

Electronically Adjustable Mixed-Mode Implementations of the Jerk Functions

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Abstract

In this paper the several structures of the third-order nonlinear oscillator with electronically adjustable parameters are presented. A brief numerical analysis including the computation of the largest Lyapunov exponent is also provided and the associated nominal state space trajectories are given. Some examples of the mixed-mode circuit realizations based on transconductors (OTA) are discussed. Theoretical assumptions are confirmed with experimental measurement.

Keywords: Chaos, integrator synthesis, jerk function, transconductor, analog oscillator, electronic adjustment

1 Introduction

It is well known that a very complex dynamical behavior can be observed in the dynamical systems with at least three degrees of freedom. This behavior is called chaos and is universal in the sense it can be observed in many distinct scientific fields where mathematical models with some time evolution are used. Thus a study of chaos is important from the theoretical viewpoint. There are some practical applications as well mainly involving the telecommunication purposes. There are two major properties of a chaotic signal. It is an extreme sensitivity to the tiny changes of the initial conditions and the broad-band continuous frequency spectrum. Electronic circuits capable to generate such signals are useful and that is the reason why is this topic covered by a lot of interesting publications.

2 Mathematical Background

As pointed in the fundamental publication [4] many scientists took up the challenge to find the simplest third-order autonomous deterministic dynamical system with associated chaotic behavior. After years of studying and simulating it seems that there are several candidates, but the algebraically simplest cases can be written in the following general form

$$\ddot{x} = a\dot{x} + b\dot{x} + cx + \psi(x, \dot{x}, \ddot{x}) + d, \quad (1)$$

where dots denote the time derivatives and ψ is the scalar function with up to four nonlinear terms of the quadratic type. It has been shown in [4] that individual members of the family (1) can be divided into eight distinct groups in the sense of non-existing linear transformation of the coordinates which directly turns one system into another. Individual systems are marked as JD₀₋₇ accordingly to its algebraical complexity, see [4] for more details. Regarding the idea of practical implementation as an electronic circuit our interest were focused on the single particular case featured by

$$a = -0.5 \quad b = -1.9 \quad c = 0 \quad d = -1 \quad \psi(x, \dot{x}, \ddot{x}) = x^2, \quad (2)$$

Following the rules of the linear algebra it is not hard to derive the positions of the fixed points or to determine their stability. The main drawback of such analysis is in fact that none eigenvalue gives us a precise information if the system is chaotic. A parametric space leading to the irregular behavior is often very narrow and surrounded by a limit cycle or a region of unbounded solution. Fortunately there exists a chaos quantifier which is accurate enough and elementary to compute. The so-called Lyapunov exponents (LE) are real numbers which measure the average ratio of the divergency of two neighborhood trajectories along some Euclidean space base. A short script in Matlab for the calculation of a spectrum of the one-dimensional LE has been made.

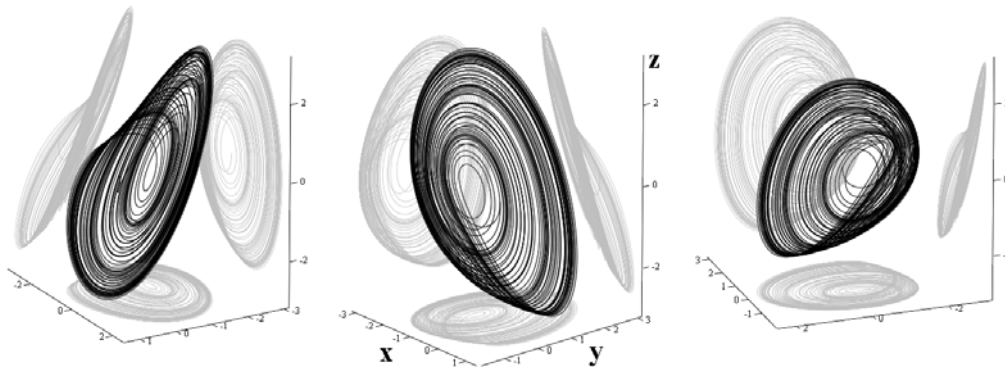


Fig. 1. Perspective views on typical chaotic attractor obtained numerically.

