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Moments Inequalities for NBUL Distributions with Hypotheses Testing Applications

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Abstract

Our goal in this paper is to establish inequalities for the moments of new better than used in Laplace transform order (NBUL) distributions. Then we use these inequalities to construct new tests for exponentiality versus NBUL. Pitman's asymptotic relative efficiency, power and critical values are employed to assess the performance of the test. The problem when the right censored data is also handled. Finally, some applications to elucidate the usefulness of the proposed test in reliability analysis are discussed.

Keywords: Moment inequalities, classes of life distributions, life testing, efficiency, censored data, NBU, NBU(2), NBUL.

1 Introduction

Notions of positive aging play an important role in reliability theory, survival analysis and some related areas, such as economics and actuarial science. As a result, a multitude of nonparametric classes of life distributions describing aging have been introduced and studied in the literature. One of the more

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commonly used classes is the new better than used in Laplace transform order (NBUL) class. Wang [20] introduced it as a large class of distributions that contains most of previously known classes like new better than used (NBU) (Barlow and Proschan, [5]) and new better than used in increasing concave order NBU(2) (Deshpand et al., [7]).

A non-negative random variable X is said to be new better than used in the Laplace transform order (denoted by $X \in$NBUL) if, and only, if

$$\int_0^\infty e^{-sx}\overline{F}(x+t)dx \leq \overline{F}(t)\int_0^\infty e^{-sx}\overline{F}(x)dx, \quad s \geq 0. \tag{1.1}$$

The NBUL expanding the NBU and NBU (2) classes to a much bigger and a more practical one. Some interpretations, properties and closure properties of the NBUL class have been discussed by Yue and Cao [19] and Gao et al. [10]. Belzunce et al. [6] showed that the NBUL class is preserved under both the pure birth shock model and Poisson shock model. Likewise, a large number of tests for exponentiality against some classes of life distributions have been proposed. The rationale for using such tests instead of omnibus tests for exponentiality is well-known. Therefore, if one has some knowledge about the class of distributions which may occur, it is reasonable to use tests that are well adapted to detect the possible alternatives. It may be even more dangerous to overly restrict the set of possible alternatives. Hence, using a fairly large class of aging distributions seems to be a reasonable compromise.

Recently, Al-Wasel et al. [4] studied further the NBUL class and investigated a test statistic for it based on U-statistic by using the expected value method and calculated Pitman asymptotic efficiency, power and critical values of the proposed statistic. However, few applications of their test was presented.

In the current investigation, moment inequalities for the NBUL class are developed in Section 2. Based on these inequalities test statistics for testing H_0 : is $exp(\mu)$ against alternatives that H_1: is NBUL and not exponential is presented. This test statistic is simple to devise, calculate and has exceptionally privet Pitman asymptotic efficiency. In that section, Monte Carlo null distribution critical points are simulated for sample sizes $n = 5(5)50$ and the power estimates are also calculated. In *Section* 3, we dealing with right-censored data and selected critical values are tabulated. Finally, in Section 4 we discuss some applications to elucidate the usefulness of the proposed test in reliability analysis.

2 Moments inequalities

The next result provides moments inequality for the NBUL distributions. In this, as well as subsequent results all moments are assumed to exist and are finite.

Theorem 2.1.

Let $\phi(s) = \int_0^\infty e^{-sx} dF(x)$, if F is $NBUL$ then for all integer $r \geq 0$

$$\begin{aligned} \frac{\mu_{(r+1)}}{s(r+1)}[1-\phi(s)] \quad &\geq \quad \frac{-(-1)^r r!}{s^{r+2}}(1-\phi(s)) \\ &+ \frac{r!}{s^{r+1}} \sum_{i=0}^{r} (-1)^i \frac{s^{r-i}}{(r-i)!} \frac{\mu_{(r-i+1)}}{(r-i+1)}. \end{aligned} \tag{2.1}$$

Proof.

