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New Tilt Optimal Angle of a 6 Dof Robot

Rebiai Cherif

Laboratory in computer-integrated manufacturing (LRP),
University of Batna, Faculty of Science and Engineering
Department of Mechanics, Batna, Algeria
rebiaicherif@yahoo.fr

Brioua Mourad

University of Batna, Faculty of Science and Engineering
Department of mechanics Batna, Algeria
Brio_Ray@hotmail.com

Abstract

This work is about the determination of a parameter of slope of a 6 dof robot. This geometrical parameter of design is related to the singularity of the mechanism. To move away from this singularity we propose a better alternative of this parameter, as being the total maximum of a function, which definite the manipulability expression, the latter which is a big factor for the kinematics performance evaluation of the robot.
The parameter founded minimizes the effects of the singularity on the whole of the configurations and softened as well as possible the movements of the articulations of the robot.

Keywords: télé-echography; Optimization; Manipulability; Modelling

1. Introduction

Because of the development of the means of communication and need for medical diagnostic in insulated zones, the tele-operation cheek a very important role. The mobile interconnectivity and the telemedicine are the important exits to have the effectiveness in the health care, since the medical information can be transmitted more quickly and the doctors can take diagnoses and faster decisions of treatment [1] and, [2].

Within the framework of the medicine of proximity, for emergency interventions or on difficult sites of access, the lack of specialists in echography makes delicate the practice of this technique not-invasive and importance growth in medical environment tele-echography brings a solution to this type of situations [6]. A considerable effort has been made to evaluate the kinematics performance of this robot [3]. In this paper a new angle of inclination of this mechanism is presented for its design in order to move it away from its singularities.

2. Robot Kinematics structure

The robot structure designed to obtain the movement of the wished probe, is a six degrees of freedom serial type with: three rotations around the concurrent axes and three translations. See figure (1). The robot is positioned on the skin of the patient and can be maintained by a no expert person in echography. The first two prismatic pairs P1 and P2 allow the precise positioning of the probe on the area of the interest. The revolute pairs R1 and R2 have concurrent axes. They allow the positioning and the orientation of the probe axis inside a conical space with half angle apex of 2α. The probe is oriented by keeping the contact with the same point of the skin; this point is the point of intersection of the two axes of revolution. The revolute pair R3 allows the rotation of the probe around its symmetric axis and enables the expert to access a longitudinal or transverse view of a given organ. The role of the last prismatic pair P3 is to modify the contact force between the probe and the skin [3].

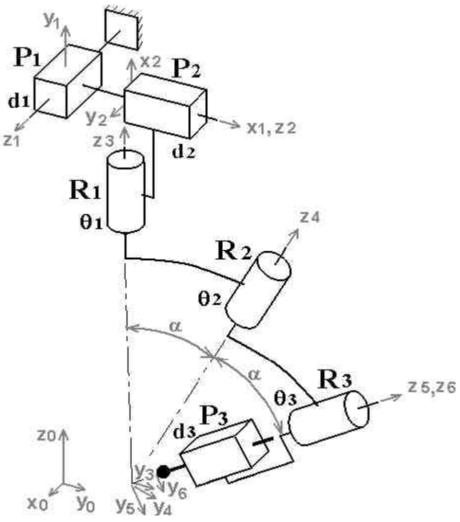

Figure1. Robot Kinematics structure [3].

3. Geometrical model.

The geometrical model is a model of transformation between operational space (probe position), and articular space (generalized coordinates). For this serial robot the model is obtained by the passage matrix:

$$ {}^{0}T_6 = {}^{0}T_1 {}^{1}T_2 {}^{2}T_3 {}^{3}T_4 {}^{4}T_5 {}^{5}T_6 \qquad (1)$$

$$\text{Such as: } {}^{0}T_6 = \begin{bmatrix} {}^{0}R_6 & {}^{0}P_6 \\ 0\ 0\ 0 & 1 \end{bmatrix} \qquad (2)$$