This program takes advantage of the repeated numerical integration process using build-in fourth-order Runge-Kutta method with fixed step size and Gram-Smith orthogonalization. The numerical integration itself showing the typical chaotic attractor is demonstrated in Fig. 1.

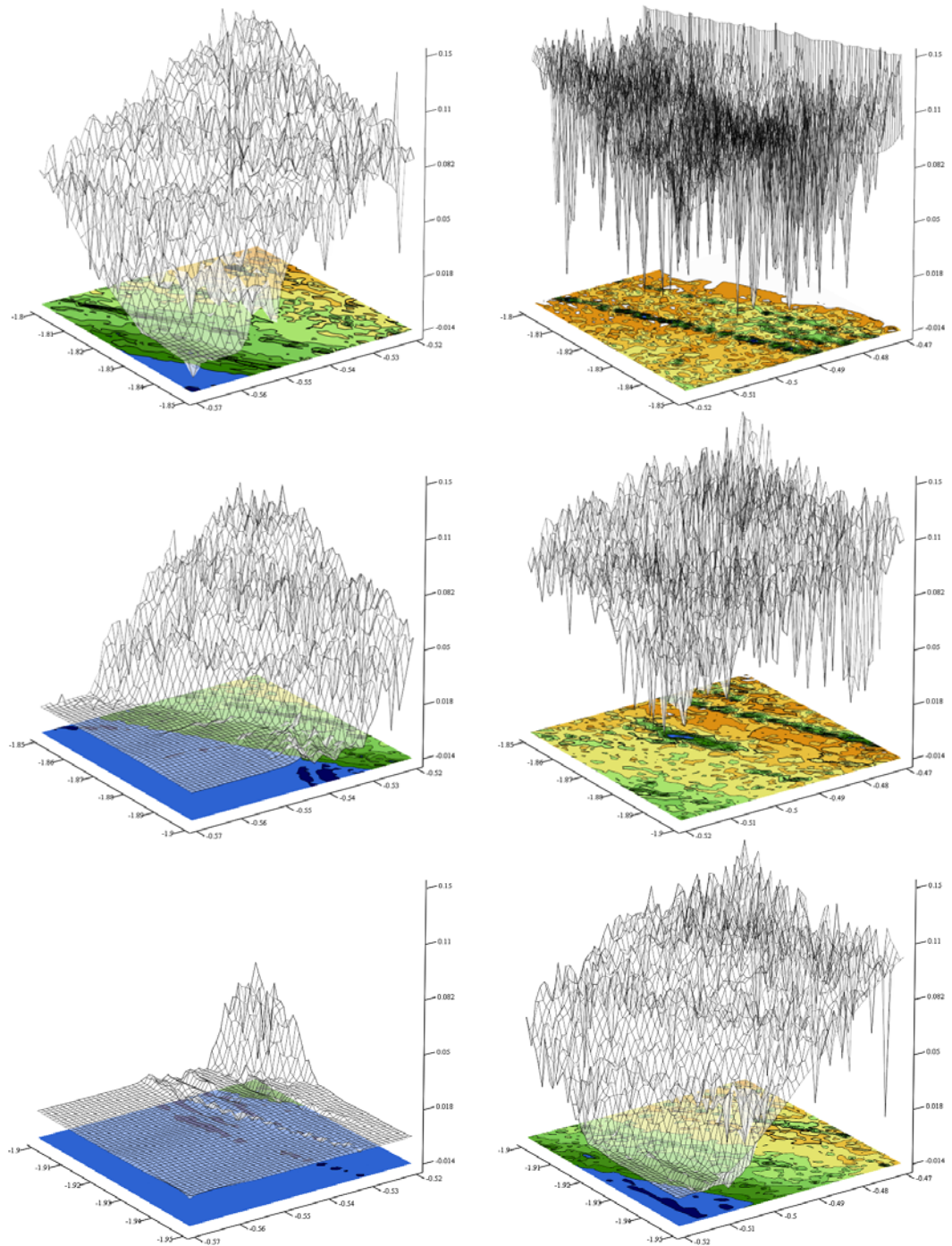


Fig. 2. Graphs of the largest LE, parameter space for the electronic adjustment.