Since F is $NBUL$, then from (1.1) we have

$$\int_0^\infty t^r \int_0^\infty e^{-su} \overline{F}(u+t)\, du dt \leq \int_0^\infty t^r \overline{F}(t)\, dt \int_0^\infty e^{-su} \overline{F}(u)\, du, \quad \text{for all integer } r \geq 0.$$

Note that

$$\begin{aligned} \int_0^\infty t^r \overline{F}(t)\, dt \quad &= \quad E\left[\int_0^\infty t^r I(X > t)\, dt\right] \\ &= \quad \frac{\mu_{(r+1)}}{(r+1)}. \end{aligned} \tag{2.2}$$

It is easily to show that

$$\begin{aligned} \int_0^\infty e^{-su} \overline{F}(u)\, du \quad &= \quad E \int_0^\infty I(X > u)\, e^{-su} du \\ &= \quad \frac{1}{s}(1-\phi(s)), \end{aligned} \tag{2.3}$$

and

$$\int_0^\infty t^r \int_0^\infty e^{-su} \overline{F}(u+t)\, du dt = \int_0^\infty e^{-st} \overline{F}(t) \int_0^t v^r e^{sv} dv dt,$$

where

$$\int_0^t v^r e^{sv} dv = \frac{r!}{s^{r+1}} \left[-(-1)^r + \sum_{i=0}^{r} (-1)^i \frac{(ts)}{(r-i)!} e^{st} \right].$$

Then

$$\int_0^\infty t^r \int_0^\infty e^{-su} \overline{F}(u+t)\, du dt = \frac{-(-1)^r r!}{s^{r+2}} (1-\phi(s)) + \frac{r!}{s^{r+1}} \sum_{i=0}^{r} \frac{(-1)^i s^{r-i}}{(r-i)!} \frac{\mu_{(r-i+1)}}{(r-i+1)}. \tag{2.4}$$

Making use of (2.2)-(2.4) the result follows. ∎

When $r = 1$, (2.1) reduces to

$$\frac{\mu_2}{2s} (1-\phi(s)) \geq \frac{1}{s^3} (1-\phi(s)) + \frac{1}{s^2} \left(\frac{s}{2} \mu_2 - \mu \right).$$

2.1 Applications to hypothesis testing

In the context of reliability and life testing, the hazard rate of a life distribution plays an important role for stochastic modeling and classification. Being a ratio of probability density function and the corresponding survival function, it uniquely determines the underlying distribution and exhibits different monotonic behaviors. The concept of the ageless notion is equivalent to the phenomenon that age has no effect on the hazard rate. Thus the ageless property is equal to constant hazard rate, that is, the distribution is exponential. Hence testing aging classes is done by testing exponentially versus some kind of classes. This applies to many aging classes such as NBU, NBU(2) and NBUL, among others. For a recent literature on testing the above classes as well as others we refer the readers to Ahmad [2], Ahmad and Mugdadi [3], Mahmoud et al. [15] and Al-Wasel et al. [4].

In view of Theorem 2.1 with $r \geq 0$ we develop a test $H_0 : F$ is exponential against an alternative that $H_1 : F$ is NBUL. We may use $\delta(s)$ as a measure of departure from exponentiality where

$$\delta(s) = \frac{\mu_{(r+1)}}{s(r+1)} (1-\phi(s)) + \frac{-(-1)^r r!}{s^{r+2}} (1-\phi(s)) - \frac{r!}{s^{r+1}} \sum_{i=0}^{r} (-1)^i \frac{s^{r-i}\ \mu_{(r-i+1)}}{(r-i)!\ (r-i+1)}.$$

If $r = 1$, then $\delta(s)$ reduce to

$$\delta(s) = \frac{1}{s^3}\left[-1 + s\mu + \left(1 - 2^{-1}s^2\mu_2\right)\phi(s)\right]. \tag{2.5}$$

Note that under $H_0 : \delta(s) = 0$, while under $H_1 : \delta(s) > 0$.

To estimate $\delta(s)$, let $X_1, X_2, ..., X_n$ be a random sample from F. So the empirical form of $\delta(s)$ in (2.5) is

$$\hat{\delta}(s) = \frac{1}{s^3 n^2}\sum_i\sum_j\left[-1 + sX_i + \left(1 - 2^{-1}sX_i^2\right)e^{-sX_j}\right]. \tag{2.6}$$

To find the limiting distribution of $\hat{\delta}(s)$ we resort to the U-statistic theory. Set

$$\phi_s(X_1, X_2) = \left[-1 + sX_1 + \left(1 - 2^{-1}sX_1^2\right)e^{-sX_2}\right],$$

and define the symmetric Kernel

$$\psi(X_1, X_2) = \frac{1}{2!}\sum\nolimits_R \Phi(X_{i_1}, X_{i_2}),$$

where the summation is over all arrangements of X_{i_1}, X_{i_2} then $\hat{\delta}(s)$ in (2.6) is equivalent to the U-statistic

$$U_n = \left[\binom{n}{2}\right]^{-1}\sum\nolimits_{i<j}\psi(X_i, X_j).$$

The next result summarizes the asymptotic normality of $\hat{\delta}(s)$.

