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A Graphical Approach to Find the Critical Path in a Project Network

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Abstract

The purpose of the critical path method (CPM) is to identify critical activities on the critical path so that resources may be concentrated on these activities in order to reduce project length time. The traditional method of finding critical path of a project network is a two pass computation. In the computation, the earliest and latest times of each activity are evaluated. Based on the times, the slack time for each activity is evaluated and the critical path of the network is obtained. In this paper, we construct a rooted tree for the project network to find all paths in the network in such a way that the longest path in the tree as the critical path. Using the critical path we rank all the paths in the project network and evaluate the float time for each activity in the network. It is established that the new graphical approach is a better alternative for dealing with project networks to find critical path.

Keywords: Critical Path Method, rooted tree, slack time

1. Introduction

Networks have proved very useful for performance evaluation of some types of projects. This evaluation includes determining certain aspects about the project, e.g., what is the least amount of time in which the project may be completed, and which individual activities should be speeded to reduce overall project length, etc. [1]. Since the activities in the network can be carried out in parallel, the minimum time to complete the project is the length of the longest path from the start of project to its finish. The longest path is the critical path. The purpose of the CPM is to identify critical activities on the critical path so that resources may be concentrated on these activities in order to reduce project lenth time. Besides, CPM has proved very valuable in evaluating project performance and identifying bottlenecks. Thus CPM is a vital tool for the planning and control of complex project. By using CPM, managers are able to obtain [3,4] a graphical display of project activities, an estimate of how long the project will take and an indication of which activities are the most critical for timely project completion.

The two-pass tabular method [2] provides a systematic tabular approach that works for any size of project. It first makes a forward pass through the projects activity list, establishing the earliest start times and earliest finish times for all activities. Then it makes a backward pass to calculate the float times from the latest start times and latest finish times. Finally, the critical path is identified by connecting all activities that have zero slack times.

In this paper, we construct a rooted tree for the project network to find all paths in the network in such a way that the longest path in the rooted tree as the critical path. Using the critical path we rank all the paths in the project network and evaluate the float time for each activity in the project network.

2. Background information

2.1 Critical path method

A project network is an acyclic digraph, where the vertices represent events, and the direct edges represent the activities, to be performed in a project. Formally, a project network is represented by N = (V,A,T). Let $V = \{v_1, v_2, \ldots, v_n\}$ be a set of vertices, where v_1 and v_n are the start and final events of the project, and each v_i belongs to some path from v_1 to v_n. Let $A \subset V \times V$ be the set of a directed edge $a_{ij} = (v_i, v_j)$, that represents the activities to be performed in the project. Activity a_{ij} is then represented by one, and only one, arrow with a tail event v_i , and a head event v_j . For each activity a_{ij}, a magnitude $t_{ij} \in T$ is defined, where t_{ij} is the time required for the completion of a_{ij} . A critical path is a longest path from v_1 to v_n, and an activity a_{ij} on a critical path is called a critical activity. Let E_i and L_i be the earliest event time, and the latest event time for event i, respectively. Let E_j and :L_j be the earliest event time, and the latest event time for event j, respectively. Let $D_j = \{i \,/\, i \in V \text{ and } a_{ij} \in A\}$ be a set of events obtained from event $j \in V$ and i < j. We then obtain E_j using the following equations

$$E_j = \max_{i \in D_j} [E_i + t_{ij}] \text{ and } E_1 = L_1 = 0. \quad (1)$$

Similarly, let $H_i = \{ j \,/\, j \in V \text{ and } a_{ij} \in A\}$ be a set of events obtained from event $i \in V$ and i < j . We obtain L_i using the following equations

$$L_i = \min_{j \in H_i} [L_j - t_{ij}] \text{ and } L_n = E_n \quad (2)$$

The interval $\{E_i , L_j]$ is the time during which a_{ij} must be completed. We write

$$T_{ij} = L_j - E_i \quad (3)$$

as the interval length available for activity a_{ij} ;$T_{ij} \geq t_{ij}$.

Activity a_{ij} is critical if
$T_{ij} = t_{ij}$, i.e., $L_j = E_i + t_{ij}$,or($E_i = L_i$ and $E_j = L_j$) . (4)
The critical path of N = (V,A,T) is obtained using (1) through (4).

2.2 Rooted tree

A labeled graph G = [V, E, Σ, L] consists of a vertex set V, an edge set E, an alphabet Σ for vertex and edge labels, and a labeling function $L : V \cup E \to \Sigma$ that assigns labels to vertices and edges. A free tree is an undirected graph that is connected and acyclic. A rooted tree is a free tree with a distinguished vertex that is called the root. In a rooted tree, if vertex v is on the path from the root to vertex w then v is an ancestor of w and w is a descendent of v. If in addition v and w are adjacent, then v is the parent of w and w is a child of v. A rooted unordered tree is a directed acyclic graph satisfying the following (i) There is a distinguished vertex called the root that has no entering edges, (ii) every other vertex has exactly one entering edge, and (iii) there is a unique path form the root to any other vertex. A rooted ordered tree is a rooted tree that has a predefined left-to right ordering among the children of each vertex.

3. Proposed method

Step 1 : Construct a project network and assign numbers to each node using Fulkerson's rule[5].

Step 2 : Construct a rooted tree by taking initial node as a root. The lower number node is taken as left child and the higher nodes (in order) are taken as right children while constructing the tree. The weights (activity times) in the tree are taken from the activity table.