The result of calculation LE for the dynamical system under inspection is shown in Fig. 2. The contour plots are square planes with bifurcation parameter's spacing $\Delta=0.001$. In this area the behavior of the dynamical system was deeply analyzed. Individual contour plots are topographically scaled, i.e. white color represents unbounded solution, yellow regions are chaotic, area marked by green covers a limit cycle and finally blue colors are more trivial solutions. The vertical range is keeping uniform. Simulation parameters were time step $t_{\text{step}}=0.01$, final time $t_{\text{max}}=500$ and zero initial conditions.

3 Circuitry Implementations

Recently, several circuitry implementations of the jerk functions have been published [5]. Many configurations are based on the parallel connection of the third-order linear admittance network with a nonlinear resistor [6]. This approach leads to the simple final circuitry but at the cost of the quite complicated relations between system parameters and circuit elements. Moreover, the parameters a and b are supposed to be electronically adjusted continuously and independently. From the practical point of view the linearity is an important factor. These factors lead to the proposition that integrator-based synthesis with some electronically adjustable devices is indeed a right choice.

Three different conceptions can be used for the design of the final chaotic oscillator. First one deals with the OTA based voltage integrators with standard voltage summation by using a voltage feedback operational amplifier (VFA). This building block can be omitted by making a current summation. Third modification represents a truly current-mode (CM) circuit. The constant term in the equation (1) is realized by means of the dc voltage source. The authors believe that any jerk function given in this paper can be implemented as an electronic circuit by following one approach mentioned above. In each particular case the integrators are realized by using OTAs making an electronic tuning of its time constant possible. These time constants can be varied independently but, after a slight modification of the circuit, also simultaneously. While speaking about individual integrators the associated capacitors are fixed on 100nF and OTA control currents is fixed on $I_{\text{set}}=50\mu\text{A}$. These voltage-mode networks contain input voltage dividers removing the drawback of the limited dynamical ranges of used OTAs. For example, the integrated circuits like LT1228 or MAX435 have this negative feature. The other parameters such as feedback gains or multiplying scale factor are adjusted accordingly to the realized differential equation.

The simplest way how to realize variable feedback gain is by using voltage controlled amplifier, for example VCA810. The equivalent solution is resistor controlled by an external voltage connected inside a basic inverting amplifier network with VFA. Electronic tuning of the resistor value is possible due to the programmable current multiplier commercially available under the notion EL2082. For the given transfer coefficients a , b and chosen values of the fixed resistors R_1 through R_5 the resulting control voltages are specified in Fig. 3.

Note that up to eight active devices are needed. Cheap and off-the-shelf

components like AD633 or MLT04 can be used in place of the voltage-mode four-quadrant analog multipliers with desired high input impedances while LM13700 or LT1228 can be successfully used as integrators.

Thanks to the possibility to make an OTA output current summation on the single node the interesting modification is provided in Fig. 4. Let compare this oscillator to the one given in Fig. 3. Note that now we are able to make an accurate electronic adjustment of the individual system coefficients a and b .

It is not hard to make OTA the only active devices needed for the construction of the final oscillator. In such a case the multiplier must be also realized by OTA. A consequence of this is a minor increasing number of the circuit elements, mainly linear resistors for exact regulation of the multiplier.

Transconductors can be advantageously used for the fully current-mode (CM) modification of the chaotic oscillator without input voltage dividers. It is also much simpler to make the operation of summation. On the other hand, commercially available integrated circuits have only a single current output instead of the desired three. This makes the final circuit much more complicated. A large amount of active blocks are needed in the integrators if they are realized by means of the common OTA. Transconductors can be also used as a current amplifiers electronically adjusting parameters a and b . The most probably there is only a single available analog current multiplier denoted as EL4083. This relative problem has been surpassed as it is shown in Fig. 5. Note that there is no need for the final oscillator to include any resistors. Moreover, only OTA building block is used. It stands to reason that this property predetermine this type of the oscillator to be implemented on chip.