Theorem 2.2.

As $n \to \infty$, $\sqrt{n}\left(\hat{\delta}(s) - \delta(s)\right)$, is asymptotically normal with zero mean and variance σ_s^2 given in (2.8). Under H_0, the variance is reduces to (2.9).

Proof.

First note that

$$\begin{aligned}\phi_{1,s}(X_1) &= E\left[\phi_s(X_1, X_2) \mid X_1\right] \\ &= \left[\frac{2 - s^2X_1^2}{2(1+s)}\right] + sX_1 - 1,\end{aligned}$$

and

$$\begin{aligned}\phi_{2,s}(X_1) &= E\left[\phi_s(X_2, X_1) \mid X_1\right] \\ &= -s^2e^{-sX_1} + e^{-sx_1} + s - 1.\end{aligned}$$

Thus, set

$$\begin{aligned}\phi_s(X_1) &= \phi_{1,s}(X_1) + \phi_{2,s}(X_1) \\ &= \left[\frac{2 - s^2 X_1^2}{2(1+s)}\right] + s(X_1+1) + e^{-sX_1}\left(-s^2+1\right) - 2.\end{aligned} \tag{2.7}$$

In view of (2.7), the variance of $\hat{\delta}(s)$ is

$$\sigma_s^2 = Var\left\{\left(\frac{2 - s^2 X_1^2}{2(1+s)}\right) + s(X_1+1) + e^{-sX_1}\left(-s^2+1\right) - 2\right\}. \tag{2.8}$$

Under H_0, it is easy to prove that $\mu_0 = E(\psi_s) = 0$, and the variance $\sigma_{0,s}^2$ reduces to

$$\sigma_{0,s}^2 = \frac{1}{(1+s)^2(1+2s)}\left[s^6 + 4s^5 - 6s^4 - 10s^3 + 10s^2 + 16s + 6\right]. \tag{2.9}$$

2.2 Asymptotic efficiency

To asses the quality of this procedure, we evaluate its *Pittman asymptotic* efficiency (PAE) for two alternatives in the class (since they are in the NBUL). These are:

(i) Linear failure rate family:

$$\overline{F}_1(x) = e^{-x-\frac{\theta}{2}x^2},\ x \geq 0,\ \theta \geq 0;$$

(ii) Makeham family:

$$\overline{F}_2(x) = e^{-x-\theta(x+e^{-x}-1)},\ x \geq 0,\ \theta \geq 0.$$

Note that under $\theta = \theta_0$, the linear failure rate and the Makeham distributions reduce to the exponential distribution. The PAE is defined by:

$$PAE(\delta(s)) = \frac{d}{d\theta}\delta(s)|_{\theta\to\theta_0}\ /\ \sigma_{0,s}.$$

In the above two cases we get the following PAE values:

(i) Linear failure rate family:

$$\begin{aligned}PAE(\hat{\delta}(s)) &= \frac{1}{\sigma_{0,s}}\left|\frac{(r+1)!}{(1+s)^2}\right| \\ &= 0.8153;\end{aligned}$$

(ii) Makeham family:

$$PAE(\hat{\delta}\,(s)) = \frac{1}{\sigma_{0,s}}\left|\frac{-3}{4\,(1+s)\,(s+2)}\right|$$
$$= 0.1529.$$

Note that the above results show that these alternatives have decreasing efficiency in $s \geq 0$ at $r = 1$ and have maximum value at $s = 0.001$.

2.3 Monte Carlo null distribution critical values

In practice, simulated percentiles for small samples are commonly used by applied statisticians and reliability analyst. Next, we simulate the Monte Carlo null distribution critical points for $\hat{\delta}\,(s)$ in (2.6) based on 5000 simulated sample 5(5)50 from the standard exponential distributions. Table (1) gives the upper percentile points of the statistic $\hat{\delta}\,(s)$ with 5000 replications at $s = 0.5$.

Table (1)
The upper percentile of $\hat{\delta}(s)$ with 5000 replications at s=0.5

n	90%	95%	99%
5	0.4711	0.6813	1.3509
10	0.3141	0.4268	0.7 050
15	0.2436	0.3300	0.5216
20	0.2035	0.2661	0.3886
25	0.1893	0.2404	0.3542
30	0.1726	0.2197	0.3120
35	0.1607	0.1997	0.2867
40	0.1546	0.1901	0.2731
45	0.1453	0.1759	0.2630
50	0.1338	0.1637	0.2328

In view of Table (1), it is noticed that the critical values are increasing as the confidence level increasing and is almost decreasing as the sample size increasing at $s = 0.5$ and $r = 1$.

