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Toward Accuracy and Efficiency Enhanced Model Reduction in Micro-Technology

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Abstract

An accuracy and efficiency enhanced method for model order reduction is proposed. The reduction technique is based a recently developed methods, namely frequency domain balanced truncation within a frequency bound. This reduction technique is developed for model reduction of singular systems which are frequently appear in analysis and simulation in Micro-technology as a result of finite element method. The proposed method is of interest for practical model order reduction because in this context it shows to keep the accuracy of the approximation as high as possible without sacrificing the computational efficiency. Numerical results show the accuracy and efficiency enhancement of the method in comparison to previous techniques in this context.

Keywords: Singular Systems, Frequency-Domain Grammian, Model Reduction, Micro-technology

1 Introduction

The ever-increasing need for accurate mathematical modelling of physical as well as artificial processes for simulation and control leads to models of high complexity. This problem demands efficient computational prototyping tools to replace such complex models by an approximate simpler models, which are capable of capturing dynamical behaviour and preserving essential properties of the complex one, either the complexity appears as high order describing dynamical system or complex nonlinear structure. Due to this fact model reduction methods have become increasingly popular over the last two decades.

Such methods are designed to extract a reduced order state space model that adequately describes the behaviour of the system in question.
In the field of simulation and analysis in micro and nanotechnology frequently we have to deal with large dynamical systems which are mostly the results of the tools for FEM (finite element method). On the other hand these results are in descriptor structure and usually singular. This problem is motivated the researchers in this context to study model reduction of singular systems. In this paper, an accuracy and efficiency enhanced method for model order reduction is presented for model reduction of singular systems and the method is compared with the previous counterpart [3].
The paper is organized as follows. In section 2, we present some system theoretic and mathematical background briefly. Section 3 consists of presenting our method in detail. In section 4 the proposed methods applied to benchmark examples for the more numerical illustration. The conclusion is presented in section 5.

2 Mathematical and System Theoretic Background

In this section some notions and notations in context of model reduction in general and model reduction of singular systems in particular are presented in brief. More details can be found in [1],[2],[3], and [4].

Definition 1. Dynamical singular systems:

$$E\dot{x} = Ax + Bu, \qquad y = Cx \qquad (1)$$

$$\tilde{E}\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}u, \qquad y = \tilde{C}\tilde{x} \qquad (2)$$

are called restricted equivalent systems if there exist non-singular P and Q such that: $x = P\tilde{x}$, $QEP = \tilde{E}$, $QAP = \tilde{A}$, $QB = \tilde{B}$, $CP = \tilde{C}$

Definition 2. Dynamical singular system (1) is regular if there exist $\alpha \in C$ such that: $\det(\alpha E + A) \neq 0 \quad or \quad \det(\alpha E - A) \neq 0$

Following we recall a theorem from [4] which is very important in reduction of dynamical singular systems.

Theorem 1. Every regular singular system has a restricted equivalent dynamical system which is described by the following equations:

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1u \qquad (3)$$

$$0 = A_{21}x_1 + A_{22}x_2 + B_2u \qquad (4)$$

$$y = C_1x_1 + C_2x_2 \qquad (5)$$

In order to get this structure we can apply SVD on E:

$$U^T EV = diag(\Sigma, 0)$$

and then by coordinate transformation:

$$x = V \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

We have:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = TU^T AV, \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = TU^T B,$$

$$[C_1 \quad C_2] = CV^T, \quad T = \begin{bmatrix} \Sigma^{-1} & 0 \\ 0 & I \end{bmatrix}$$

This kind of transformation enables us to get non-singular system out of a singular structure (by expressing x_2 in terms of x_1 and eliminating x_2).

3 Frequency-Domain Balanced Truncation within Frequency bound for Singular System

In this section we first briefly present frequency domain balanced truncation method and then the procedure for the developing the proposed method for reduction of singular systems.
There are huge amount of reduction techniques in literature mainly from two categories, moment matching based reduction techniques e.g. [5],[6] or SVD based methods e.g.[7]. One commonly used and globally more accurate approach for model reduction is the so-called Balanced Model Reduction first introduced by Moore [7]. In this method, the system is transformed to a basis where the states which are difficult to reach are simultaneously difficult to observe. Then, the reduced model is obtained simply by truncating the states which have this property. Because of being operational the system within frequency bound and outside that it is not important to have an accurate approximation the accuracy can be improved by applying balanced model reduction in the specified frequency band [8],[9].
Controllability and observability Gramians in terms of w over a frequency band $[w_1, w_2]$ are defined as:

$$W_{cf} = \frac{1}{2\pi} \int_{w_2}^{w_1} (Ijw - A)^{-1} BB^* (-Ijw - A^*)^{-1} dw$$

$$W_{of} = \frac{1}{2\pi} \int_{w_2}^{w_1} (-Ijw - A^*)^{-1} C^* C (Ijw - A)^{-1} dw$$

(6)

With an appropriate similarity transformation T and change of the basis, system realization can be transformed to a new balanced realization, so that the Gramians are equal and diagonal (in decreasing diagonal elements):

$$\bar{W}_{cf} = \bar{W}_{of} = diag(\sigma_1, \sigma_2, ..., \sigma_n) \qquad (7)$$

The reduced order model is obtained by truncating the states which are related to the set of least diagonal elements. This model reduction technique is called frequency domain balanced truncation within a frequency bound (FDBT). This method is stability preserving and shows to provide better more efficient approximation with less error comparing to the ordinary balanced truncation.

