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A Wavelets Algorithm for the Seismic Waves Alignment

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Abstract

The seismic waves alignment is a necessary operation in the pretreatment step for geophysics data interpretation, in particular for a certain number of separation methods of seismic waves. In fact, the classical filtering technics of seismic signals are sensitive to a good waves alignment (infinite apparent velocity of propagation on the array sensor).

We propose in this work, a new algorithm based on the wavelet transform concept (time-frequency representation) applied on the seismic trace. The objective of the algorithm is to perform the waves alignment. This method is adapted to a complex geological field case where the waves are generating delays and phase shift. Our calculation methodology is presented. Furthermore, We have simulated this method on Matlab and The simulations tests are numerically satisfactory.

Keywords: Seismic Profile,waves alignment, modeling, Seismic trace, wavelet transform, time-delay, phase shift

1 Introduction

The seismic waves alignment is a fundamental step in various applications in multidimensional signals treatment, in particular for seismic marine, acoustics, biomedical, EMG and ECG signals,etc.

In geophysics, the propagation of waves takes place in a complex field, which generates delayed waves (velocity), phase shift, and sometimes dispersal (surface waves). Therefore, the estimation of the delay and phase shift is a step of the seismic profile pre-treatment, and usually a means to separate the waves.

Actually, the waves seismic separation technics are based on the "good" waves alignment who will be extracted. As an example, the method of separation with the singular value decomposition SVD (see [1]), f-k filtering or f-k median filtering (see[2]) are "sensitive" to the "good" waves alignment.

In this paper, We propose a new method for seismic waves alignment based on the estimation of time delay and phase shift by the concept of the continuous wavelet transform. We demonstrate that through modeling the seismic signal, choosing a wavelet analysis and a scale factor, it will be possible to do the denoising and meanwhile give an estimation of the delay time and the phase shift of the seismic waves, which enable to achieve a fine wave alignment, that is more adapted to a propagation in a complex field. We also demonstrate, basing on synthetic signals and in the presence of noise of diverse powers, the interest and performance of this approach.

Moreover, We represent our Alignment algorithm that illustrates our calculation methodology. Finally, we present the result of our simulation tests on Matlab.

2 A seismic signal Modelling

The seismic exploration methods consist of provoking a shock in the substratum through an explosion, vibrating truck, weight fall,...etc, and observing superficially the reflected waves on the geologic or refracted layers along certain interfaces (see [3]).

A seismic signal, also called a seismic trace, is obtained by the reflexing of a seismic wave on the different layers of rock; the noted seismic trace $f(t)$ is the response registered by a sensor (geophone or hydrophone).

Generally, for N sources emitting $s_k(t)$ signals (impulsions) with $k = 1, .., N$ and after a propagation in a supposedly homogeneous and isotropic complex field (there is no dissipation of energy and there is a presence of delays and phase shift), the received seismic on a given sensor can b written as the following (see [4] and [5]):

$$f(t) = \sum_{k=1}^{N} a_k \delta(t - \tau_k) e^{i\varphi_k} * s_k(t) + b_k(t) \tag{1}$$

with: τ_k and φ_k are respectively the delay and the phase shift of the k^{rd} wave on the sensor, a_k represents the gain in amplitude and b_k the additive noise.

To simplify, we consider the case of a single wave received on two different sensors.

Throughout our analysis, we suppose that we have one source emitting the wave $s(t)$ supposed not to be dispersed, and respectively noting by $X(t)$ and $Y(t)$ the signal received on the first sensor and the same signal received on the second sensor,and which is delayed and defased (phase shift) of the first one:

$$X(t) = s(t) + b_x(t) \tag{2}$$

$$Y(t) = s(t - \tau) e^{i\varphi} + b_y(t) \tag{3}$$

wherein,

- $S(t)$ represents the unknown reference signal.
- $s(t - \tau)e^{i\varphi}$ is the same delayed signal of τ and the dephased one of φ.
- $b_x(t)$ and $b_y(t)$ are two noise sources supposed to be of high frequencies (see[3]).

For the alignment of the second trace, the problem lies in estimating the delay τ and the phase shift φ.

3 The Continuous Wavelet Transform

During this work, we consider the Hilbert space (space of the signals of finite energy):

$$L^2(R) = \{f : R \to R / \int_R |f(t)|^2 \, dt < +\infty\}.$$

A function $\psi \in L^2(R) \cap L^1(R)$ is a wavelet if she verified the admissibility condition:

$$C_\psi = \int_{-\infty}^{+\infty} \frac{\left|\widehat{\psi}(\xi)\right|^2}{|\xi|} d\xi < \infty$$

ψ is called a analyzing wavelet or admissible.