Is the matrix of transformation between the reference probe mark and the basic reference mark, and it is given by:

$$S_x = C_3(C_1C_2 - C\alpha S_1S_2) - C\alpha S_3(C_1S_2 + C\alpha C_2S_1) + S^2\alpha S_1S_3 \qquad (3)$$

$$n_x = S^2\alpha S_1C_3 - C\alpha C_3(C_1S_2 + C\alpha C_2S_1) - S_3(C_1C_2 - C\alpha S_1S_2) \qquad (4)$$

$$a_x = S\alpha(C_1S_2 + C\alpha C_2S_1 + C\alpha S_1) \qquad (5)$$

$$P_x = d_1 + d_3a_x \qquad (6)$$

$$S_y = C_3(S_1C_2 + C\alpha C_1S_2) - C\alpha S_3(S_1S_2 + C\alpha C_2C_1) - S\alpha C_1S_3 \qquad (7)$$

$$n_Y = -S_3(S_1C_2 + C\alpha C_1S_2) - C\alpha C_3(S_1S_2 - C\alpha C_2C_1) - S\alpha C_1C_3 \qquad (8)$$

$$a_y = S\alpha(S_1S_2 - C\alpha C_2C_1 - C\alpha C_1) \qquad (9)$$

$$P_y = d_2 + d_3a_y \qquad (10)$$

$$S_Z = S\alpha(C_3S_2 + C\alpha\left[S_3 + C_2S_3\right]\ ) \qquad (11)$$

$$n_Z = C\alpha S\alpha\left[C_3 + C_2C_3\right] - S\alpha S_2S_3 \qquad (12)$$

$$a_Z = C^2\alpha - S^2\alpha C_2 \qquad (13)$$

$$P_Z = d_3a_Z \qquad (14)$$

With $C_1 : \cos\theta_1 ; S_1 : \sin\theta_1 ; C\alpha : \cos\alpha ; S\alpha : \sin\alpha$
$S_2 : \sin\theta_2\ ; C_2 : \cos\theta_2 ; S_3 : \sin\theta_3\ ; C_3 : \cos\theta_3$

3.1 Opposite geometrical model.

The opposite model consists in calculating the articular coordinate's correspondents with a given situation of the final body. We express the probe position in a situation wished by the matrix:

$$U_0 = \begin{bmatrix} S_x & n_x & a_x & P_x \\ S_y & n_y & a_y & P_y \\ S_z & n_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (15)$$

Or s, n, a, are the directors cosine, and P the coordinates of the probe point end. Thus one seeks to solve the system of equations according to:

$$U_0 = {}^0T_1(q_1)\,{}^1T_2(q_2)\,{}^2T_3(q_3)\,{}^3T_4(q_4)\,{}^4T_5(q_5)\,{}^5T_6(q_6) \qquad (16)$$

The opposite geometrical model is deduced by solving a system from equation to 6 unknown factors, and it is given by:

$$d_1 = P_x - a_x d_3 \qquad (17)$$

$$d_2 = P_y - a_y d_3 \qquad (18)$$

$$d_3 = p_z / a_z \qquad (19)$$

$$\cos\theta_2 = \frac{\cos^2\alpha - a_z}{\sin^2\alpha} \qquad (20)$$

$$\frac{-a_x}{a_y} = tg(\theta_1 + \frac{\theta_2}{2\cos\alpha}) \qquad (21)$$

$$\frac{-s_z}{n_z} = tg(\theta_3 + \frac{\theta_2}{2\cos\alpha}) \qquad (22)$$

4. Kinematics model

4.1 Kinematics model direct

The robot kinematics direct model can be summarized by obtaining the Jacobienne matrix J defined by:

$$\dot{X} = \begin{bmatrix} W_6 \\ V_6 \end{bmatrix} = J\ \dot{\mathrm{q}} \qquad (23)$$

Where $\begin{bmatrix} W_6 \\ V_6 \end{bmatrix}$: Are the Vector of rotation and of translation of the probe and the vector containing generalized speeds (linear or angular) of each articulation.

And as our robot has six degrees of freedom ,3 Rotations and 3 translations, the general form of Jacobien is that described by the equation (24):

$$J = \begin{pmatrix} A & 0_3 \\ B & C \end{pmatrix} \qquad (24)$$

With A, B, C matrices of dimensions 3×3 and 0_3 a null matrix.