At this point we are ready to develop this model reduction technique for reduction of singular systems. In the first step we use theorem 1 to the general structure: $E\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t), \quad t \geq 0$ and we achieve non-singular system:

$$\dot{x}_1 = (A_{11} - A_{12}A_{22}^{-1}A_{21})x_1 + (B_1 - A_{12}A_{22}^{-1}B_2)u$$
$$y = (C_1 - C_2A_{22}^{-1}A_{21})x_1 + (-C_2A_{22}^{-1}B_2)u$$

which is ready for reduction by frequency-domain balanced truncation. As we will see in the next section, the method works very well and also better than previous methods in this context.

4 Numerical Example

In this section we apply our method to a benchmark example and we compare the method with the method in [3].

Consider singular system of [3]:

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \dot{x} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ -1 & -2 & -1 & -1 \\ 50 & 0 & -1 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 20 \\ 2 \end{bmatrix} u$$
$$y = \begin{bmatrix} -40 & -1 & 1 & 3 \end{bmatrix} x$$

As result of the first step of the framework we have:

$$\dot{x}_1 = \begin{bmatrix} -10 & 0.2 & 0.2 \\ 0 & 0 & 10 \\ 49 & -2 & -2 \end{bmatrix} x_1 + \begin{bmatrix} -0.4 \\ 0 \\ 22 \end{bmatrix} u$$
$$y = [-190 \quad -1 \quad 4] x_1 - 6u$$

as an equivalent of the singular system.

The approximation error of the reduction to second order dynamical system applying the proposed method and the method in [3] is shown in Figure 1-2 for two different frequencies bound. Figure (3) shows the step responses of original method and the reduced systems using the method in [3] and the proposed method. It can be seen that the results of the model reduction in the proposed method is more accurate than the method in the results in [3].

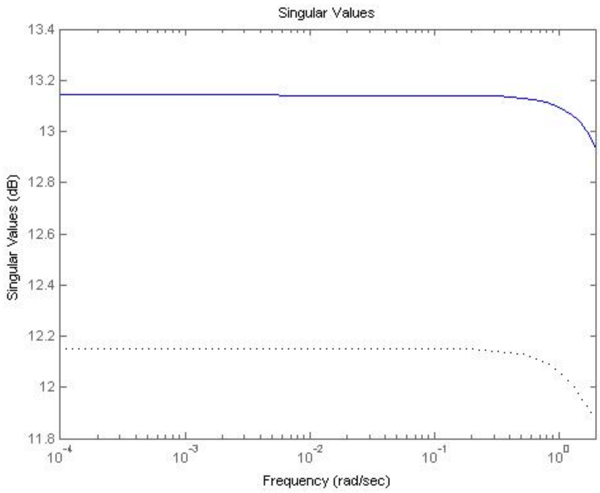

Figure 1. Error in the proposed method (dotted) and the error in the method of [3] (solid)

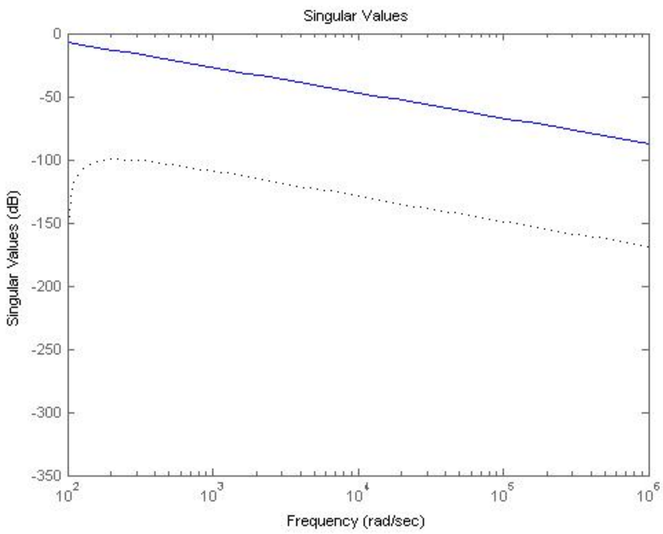

Figure 2. Error in the proposed method (dotted) and the error in the method of [3] (solid)

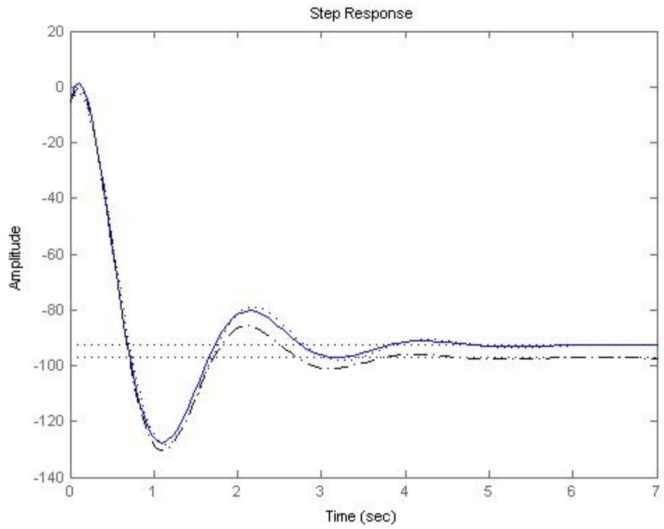

Figure 3. Step responses: original model (solid), reduced model using [3] (dash-dotted) and reduced model using the proposed framework within [1, 1000] frequency bound (dotted)

5 Conclusion and future work

In this paper a framework for model order reduction of singular system is proposed. The reduction technique is based a recently developed methods, namely frequency domain balanced truncation within a frequency bound. This reduction technique is developed for model reduction of singular systems which are frequently appear in analysis and simulation in Micro-technology as a result of finite element method. Numerical results show the accuracy and efficiency enhancement of the method in comparison to previous techniques in this context. The next to do is to apply the method on real industrial examples from micro-technology.

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