The Continues Wavelet Transform (CWT) of the function $f \in L^2(R)$ associated to the wavelet ψ noted T^ψ is defined by

$$T^\psi f(a,b) = \frac{1}{\sqrt{C_\psi}} \int_{-\infty}^{+\infty} f(t)\overline{\psi}_{a,b}(t)dt$$

By Parseval equality this formula is equivalent to the CWT formula in Fourier domain:

$$T^\psi f(a,b) = \frac{\sqrt{a}}{\sqrt{2\pi C_\psi}} \int_{-\infty}^{+\infty} e^{i\xi b}\widehat{\overline{\psi}}(a\xi)\widehat{f}(\xi)d\xi \tag{4}$$

A formulation of the Continuous Wavelets Transform in the Fourier domain leads to a calculating algorithm which is less costly (in terms of the number of operations) .

4 The Alignment Algorithm

In this section, we give a recall of some obtained results of the seismic parameters estimation based on the trace seismic modelling and the continuous wavelets transform. For the integral demonstration, see the paper [8].

In particular we are proof that, by the calculation of the wavelets transform of the seismic signal, we obtain an equation of the type:

$$Wy(a,b) = \sum_{k=0}^{N} a_k e^{i\varphi_k} Ws(a, b - \tau_k)$$

This wavelet transform is defined in this case in the Fourier domain to ensure the speed of the algorithm because the complexity is order O(n) instead of O(nlog (n)) in the case of the transform defined in space time. Thus, the estimation of time delay and phase shift depends on calculating the local maximum of the wavelet transform of the signal $y(t)$.

We proof in [8], that the adapted wavelet for the time delay and phase shift estimation of seismic signals must be a "close form" to that of the seismic impulsion $s(t)$, of a compact support, symmetrical, and without oscillations.

In particular, we have demonstrated that if the wavelet ψ is chosen from the impulsion s' "close" form, and for choosing a scale factor $a \approx 1$, the wavelet transform of seismic trace $Wy(a,b)$ reaches its maximum for the value $b = \tau$.

Figure 1 and Figure 2 bellow show respectively our methodology of calculation and give the illustration of the alignment algorithm of a wave recorded on n geophones:

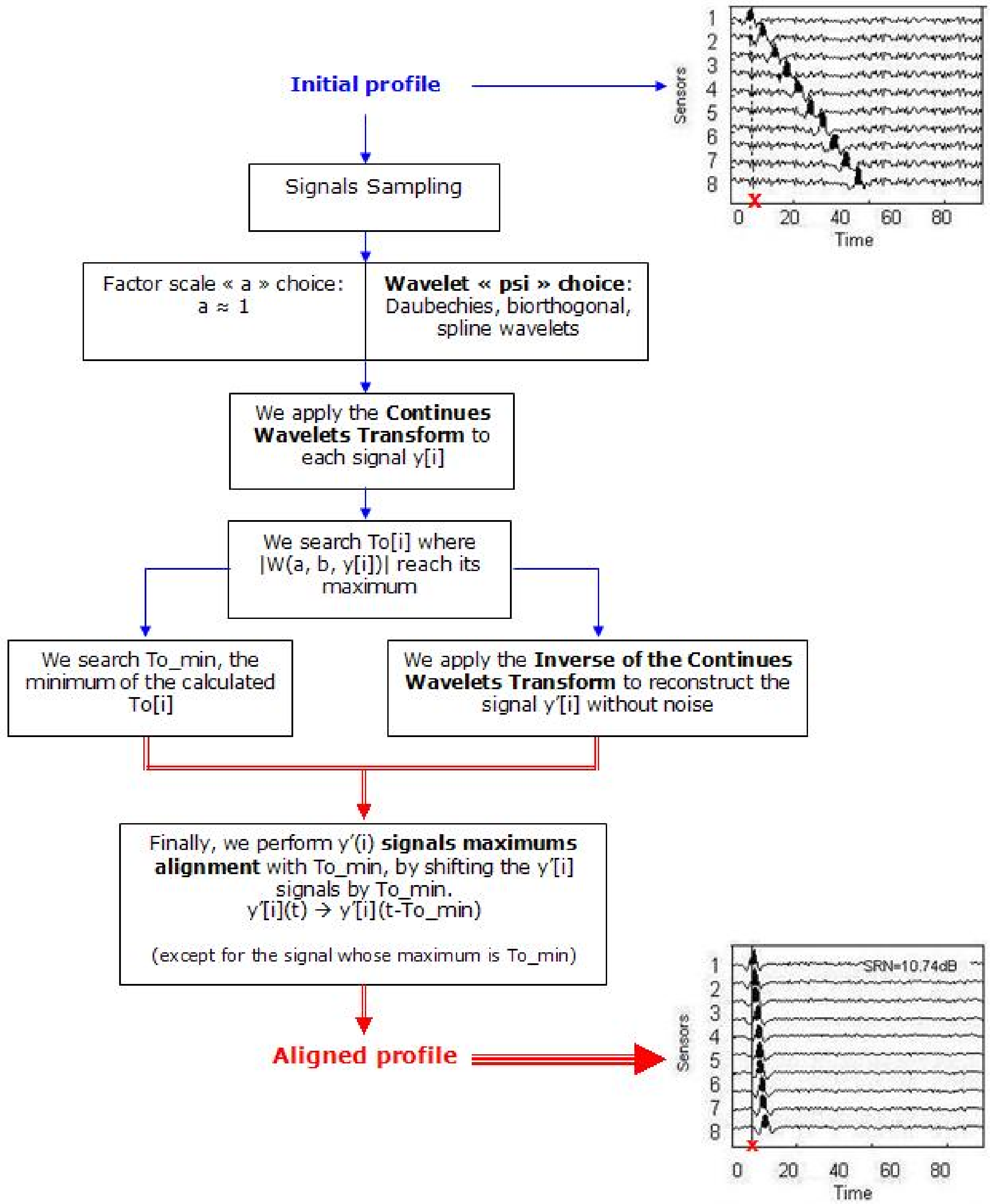


Figure 1: Calculation Methodology

```
/* The Continues Wavelets Transform definition */
a = 1;
/* we choose psi wavelet to use by the Transform to be able to eliminate */
/* the noise; (Daubechies for N=2; Biorthognal and spline wavelets).      */

For 1 <= i <= n:
{
      /* We apply the Continues Wavelets Transform to each signal yi */
      /* And we search To[i] where |W(a, b, y[i])| reach its maximum */

      Wy[i] := W(a, b, y[i]) ;
      Recherche_max(|W(a, b, y[i]|) ;
      Recherche b = To[i] with |W(a, To[i], y[i])|  = max(|W(a, b, y[i]|) ;

      /* We apply the Inverse of the Continues Wavelets Transform to */
      /* reconstruct the signal y[i] without noise.                   */

      y_prime[i] = W-1(a, b, y[i]) ;
}

/* To[i] minimum search, for i between 1 and n                        */

To_min = To[1];
I_min = 1;
For 2 ≤ i ≤ n:
{
      If ( To[i] < To_min)
      {
            To_min = To[i];
            I_min = i;
      }
}

/* Signals maximum alignment with the maximum of y[I_min] signal      */
For 1 <= i <= n:
{
      If (i ≠ I_min)
            y_prime[i] (t) = y_prime[i] (t – To_min)
            /* t is a variable representing time */
}
```

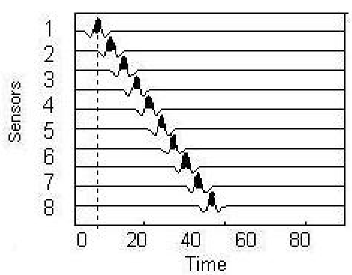
Figure 2: Alignement Algorithm

5 The Algorithm Tests: synthetic profile

We have seen that the wavelet adapted to the estimation of the time delay and phase shift of the seismic signals must be a "close form" to that of the seismic impulsion $s(t)$, of a compact support, symmetrical, and without oscillations.

We have tested our method on synthetic signals using diverse classical wavelets, which verify the seismic impulsion properties (Daubechies, biorthogonal and spline wavelet). The obtained results are numerically satisfactory.

To simplify, we consider the case of two traces (simulation by a function of the Mexican hat type), the second being delayed of 3.14 and defased of $0.41 rad$ in regard to the first trace, and we suppose that shannon theorem is well verified. Figure 3 shows the obtained result.



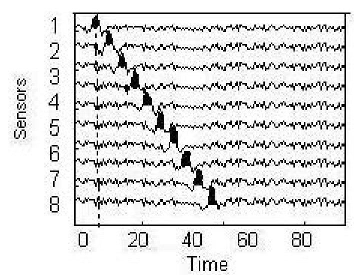


Figure 3: The Synthetic Profile no and with noise (SRN=-3dB, the case of with only delay)

We introduce the average quadratic errors of the delay and phase shift respectively noted by Err_τ and Err_φ, and defined in the general case by:

$$
\begin{aligned}
Err\_\tau &= \frac{1}{N_c - 1} \sum_{i=1}^{N_c - 1} \left[\overline{\tau_i} - \tau_i\right]^2 \\
Err\_\varphi &= \frac{1}{N_c - 1} \sum_{i=1}^{N_c - 1} \left[\overline{\varphi_i} - \varphi_i\right]^2
\end{aligned}
$$

With

τ_i :the real delay value

$\overline{\tau_i}$:the approximate delay value

φ_i :the real phase shift value

$\overline{\varphi_i}$:the approximate phase shift value

N_c :number of sensors

This quality criterion will be used to judge the quality of alignment and performance of the suggested method.

