begin{pmatrix} \bar{u} = \begin{pmatrix} \bar{u}^l \\ \bar{u}^u \end{pmatrix}_{2\times 1} \\ \bar{v} = \begin{pmatrix} \bar{v}^l \\ \bar{v}^u \end{pmatrix}_{2\times 1} \end{pmatrix}_{2\times 1} - \begin{pmatrix} 0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{2\times 2} & A = \begin{pmatrix} A^l & 0 \\ 0 & A^u \end{pmatrix}_{2\times 2} \\ B^T = \begin{pmatrix} {B^l}^T & 0^T \\ 0^T & {B^u}^T \end{pmatrix}_{2\times 2} & 0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{2\times 2} \end{pmatrix}_{2\times 2} \begin{pmatrix} \bar{\xi} = \begin{pmatrix} \bar{\xi}^l \\ \bar{\xi}^u \end{pmatrix} \\ \bar{\eta} = \begin{pmatrix} \bar{\eta}^l \\ \bar{\eta}^u \end{pmatrix} \end{pmatrix} = \begin{pmatrix} -e_{2m} \\ -e_{2N} \end{pmatrix} \tag{21}$$

$$\begin{pmatrix} \bar{u} \\ \bar{v} \end{pmatrix} \geq 0 \ , \ \begin{pmatrix} \bar{\xi} \\ \bar{\eta} \end{pmatrix} \geq 0 \ , \ \begin{pmatrix} \bar{u} \\ \bar{v} \end{pmatrix}\begin{pmatrix} \bar{\xi} \\ \bar{\eta} \end{pmatrix} = 0$$

Above problem is an LCP.

Conversely, now let $(\bar{u}, \bar{v}, \bar{\xi}, \bar{\eta})$ be a solution of LCP (21), by defining

$$\begin{cases} \bar{x}_p^l = \dfrac{\bar{\xi}^l}{\sum(\bar{\xi}_i^l + \bar{\xi}_i^u)} \\ \bar{x}_p^u = \dfrac{\bar{\xi}^u}{\sum(\bar{\xi}_i^l + \bar{\xi}_i^u)} \end{cases} \quad \text{and} \quad \begin{cases} \bar{y}_p^l = \dfrac{\bar{\eta}^l}{\sum(\bar{\eta}_j^l + \bar{\eta}_j^u)} \\ \bar{y}_p^u = \dfrac{\bar{\eta}^u}{\sum(\bar{\eta}_j^l + \bar{\eta}_j^u)} \end{cases} \tag{22}$$

We will have

$$\begin{pmatrix} 0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{2\times 2} & A = \begin{pmatrix} A^l & 0 \\ 0 & A^u \end{pmatrix}_{2\times 2} \\ B^T = \begin{pmatrix} {B^l}^T & 0^T \\ 0^T & {B^u}^T \end{pmatrix}_{2\times 2} & 0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{2\times 2} \end{pmatrix}_{2\times 2} \begin{pmatrix} \bar{\xi} = \begin{pmatrix} \bar{\xi}^l \\ \bar{\xi}^u \end{pmatrix} \\ \bar{\eta} = \begin{pmatrix} \bar{\eta}^l \\ \bar{\eta}^u \end{pmatrix} \end{pmatrix} \geq \begin{pmatrix} -e_{2m} \\ -e_{2N} \end{pmatrix} \tag{23}$$

By multiplying the above inequalities, we have

$$\begin{aligned} & A^l \bar{\eta}^l + A^u \bar{\eta}^u \geq e_{2m} \\ & B^{l^T} \bar{\xi}^l + B^{u^T} \bar{\xi}^u \geq e_{2N} \end{aligned} \tag{24}$$

Consequently we have,

$$\begin{aligned} & A^l \bar{y}_p^l \left(\sum(\bar{\eta}_j^l + \bar{\eta}_j^u)\right) + A^u \bar{y}_p^u \left(\sum(\bar{\eta}_j^l + \bar{\eta}_j^u)\right) \geq e_{2m} \\ & B^{l^T} \bar{x}_p^l \left(\sum(\bar{\xi}_i^l + \bar{\xi}_i^u)\right) + B^{u^T} \bar{x}_p^u \left(\sum(\bar{\xi}_i^l + \bar{\xi}_i^u)\right) \geq e_{2N} \end{aligned} \tag{25}$$

By defining:

$$\begin{cases} \dfrac{1}{\sum(\overline{\eta}_j^{\,l} + \overline{\eta}_j^{\,u})} = \overline{x}_p^{l^T} A^l \, \overline{y}_p^l + \overline{x}_p^{u^T} A^u \, \overline{y}_p^u \\ \dfrac{1}{\sum(\overline{\xi}_i^{\,l} + \overline{\xi}_i^{\,u})} = \overline{x}_p^{l^T} \overline{B}^l \, \overline{y}_p^l + \overline{x}_p^{u^T} \overline{B}^u \, \overline{y}_p^u \end{cases} \tag{26}$$