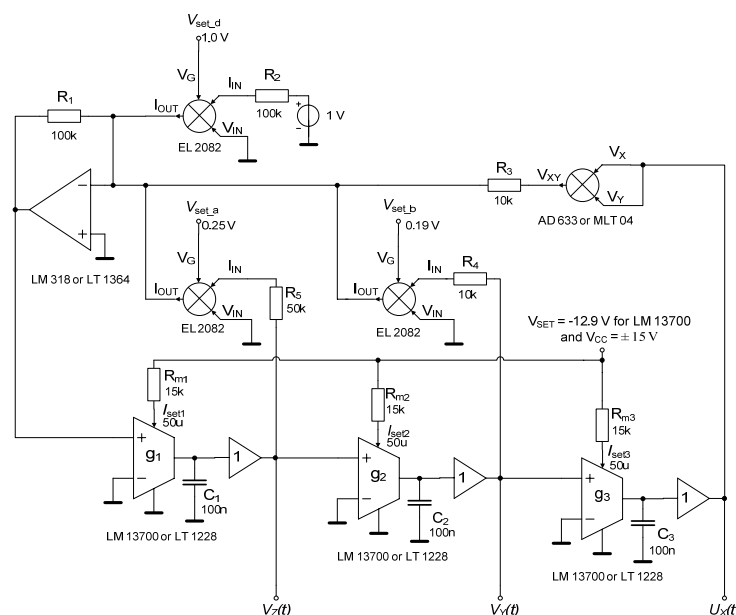


Fig. 3. Voltage-mode implementation with experimentally confirmed chaos.

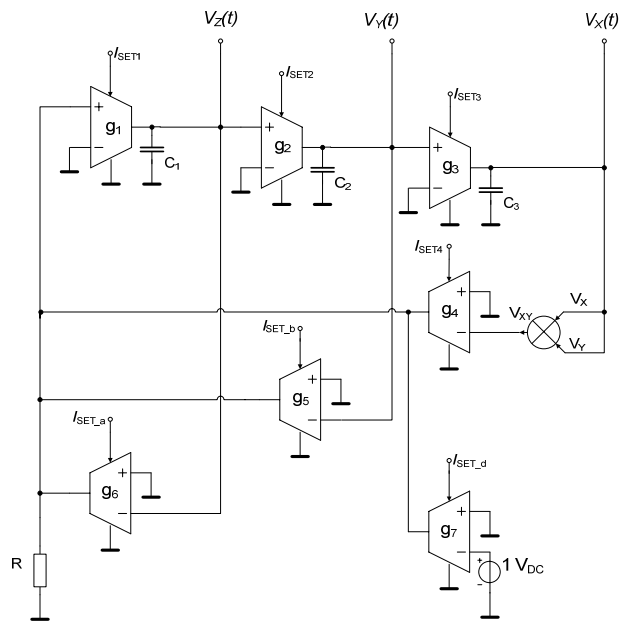


Fig. 4. Voltage-mode implementation with current summation.

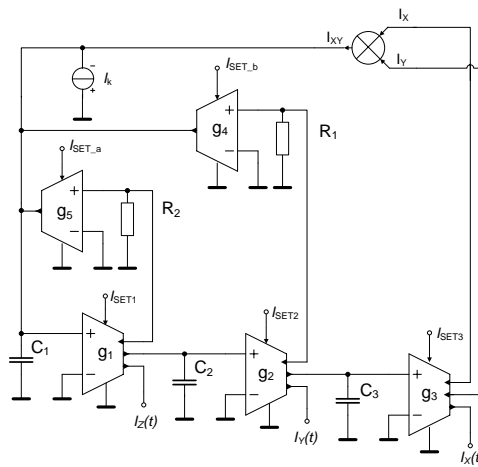


Fig. 5. Current-mode implementation of the chaotic oscillator.