Step 3 : Find all the paths from the root to last node of the network in rooted tree.

Step 4 : Add all the weights in each path which gives path length. The path with the largest path length is considered as the critical path.

Step 5 : Calculate the slack time of each path using the formula
Slack time of a path = critical path length – concerned path length.

Step 6 : Rank the paths by arranging the slack times of paths in ascending order.

Step 7 : Assign the slack time of first path for all activities on the path and next consider the second path and assign the slack time of it for remaining activities and so on.
Continue this process until all the activities assigned the slack time.

4. Numerical Example

Step 1 : Project network is presented in Fig. 1 .

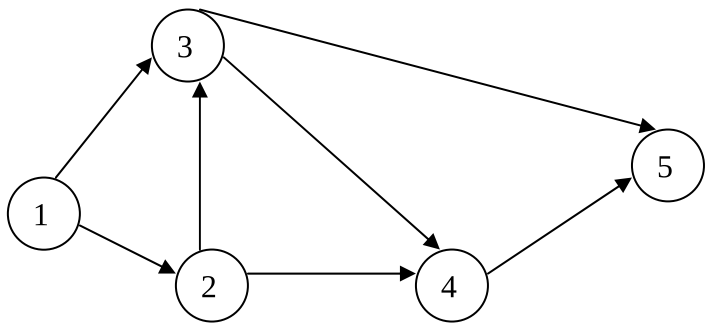

Fig. 1 Project Network

Step 2 : Activity table is presented in Table I and rooted tree for the project network is presented in Fig 2.

Table I : Activity table

Activity	Activity time (days)
1-2	5.1
1-3	7.2
2-3	4.5
2-4	3
3-4	0
3-5	15.8
4-5	2.5

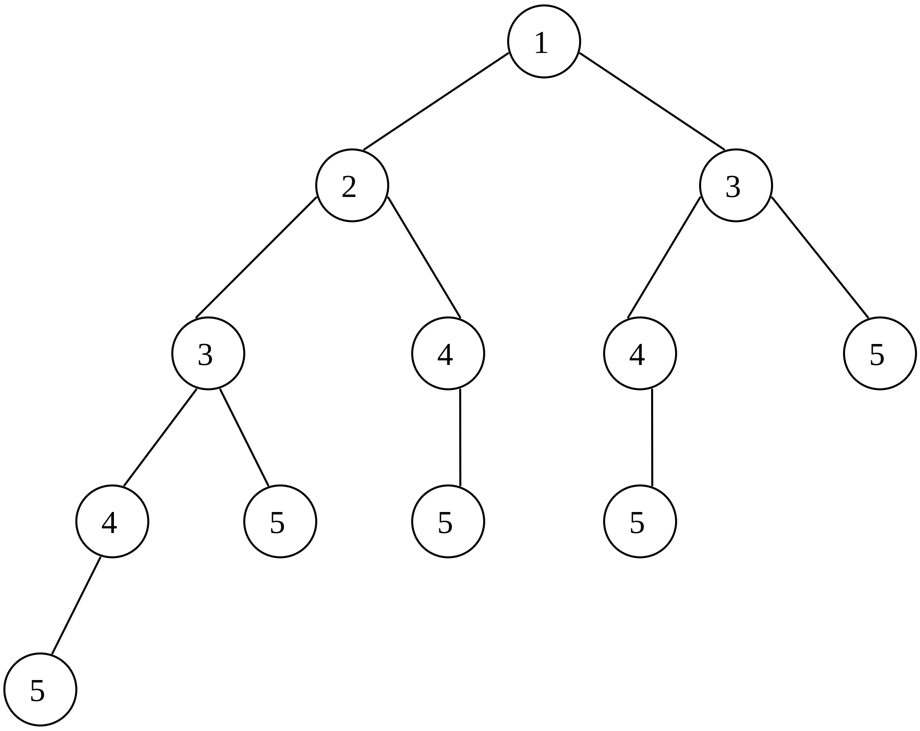

Fig. 2 Rooted tree for the project network

Step 3 : Paths obtained from Fig. 2 are 1-2-3-4-5 , 1-2-3-5, 1-2-4-5, 1-3-4-5, 1-3-5.

Step 4 : (i) Length of the path 1-2-3-4-5 is 12.1
(ii)Length of the path 1-2-3-5 is 25.4
(iii)Length of the path 1-2-4-5 is 13.6
(iv) Length of the path 1-3-4-5 is 9.7
(v) Length of the path 1-3-5 is 23
critical path on project network is 1-2-3-5

Step 5 : Float time of path 1-2-3-4-5 is 13.3
Float time of path 1-2-4-5 is 11.8
Float time of path 1-3-4-5 is 15.7
Float time of path 1-3-5 is 2.4
Float time of path 1-2-3-5 is 0

Step 6 : Rank of path 1-2-3-5 is 1
Rank of path 1-3-5 is 2
Rank of path 1-2-4-5 is 3
Rank of path 1-2-3-4-5 is 4
Rank of path 1-3-4-5 is 5

Step 7: Float time of each activity is presented in Table – II .

Table II : Slack time of each activity

Activity	Float time (days)
1-2	0
1-3	2.4
2-3	0
2-4	11.8
3-4	13.3
3-5	0
4-5	11.8

Conclusion

A new graphical approach to find critical path in a project network has presented. Float time for each activity is calculated by constructing a rooted tree for the project network. This method is easy and simple as compared to the traditional method. Finding float times for all activities and also the float for all the different paths is useful for monitoring the project network.

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