We have applied this method on wavelets which have a form close to the seismic wavelet: a biorthogonal wavelet, and the Daubechies's wavelet in two null moments and the phase shift estimation in this case is exact, and the Daubechies, biorthognal and spline wavelets are well adapted in this case for the delay and phase shift estimation.

To study the evolution of the average quadratic errors of the delay and phase shift, we apply noises of more or less great amplitudes different signal report on noise SRN.
The pic of the seismic wavelet transform, is attained at the time delay value with an weak average quadratic error which validates the theorical obtained result.

We have realized the same tests on Daubechies wavelets and we have noticed that the case of Daubechies wavelets in two null moments ($N = 2$) is more adapted (weak average quadratic error).

We have also tested the biorthogonal wavelets, and with the same noise amplitude, and we have noticed that the pic of the wavelets transform is altered in relation to the delay, witch can be theoretically explained by the unsymmetry of the Daubechies wavelets in two null moments. The spline and biorthogonal wavelets are symmetrical, and should not bring about the shifting; still, when it comes to practice, they have the same performance.

$SRN(dB)$	Err_τ	Err_φ
10	$0.53.10^{-7}$	$0.35.10^{-7}$
-3	$0.36.10^{-5}$	$0.77.10^{-6}$
-10	$0.28.10^{-5}$	0.7910^{-6}
-12	$0.42.10^{-2}$	$0.50.10^{-6}$
-15	$0.51.10^{-2}$	$0.52.10^{-6}$
-17	$0.55.10^{-2}$	$0.46.10^{-4}$
-19	$0.19.10^{-1}$	$0.26.10^{-3}$
-20	$0.20.10^{-1}$	$0.28.10^{-3}$
-22	$0.30.10^{-1}$	$0.31.10^{-1}$
-25	$0.80.10^{-1}$	$0.40.10^{-1}$
-27	$0.99.10^{-1}$	$0.51.10^{-1}$
-30	0.15	0.11

TAB.4.1 The obtained results by Daubechies's wavelets

To evaluate this method for alignment of seismic waves by wavelet transform, on the Matlab, we tested this method on a different synthetic seismic profiles composed of one wave and for two situations: case of delayed profile and case of delayed and phase shifted profile. The results are presented in the following figures (the synthetic profile with noise SRN=-3dB).

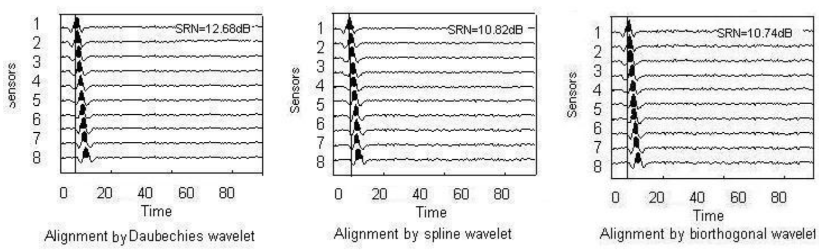


Figure 4: The obtained results by different wavelets (no phase shift)

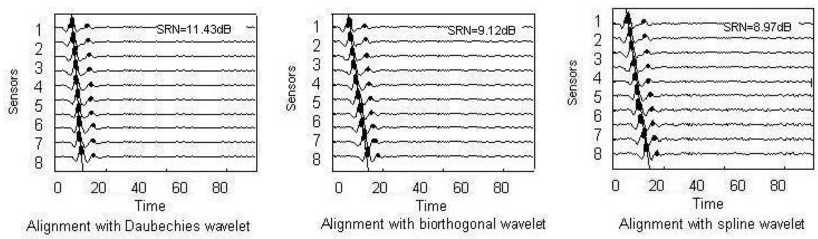


Figure 5: The obtained results by different wavelets (presence of delay and phase shift)

6 Conclusion

While realizing our tests, we have noticed that the Daubechies wavelets gives a numerically satisfactory estimation of the delay and the phase shift in regard to biorthognal and spline wavelets. Therefore, it is more adapted for our problem.

The Daubechies wavelet even if it gives a "good" estimations (weak average quadratic errors), because it is of a compact support, represents a certain shifting due to the fact that it is not symmetrical but it gives a numerically satisfactory results for the denoising (weak SRN).

We have chose to present a continuous wavelet transform, in the time frequency domain not only for its frequencies quality, but also for its calculating time which is less elevated (in terms of operations) in regard to the direct calculating algorithm formula specially in the case of a seismic profile registering several waves on different sensors.

Our approach for alignment of seismic waves by wavelet transform, is based on the estimation of time delay and phase shift trace by trace and on comparisons between the coefficients of wavelets. For the numerical evaluation the discreet wavelets are more interesting. Therefore, we have chosen for calculating the wavelet transform, the unvarying "trous" algorithm in translation by entire values ($a = a_j = 2^j$ and $b = b_k = k$ with $j, k \in Z$).

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