To calculate Jacobien of our robot, we proceeded in the following way: We have a decoupling structure, three revolute pairs of convergent axes and three prismatic pairs.

For the rotation part of Jacobien:

$$J_i = [{}^0 z_{i-1}] \tag{25}$$

For the prismatic part of Jacobien:

$$J = [\partial Pi / \partial q_j] \tag{26}$$

With pi: components of the position vector of the GMD (geometrical model direct).

q_j : articular variables of the Robot.

Our robot Jacobien is made of three no null matrices A, B, C which are given by:

$$A = \begin{bmatrix} 0 & s\alpha s\theta_1 & s\alpha\left[c\theta_1 s\theta_2 + c\alpha s\theta_1 (c\theta_2 + 1)\right] \\ 0 & -s\alpha c\theta_1 & s\alpha\left[\ddot{}s\theta_1 s\theta_2 - c\alpha c\theta_1 (c\theta_2 + 1)\right] \\ 1 & c\alpha & -s^2\alpha c\theta_2 + c^2\alpha \end{bmatrix} \tag{27}$$

$$B = \begin{bmatrix} -d_3 s\alpha\left[s\theta_1 s\theta_2 - c\alpha c\theta_1 (c\theta_2 + 1)\right] & d_3 s\alpha\left[c\theta_1 c\theta_2 - c\alpha s\theta_1 s\theta_2\right] & 0 \\ d_3 s\alpha\left[c\theta_1 s\theta_2 + c\alpha s\theta_1 (c\theta_2 + 1)\right] & d_3 s\alpha\left[s\theta_1 c\theta_2 - c\alpha c\theta_1 s\theta_2\right] & 0 \\ 0 & d_3 s^2\alpha s\theta_2 & 0 \end{bmatrix} \tag{28}$$

$$C = \begin{bmatrix} 1 & 0 & s\alpha\left[c\theta_1 s\theta_2 + c\alpha s\theta_1 (c\theta_2 + 1)\right] \\ 0 & 1 & s\alpha\left[s\theta_1 s\theta_2 - c\alpha c\theta_1 (c\theta_2 + 1)\right] \\ 0 & 0 & -s^2\alpha c\theta_2 + c^2\alpha \end{bmatrix} \tag{29}$$

We notice that the row of the Jacobienne matrix is equal to 6 (equal to the number of pairs). However our robot has 6 dof, therefore this one is no redundant.

4.2 Opposite kinematics model

The objective of the opposite kinematics model is to calculate, starting from a given configuration Q, articular speeds which ensure the final reference mark an imposed operational speed.

The opposite kinematics model, is:

$$\overset{\square}{q} = J^{-1} \begin{bmatrix} w_6 \\ V_6 \end{bmatrix} \tag{30}$$

And as the Jacobienne matrix with the form equation (24), the matrices A and C are invertible and the reverse of this matrix is written:

$$J^{-1} = \begin{bmatrix} A^{-1} & 0_3 \\ -C^{-1}BA^{-1} & C^{-1} \end{bmatrix} \tag{31}$$

The resolution of the problem is thus reduced to the inversion much simple of two matrices of less dimensions [4].

4.3 Singularities

To characterize the robot singularities it is necessary to study the determinant of its Jacobienne matrix. For this kind of robots [5] one has:

$$\det(J) = \det(A)\det(C) \tag{32}$$

$$\det(J) = \sin^2\alpha(-\sin^2\alpha\cos\theta_2 + \cos^2\alpha)\sin\theta_2 \tag{33}$$

det (J)=0 the study of the determinant at summer started by refers [3] which with utilised the index of isotropy like a critter of performance for the evaluation of the geometrical parameter.

The robot has the singular configurations in two situations: ref [3]
Here one is interested in this situation:

$$\cos^2\alpha - \sin^2\alpha\cos\theta_2 = 0 \rightarrow \cos\theta_2 = 1/tg^2\alpha \tag{34}$$

It is noticed that for $\alpha \le \pi/4$ the previous equation does not have solution.

The robot presents a singular configuration so that the prismatic part P3 is parallel to the prismatic part P1 fig. 5.