2.4 The power of the proposed test

The power of the proposed test at a significance level α with respect to the alternatives F_1, F_2 and F_3 is calculated based on simulation data. Here we use the significance level $\alpha = .05$ and the next commonly used alternative distributions in reliability theory:

(i) Linear failure rate family:

$$\overline{F}_1(x) = e^{-x-\frac{\theta}{2}x^2}, \; x \geq 0, \; \theta \geq 0;$$

(ii) Gamma family:

$$\overline{F}_2(x) = \int_x^\infty e^{-u}u^{\theta-1}du/\Gamma(\theta), \; x > 0, \; \theta \geq 0;$$

(iii) Weibull family:

$$\overline{F}_3(x) = exp(-x^\theta), \; x > 0, \; \theta \geq 0.$$

Based on 5000 replications, Table (2) below shows the power estimate using $\alpha = .05$ with parameter $\theta = 1, 2$ and 3 at $n = 10, 20$ and 30.

Table (2)

Alternative Distributions: LFR, Gamma, Weibull

n	θ	LFR	Gamma	Weibull
	1	1.000	0.989	0.950
10	2	1.000	1.000	1.000
	3	1.000	1.000	1.000
	1	1.000	0.997	0.950
20	2	1.000	1.000	1.000
	3	1.000	1.000	1.000
	1	1.000	0.998	0.952
30	2	1.000	1.000	1.000
	3	1.000	1.000	1.000

It is clear from the above table that our test has good powers for all alternatives and the powers increases as the sample size increases. The power is getting as smaller as the NBUL approaches the exponential distribution.

3 Testing for censored data

In this section, a test statistic is proposed to test H_0 versus H_1 with randomly right-censored data. Such a censored data is usually the only information available in a life-testing model or in a clinical study where patients may be lost (censored) before the completion of a study. This experimental situation can formally be modeled as follows. Suppose n objects are put on test, and $X_1, X_2, ..., X_n$ denote their true life time. We assume that $X_1, X_2, ..., X_n$ be independent, identically distributed (i.i.d.) according to a continuous life distribution F. Let $Y_1, Y_2, ..., Y_n$ be (i.i.d.) according to a continuous life distribution G. Also we assume that X's and Y's are independent. In the randomly right-censored model, we observe the pairs (Z_j, δ_j), $j = 1, ..., n$, where $Z_j = \min(X_j, Y_j)$ and

$$\delta_j = \begin{cases} 1, & \text{if } Z_j = X_j \ \ (j\text{-th observation is uncensored}) \\ 0, & \text{if } Z_j = Y_j \ \ (j\text{-th observation is censored}). \end{cases}$$

Let $Z_{(0)} = 0 < Z_{(1)} < Z_{(2)} < ... < Z_{(n)}$ denote the ordered Z's and $\delta_{(j)}$ is δ_j corresponding to $Z_{(j)}$. Using the censored data (Z_j, δ_j), $j = 1, ..., n$. Kaplan and Meier[12] proposed the product limit estimator,

$$\overline{F}_n(X) = \prod_{[j\,:\,Z_{(j)} \leq X]} \{(n-j)/(n-j+1)\}^{\delta_{(j)}}, X \in [0, Z_{(n)}]$$

Now, for testing $H_0 : \delta(s) = 0$ against $H_1 : \delta(s) > 0$, using the randomly right censored data, we propose the following test statistic

$$\hat{\delta}_c(s) = \frac{1}{s^3}\left[-1 + s\mu + \left(1 - 2^{-1}s^2\mu_2\right)\phi(s)\right].$$

where, $\phi(s) = \int_0^\infty e^{-su} dF_n(u)$. For computational purposes, $\hat{\delta}_c(s)$ may be rewritten as

$$\hat{\delta}_c(s) = \frac{1}{s^3}\left[-1 + s\ \Phi + \left(1 - 2^{-1}s^2\ \Omega\right)\ \Theta\right],$$

where

$$\Phi = \sum_{k=1}^{n} \prod_{m=1}^{k-1} C_m^{\delta(m)} \left(Z_{(K)} - Z_{(K-1)}\right),$$

$$\Theta = \sum_{j=1}^{n} e^{-sz(j)} \left[\prod_{p=1}^{j-2} C_p^{\delta(p)} - \prod_{p=1}^{j-1} C_p^{\delta(p)}\right],$$

$$\Omega = 2\sum_{i=1}^{n}\prod_{v=1}^{i-1} Z_{(i)}C_v^{\delta(v)}\left(Z_{(i)} - Z_{(i-1)}\right),$$

and

$$\begin{aligned} dF_n\left(Z_j\right) &= \overline{F}_n\left(Z_{j-1}\right) - \overline{F}_n\left(Z\right), \quad C_k \\ &= \left[n-k\right]\left[n-k+1\right]^{-1}. \end{aligned}$$