We have the following inequalities:

$$\begin{aligned} &A^l \, \overline{y}_p^l + A^u \, \overline{y}_p^u \geq (\overline{x}_p^{l^T} A^l \, \overline{y}_p^l + \overline{x}_p^{u^T} A^u \, \overline{y}_p^u) e_{2m} \\ &B^{l^T} \overline{x}_p^l + B^{u^T} \overline{x}_p^u \geq (\overline{x}_p^{l^T} \overline{B}^l \, \overline{y}_p^l + \overline{x}_p^{u^T} \overline{B}^u \, \overline{y}_p^u) e_{2N} \end{aligned} \tag{27}$$

and it means that:

$$\begin{cases} (\overline{x}_p^{l^T} \; \overline{x}_p^{u^T}) \begin{pmatrix} A^l & 0 \\ 0 & A^u \end{pmatrix} \begin{pmatrix} \overline{y}_p^l \\ \overline{y}_p^u \end{pmatrix} \leq (x^{l^T} \; x^{u^T}) \begin{pmatrix} A^l & 0 \\ 0 & A^u \end{pmatrix} \begin{pmatrix} \overline{y}_p^l \\ \overline{y}_p^u \end{pmatrix} & , \forall x \in R^{2m} \\ (\overline{x}_p^{l^T} \; \overline{x}_p^{u^T}) \begin{pmatrix} B^l & 0 \\ 0 & B^u \end{pmatrix} \begin{pmatrix} \overline{y}_p^l \\ \overline{y}_p^u \end{pmatrix} \leq (\overline{x}_p^{l^T} \; \overline{x}_p^{u^T}) \begin{pmatrix} B^l & 0 \\ 0 & B^u \end{pmatrix} \begin{pmatrix} y^l \\ y^u \end{pmatrix} & , \forall y \in R^{2N} \end{cases} \tag{28}$$

eventually, it has been considered that:

$$\begin{cases} \overline{x}_p^T A \, \overline{y}_p \leq x^T A \, \overline{y}_p & \forall x \in R^{2m} \\ \overline{x}_p^T B \, \overline{y}_p \leq \overline{x}_p^T B \, y & \forall y \in R^{2N} \end{cases} \tag{29}$$

Therefore, the conversely part has also been approved.

Consequently, we established that two-person games with fuzzy data may be solved by LCP. It remains that we show how one may use the above mentioned procedure in practice. The equilibrium pair of strategies $(\overline{x}, \overline{y})$ may be defined based on the equilibrium pair of primary strategies $(\overline{x}_p, \overline{y}_p)$ in the following manner:

$$(\overline{x}, \overline{y}) = \begin{pmatrix} \overline{x}_1 + \overline{x}_{m+1} \\ \vdots \\ \overline{x}_m + \overline{x}_{2m} \\ \overline{y}_1 + \overline{y}_{N+1} \\ \vdots \\ \overline{y}_N + \overline{y}_{2N} \end{pmatrix}^T_{(m+N)\times 1} \tag{30}$$

Therefore we utilize the equilibrium pair of strategies $(\overline{x}, \overline{y})$ in practical problems. It is obvious that above mentioned procedure is an approximation and one may define different approximation.

4 Numerical example

Consider a game in which the fuzzy (α=0) loss matrices are

$$A^L = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad , A^u = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 5 & 4 \end{bmatrix}$$

$$B^L = \begin{bmatrix} 2 & 3 & 2 \\ 3 & 1 & 3 \end{bmatrix} \quad , B^u = \begin{bmatrix} 2 & 4 & 3 \\ 6 & 2 & 4 \end{bmatrix}$$

Let the following LCP be the equations based on above mentioned loss matrices:

$$\begin{bmatrix} \bar{u}_1^l \\ \bar{u}_2^l \\ \bar{u}_1^u \\ \bar{u}_2^u \\ \bar{v}_1^l \\ \bar{v}_2^l \\ \bar{v}_3^l \\ \bar{v}_1^u \\ \bar{v}_2^u \\ \bar{v}_3^u \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 5 & 4 \\ 2 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{\xi}_1^l \\ \bar{\xi}_2^l \\ \bar{\xi}_1^u \\ \bar{\xi}_2^u \\ \bar{\eta}_1^l \\ \bar{\eta}_2^l \\ \bar{\eta}_3^l \\ \bar{\eta}_1^u \\ \bar{\eta}_2^u \\ \bar{\eta}_3^u \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} \qquad (31)$$

s.t

$$\left[\bar{u}_1^l, \bar{u}_2^l, \bar{u}_1^u, \bar{u}_2^u, \bar{v}_1^l, \bar{v}_2^l, \bar{v}_3^l, \bar{v}_1^u, \bar{v}_2^u, \bar{v}_3^u\right]^T \geq 0$$

$$\left[\bar{\xi}_1^l, \bar{\xi}_2^l, \bar{\xi}_1^u, \bar{\xi}_2^u, \bar{\eta}_1^l, \bar{\eta}_2^l, \bar{\eta}_3^l, \bar{\eta}_1^u, \bar{\eta}_2^u, \bar{\eta}_3^u\right]^T \geq 0$$

$$\left[\bar{u}_1^l, \bar{u}_2^l, \bar{u}_1^u, \bar{u}_2^u, \bar{v}_1^l, \bar{v}_2^l, \bar{v}_3^l, \bar{v}_1^u, \bar{v}_2^u, \bar{v}_3^u\right]^T \left[\bar{\xi}_1^l, \bar{\xi}_2^l, \bar{\xi}_1^u, \bar{\xi}_2^u, \bar{\eta}_1^l, \bar{\eta}_2^l, \bar{\eta}_3^l, \bar{\eta}_1^u, \bar{\eta}_2^u, \bar{\eta}_3^u\right]^T = 0$$