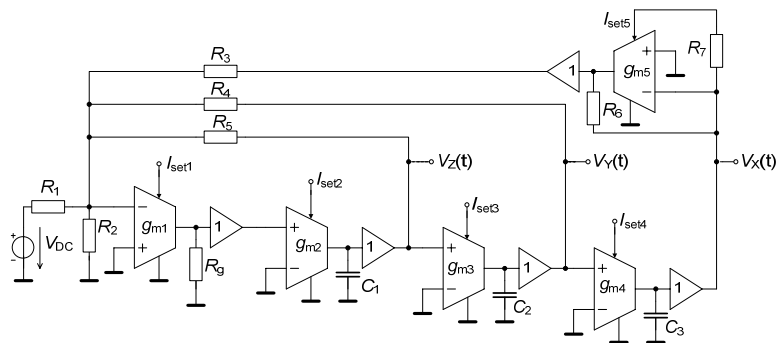


Fig. 6. Principal configuration with quadrator based on OTA.

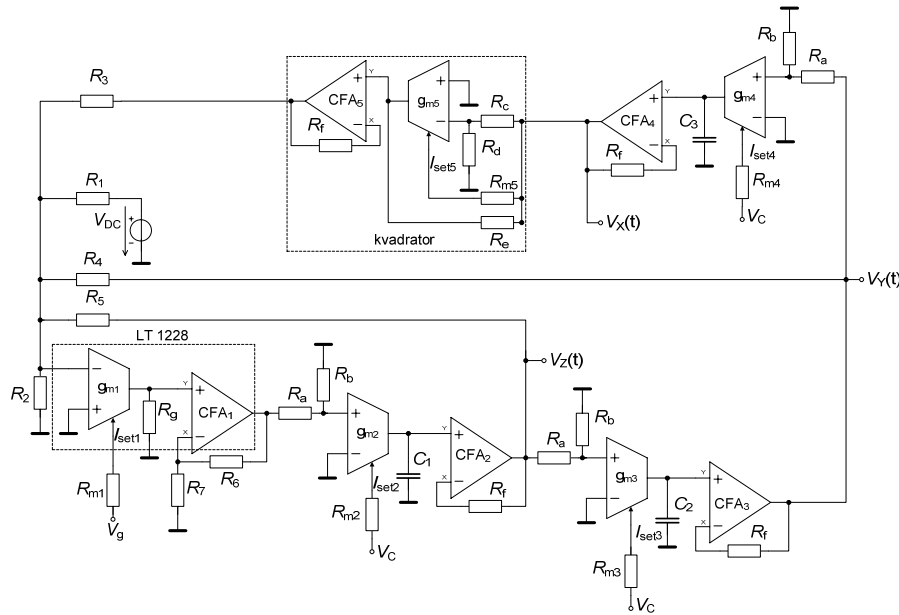


Fig. 7. Complete structure of the chaotic oscillator.

4 Experimental Verification

Certain chaotic oscillator's configurations have been verified by a number of the laboratory measurements, for example network shown in Fig. 3 or Fig. 7. Integrated circuits LM13700 (OTA), LM318 (VFA) and multiplier AD633 have been used for the obvious matter – easy accessibility and reasonable price. If the frequency scaling factor is chosen properly these devices are susceptible for practical verification. For high frequency applications other devices must be found, for example OPA860. The typical chaotic attractor for the chaotic oscillator's low frequency setup is demonstrated in Fig. 8. Frequency spectrum of the $x(t)$ state variable measured by means of the digital oscilloscope is shown in Fig. 8. The most energy is contained in the low frequency components up to few kHz so that this oscillator can be considered as low frequency (large time constant of the integrator were accepted for the testing purpose). The natural frequency of the chaotic oscillators can be lifted up by using active components with better frequency response. This proposition is proved by the only OTA configuration. Fig. 9 gives the chaotic signals in the time domain with vertical scale $2V/div.$, horizontal scale $0.1ms/div.$ and $10\mu s/div.$ respectively. The plane projections of the selected chaotic attractors measured by means of the analog oscilloscope BM538 is provided in Fig. 10. The listing of the real circuit elements is following

$$\begin{aligned}
 R_1 = R_6 = R_a = R_c = R_{m5} = 10k\Omega \quad R_2 = R_7 = R_b = 100\Omega \quad R_3 \in (0, 4.7)k\Omega \\
 R_4 \in (0, 5.2)k\Omega \quad R_5 \in (0, 21)k\Omega \quad R_f = 1k\Omega \quad R_d = 470\Omega \quad R_e = 14.7k\Omega \quad (3) \\
 R_{m1} = R_{m2} = R_{m3} = R_{m4} = 15k\Omega \quad C_1 = C_2 = C_3 = 1nF \quad V_g = -8.5V \quad V_{dc} = 1V
 \end{aligned}$$

For all laboratory measurements the supply voltage was $\pm 15\text{V}$. It is worth nothing that also Pspice circuit simulator gives the same results.