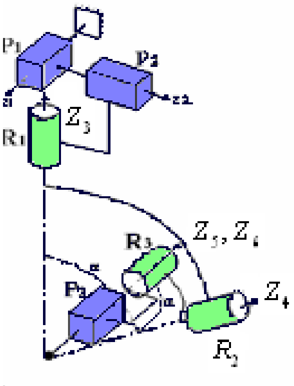

Figure 5: Singular configurations of the robot [3].

This singular configuration fig.5 can be avoided by choosing for α max a value lower than 45°.

The figures (6) watch variation of the determinant of the Jacobienne matrix according to the value of the angle θ_2 for four values of α

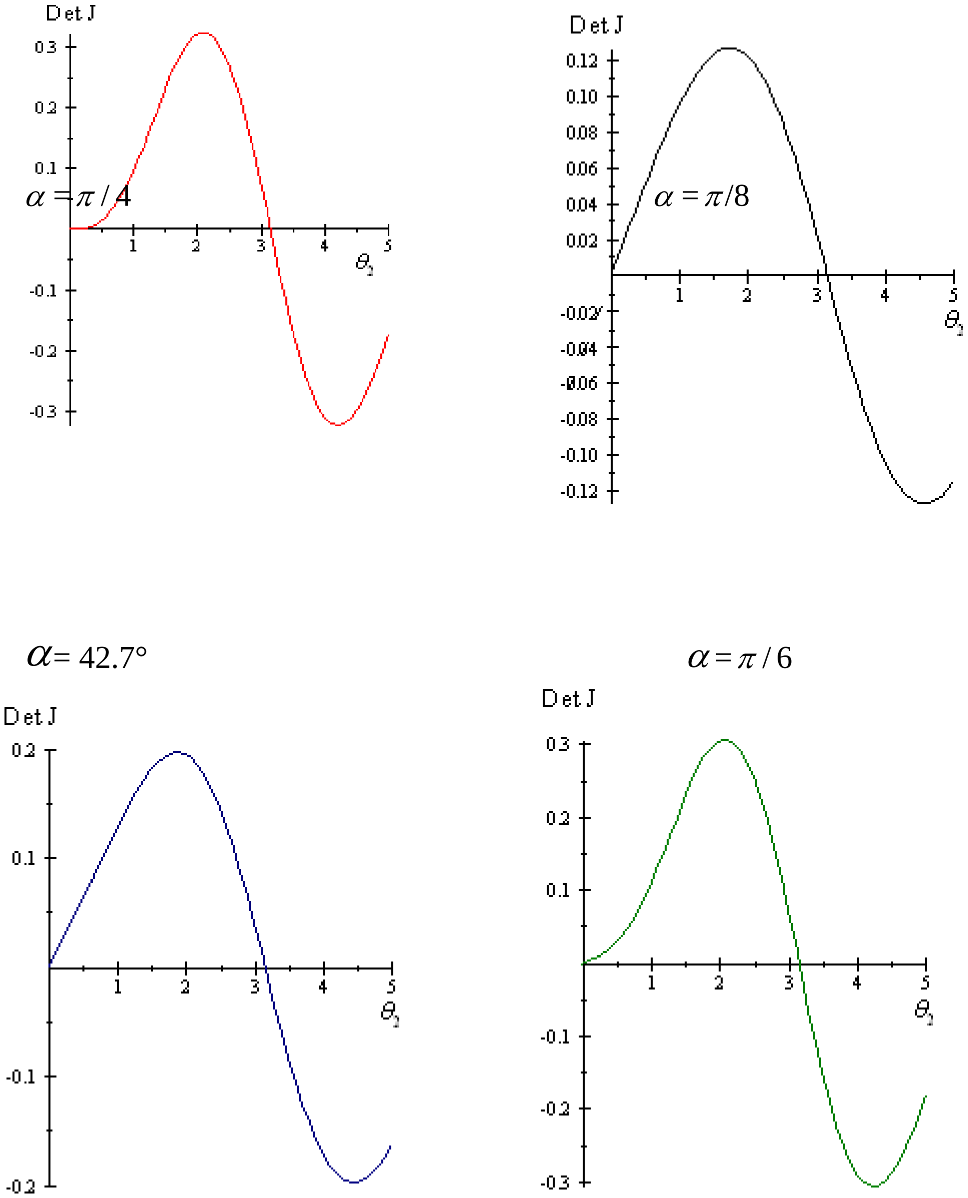

Fig.6 variation of the determinant of J according to the angle θ_2 for four values of the angle. α

5. Optimization of the geometrical parameter

The goal of this optimization is to determine an optimal value of the angle of inclination characteristic of the robot, in order to improve its kinematics performance near the singular configurations and thus to follow the best movement of the probe. This choice of optimum parameter will enable us to minimize displacements in articular space for any displacement in the vicinity of the singularities, which will increase the stability of the mechanism.