Table (3) below gives the critical values percentiles of $\hat{\delta}_c\left(s\right)$ test for sample sizes $n = 5\left(5\right)30\left(10\right), 81, 86$.

Table (3)

The upper percentile of $\hat{\delta}c(s)$ with 5000 replications at s=0.5

n	90%	95%	99%
5	-0.5934	-0.0300	1.5503
10	0.2540	0.7630	2.2252
15	0.5585	1.0243	3.3175
20	0.7487	1.1692	2.2221
25	0.8886	1.2674	2.2044
30	0.9503	1.3310	2.2028
40	1.0646	1.3453	2.1455
50	1.1277	1.4029	2.1174
60	1.2173	1.5030	2.1140
70	1.2085	1.4720	2.0664
81	1.2704	1.5023	2.1272
86	1.3402	1.5601	2.2345

It can be noticed from Table (3) that the critical values are increasing as the confidence level increasing and decreasing as the sample size increasing.

4 Some applications

In this section, we apply the test on some data-sets to elucidate the applications of the NBUL in the both non censored and censored data at 95% confidence level.

4.1 Non censored data

Data-set #1.

Consider the data-set in Abouammoh et al. [1], these data represent set of 40 patients suffering from blood cancer (leukemia) from one of ministry of health hospitals in Saudi Arabia. In this case, we get $\hat{\delta}\,(s) = 3.596123$ which is greater than the critical value of the Table (1). Then we accept H_1 the alternative hypotheses which states that the data set has NBUL property and not exponential.

Data-set # 2.

Consider the data-set given in Grubs [11] and have been used in Ebrahim et al. [8] and Shapiro [17]. This data set gives the times between arrivals of 25 customers at a facility. In this case, we get $\hat{\delta}\,(s) = 6.61722$ which is greater than the critical value of the Table (1). Hence we accept H_1 which states that the data set have NBUL property and not exponential.

Data-set #3.

Consider the data-set given in Lawless [14]. These data-set represent failure times in hours, for a specific type of electrical insulation in an experiment in which the insulation was subjected to a continuously increasing voltage stress. Here, we get $\hat{\delta}\,(s) = 0.02114$ which is less than the critical value of the Table (1). Hence we accept the null hypothesis.

Data-set #4.

Consider the data-set given in Fisher [9] which represent the differences in heights between cross- and self- fertilized plants of the same pair grown together in one pot. In this case, we get $\hat{\delta}\,(s) = -27.5858$ which is less than the critical value of the Table (1). Hence we conclude that this data set have exponential distribution.

Data-set #5.

Consider the data-set given in *Kots* and *Johnson* [13] which represents the survival times (in years) after diagnosis of 43 patients with a certain kind of leukemia. In this case, we get $\hat{\delta}\,(s) = 1.3202$ which is greater than the critical value of the Table (1). Hence we accept H_1 which states that the data set have NBUL property and not exponential.

4.2 Censored data

Data-set #6.

Consider the data from *Susarla* and *Vanryzin* [18], which represent 81 survival times (in months) of patients of melanoma. Out of these 46 represents non-censored data, and the ordered values. Taking into account the whole set of survival data (both censored and uncensored), and computing the statistic from (4) censored data, we get $\hat{\delta}_c(s) = 12.2667$ which is greater than the critical value of the Table (3) at 95% upper percentile. Then, we accept H_1 which states that the data set have NBUL property and not exponential.

Data-set #7.

On the basis of right censored data for lung cancer patients from *Pena* [16] These data consists of 86 survival times (in month) with 22 right censored. Taking into account the whole set of survival data (both censored and uncensored), and computing the statistic from (4) censored data, we get $\hat{\delta}_c(s) = 3.368$ which is greater than the critical value of the Table (3) at 95% upper percentile. Then, we accept H_1,which states that the data set have $NBUL$ property and not exponential.

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