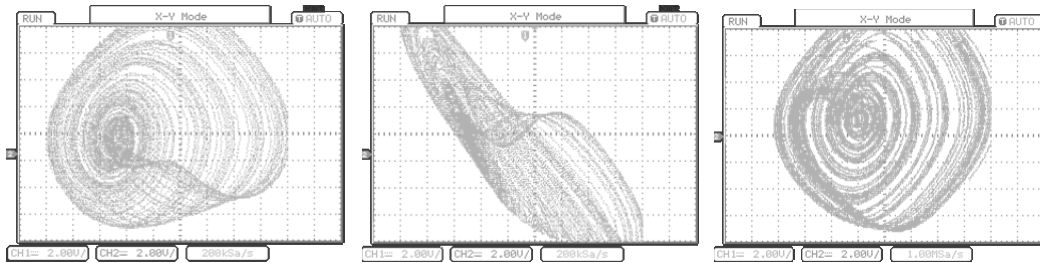


Fig. 7. Plane projections of the single-scroll chaotic attractor.

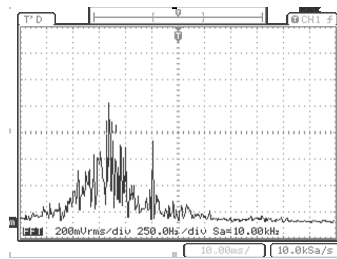


Fig. 8. Frequency spectrum of the observed chaotic signal.

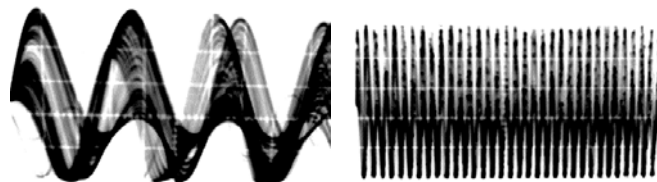


Fig. 9. Chaotic signal generated by only OTA configuration, see text.

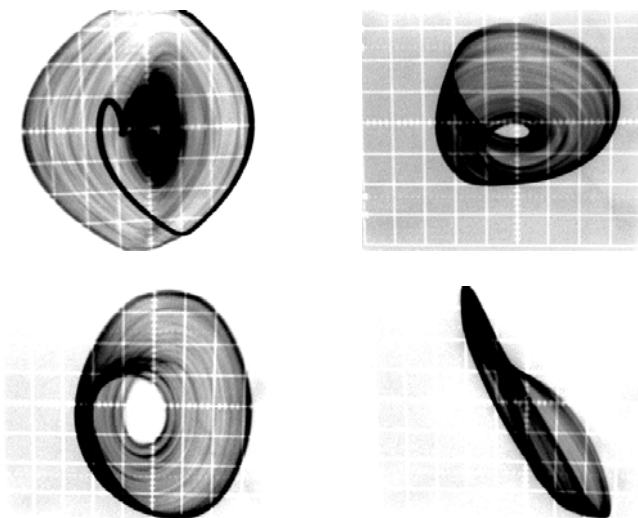


Fig. 10. Plane projections of the chaotic signals, only OTA configuration, see text.

5 Conclusion

Several configurations of the chaotic oscillators working in the three different regimes have been briefly discussed. Given technique is a utilization of the filter design methods [1], [3], [7] and can be easily extended to any dynamical system described by a single differential equation of arbitrary order. The very good final agreements between theoretical expectations and the laboratory measurements have been confirmed. By using some hypothetical active building block [2] other simpler modifications or even novel structures of the presented circuits can be achieved. The approach to do this can be adopted from [8]. This is an interesting topic for further study.

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