5.1 Manipulability:

It characterizes the ability of the manipulator to change the position and the orientation of its final body. It is null for a singular posture. Of or the idea of Yoshikawa to say that best posture is overall that which is most distant away from the singular postures. Thus it is appreciable to maximize it for a given site of the robot in the site of work. The manipulability is one of the criteria most used to evaluate the robot kinematics performance. It is used in various applications: when designing mechanism, the planning of the trajectories, or orders it mechanical systems [7].

One of measurements of the most current manipulability derived from the ellipsoid of manipulability is:

$$W_B = \sqrt{J_B(q_B).J_B{}^T(q_B)} \tag{35}$$

In the case of the none redundant robots, the expression of the manipulability W_B is reduced to:

$$W_B = \left|\det(J(q_B))\right| \tag{36}$$

And it is given as in our case by:

$$W_B = \left|\det(J(q_B))\right| = \left|\sin^2\alpha\,(-\sin^2\alpha\cos\theta_2 + \cos^2\alpha)\,\sin\theta_2\right| \tag{37}$$

The pace of this function in three dimensions is given by the curve appears fig (7).

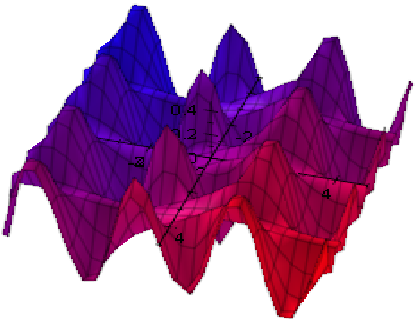

Fig (7): Shape of the curve of Manipulability.

The index of performance such as it's defined depends on the variable θ_2 and the angle α to be determined. To have a powerful index one must maximize the function W_B under constraints:

$\alpha \le \pi/4$ To avoid more singular configurations.

$\alpha \ge \pi/8$ to obtain a probe slope of $\pi/4$ at least. (Specifications of the medical experts) with a skirting of the configurations of the probe of $\theta_2 \in [0 \quad 2\pi]$

6.2 Results

The method of sequential programming quadratic is here selected for optimization.

We obtain an angle of inclination α = 44.9° with a skirting of $\theta_2 = 2\pi/3$ with the use of the tool for optimization of the Maple software.

We notice that the value of the maximum manipulability is of 0,324 for an angle of inclination $\alpha = 44.9$ ° and a probe rotation of $\theta_2 = 2\pi/3$ figure (8)

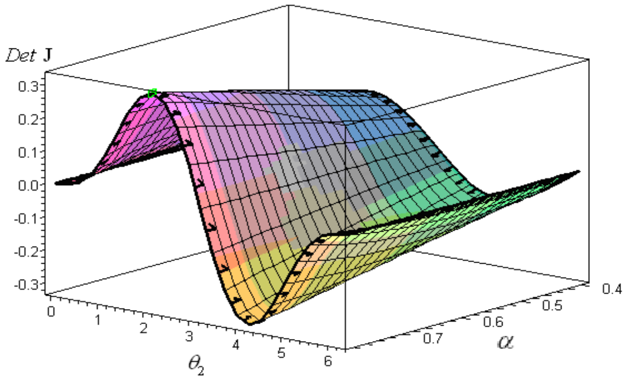

Fig (8) Robot manipulability measures

6. Conclusion

In this article we present the optimal kinematics performance of a 6 dof robot it was sought by the maximization of the index of manipulability. The result obtained minimizes the effects of the singularity on the whole of the possible configurations, and seems more promising as well as possible to approach the ultimate phase of follow-up of the robot movements the like its order.

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