By setting:

$$\left[\bar{u}_1^l, \bar{u}_2^l, \bar{u}_1^u, \bar{u}_2^u, \bar{v}_1^l, \bar{v}_2^l, \bar{v}_3^l, \bar{v}_1^u, \bar{v}_2^u, \bar{v}_3^u\right]^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \qquad (32)$$

Then

$$\bar{\eta}_1^l = \bar{\eta}_3^l = 0 \quad , \quad \bar{\eta}_2^l = 1 \quad , \quad \bar{\eta}_1^u = \bar{\eta}_3^u = \frac{1}{6} \quad , \quad \bar{\eta}_2^u = 0$$

$$\bar{\xi}_1^l = \frac{2}{3} \quad , \quad \bar{\xi}_2^l = \frac{1}{7} \quad , \quad \bar{\xi}_1^u = \frac{2}{10} \quad , \quad \bar{\xi}_2^u = \frac{1}{10} \qquad (33)$$

With respect to (21),

$$\overline{x}_1^l = \frac{20}{51} \quad , \quad \overline{x}_2^l = \frac{10}{51} \quad , \quad \overline{x}_1^u = \frac{14}{51} \quad , \quad \overline{x}_2^u = \frac{7}{51}$$
$$\overline{y}_1^l = \overline{y}_3^l = \overline{y}_2^u = 0 \quad , \quad \overline{y}_1^u = \frac{1}{8} \quad , \quad \overline{y}_2^l = \frac{3}{4} \quad , \quad \overline{y}_3^u = \frac{1}{8} \tag{34}$$

Therefore with respect to (13) , (14),

$$\overline{x}_1 = \overline{x}_1^l + \overline{x}_1^u = \frac{34}{51} \quad , \quad \overline{x}_2 = \overline{x}_2^l + \overline{x}_2^u = \frac{17}{51}$$
$$\overline{x} = (\overline{x}_1, \overline{x}_2) = (\frac{34}{51}, \frac{17}{51}) \tag{35}$$

By the similar way,

$$\overline{y}_1 = \frac{1}{8} \quad , \quad \overline{y}_2 = \frac{3}{4} \quad , \quad \overline{y}_3 = \frac{1}{8}$$
$$\overline{y} = (\overline{y}_1, \overline{y}_2, \overline{y}_3) = (\frac{1}{8}, \frac{3}{4}, \frac{1}{8}) \tag{36}$$

And $(\overline{x}, \overline{y})$ is an equilibrium pair of strategies.

5 Conclusion

The traditional game theory is based on known payoffs. In the realistic situations, almost all the times the payoffs are not known and may be considered as fuzzy. In this paper, the two-person games with fuzzy data have been investigated. It has been shown that the two-person games with fuzzy data can be transformed to LCP. Therefore the two-person games with fuzzy data may be solved by the structure of LCP. The probability structure of equilibrium pair of strategies have been modified by the probability structure of equilibrium pair of primary strategies, because of some technical difficulties without losing the probability structure of equilibrium pair of strategies. We strongly emphasize that the procedure introduced in this paper, is an approximation and one may define different approximation. We hope that our extension may be useful.

References

[1] Bazaraa, M, S., Jarvis, J, J., & Sherali, H.D. (1990). *Linear Programming and Network Flows* (2nd edition). New York: John Wiley & Sons.

[2] Kumar, S., Reddy, D., S., N., (1999). *Graphical Solution of (n×m) matrix of a game theory*. European Journal of Operational Research, 112, 467-471.

[3] Liu, S.-T., Kao, C., (2008). *Matrix games with interval data*. Computers &Industrial Engineering, 56, 1697-1700

[4] Murty, Katta, G. (1983) *Linear Programming*. New York: John Wiley & Sons.

[5] Murty, Katta, G., Yu, F-T. (1997). *Linear Complementarity, Linear and Nonlinear Programming* (Internet edition)

[6] Nair, K., G., K., Ranjith, G. (1999). *Solution of (3×3) games using graphical method*. European Journal of Operational Research, 112, 472-478.

[7] Serafini, P. (2005) *Linear programming with variable matrix entries*. Operations Research Letters, 33, 165-170.

[8] Vijay, V., Chandra, S., Bector, C., R. (2005). *Matrix games with fuzzy goals and fuzzy pay offs*. Omega, 33, 425-429.

[9] Von Neumann, John., Morgenstern, Oskar., (1953). *Theory of Games and Economic Behavior* (3rd Edition). Princeton: Princeton University Press

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