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Point Availability of a Robot with Internal Safety Device

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Abstract

We introduce a robot–safety device system composed of a robot with internal (built–in) safety device. The system is characterized by a safety shut–down rule and by the natural feature of standby. In order to obtain the point availability of the twin–system, we introduce a stochastic process endowed with a probability measure satisfying a (non–standard) integral equation. The explicit solution procedure requires the (new) notion of virtual lifetime - versus - effective lifetime of the robot. The analysis of the long–run availability requires the introduction of a signed measure. As an example, we display a computer–plotted graph of the point availability obtained by inversion of the corresponding Laplace–transform.

Keywords: robot, safety device, virtual lifetime, effective lifetime, probability measure, signed measure

1 Introduction

Up–to–date robots are often connected with a *safety* device, e.g. Vanderperre et al. (2000). Such a device prevents possible damage, caused by a robot

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failure, in the neighbouring environment. The usual “bugbears” are software failures, e.g. Gaskill et al. (1996), common–cause failures, e.g. Dhillon et al. (1997) and physical failures, e.g. Birolini (1994), Gnedenko et al. (1994). Moreover, the random behaviour of the entire system (robot, safety unit, repair facility) requires some additional measures to ensure the safety of man–machine interactions, Gaskill et al. (1996). In the previous Literature, we have considered a robot with (external) safety device, called the **T**–system, e.g. Vanderperre (2000), Vanderperre et al. (2005). The **T**–system is characterized by the natural feature of standby for the safety device and by an admissible risky state, Vanderperre et al. (2002). As a variant, we introduce a robot with *internal* (built–in) safety unit, henceforth called the **S**–system, subjected to the following safety shut–down rule : *“Any repair of the failed safety device requires a shut–down of the operative robot.”*
On the other hand, the safety unit need not to operate if the robot is in repair. Consequently, upon failure of the robot (safety device) the operative safety device (robot) is put in standby until the repair of the robot (safety device) has been completed. The **S**–system is attended by two different repairmen. Repairman R_s is skilled in repairing the safety unit, whereas repairman R is assumed to be an expert in repairing the robot. Any repair is supposed to be perfect and general.
Apart from a statistical generalization of the **T**–system with regard to the previous (restrictive) assumption of a *constant* failure rate of the robot, we also introduce the notion of virtual lifetime - versus - effective lifetime of the robot (**S**–system).
In order to obtain the point availability and the long–run availability of the **S**–system, we introduce a stochastic process endowed with a probability measure satisfying a (non–standard) integral equation. The explicit solution procedure requires the distribution of the robot’s virtual lifetime and the introduction of a *signed* measure.
As an example, we display a computer–plotted graph of the point availability obtained by inversion of the corresponding Laplace–transform.

2 Formulation

Consider the **S**–system satisfying the following conditions.

- The *operative* safety device has a constant failure rate λ_s and a general repair time distribution $R_s(\cdot), R_s(0) = 0$. Let f_s be the random variable corresponding to the failure rate λ_s. Clearly, f_s is exponentially distributed with mean λ_s^{-1}. The repair time is denoted by r_s.
- In order to define the virtual lifetime of the robot, we first consider a robot *without* a safety device, starting to operate at some time origin

$t = 0$. The lifetime of the robot is denoted by f with general distribution $F(\cdot), F(0) = 0$. Clearly, f is the time measured from $t = 0$ onwards until the robot fails. Next, we consider the **S**–system. Let

$$v_f := \begin{cases} f + \sum_{i=1}^{n_f} r_{s,i} & , \text{if } n_f > 0, \\ f & , \text{if } n_f = 0, \end{cases}$$

where $r_{s,i}; i = 1, 2, \ldots$ denotes the i–th repair time of the safety unit and n_f the number of λ_s–failures *during* f. The random variable v_f is called the *virtual* life time of the robot. Clearly, v_f reduces to f if $n_f = 0$. Therefore, we call f the *effective* lifetime of the robot. The repair time of the robot is denoted by r with general distribution $R(\cdot), R(0) = 0$. Finally, let $F_v(\cdot) := \mathbf{P}\{v_f \leq \cdot\}$.

- The variables f, f_s, r, r_s are supposed to be *statistically* independent with finite mean and any repair is perfect.
- Finally, we assume that both the robot and the safety unit are free from standby failures (the so–called "cold" standby mode).

In order to describe the random behaviour of the **S**–system, we introduce a stochastic process $\{N_t, t \geq 0\}, N_0 = 0$ **P**–a.s., with state space $\{A, B, C\} \subset [0, \infty)$ characterized by the following mutually exclusive events :

$\{N_t = A\}$: "The robot and the safety device are both operative at time t."

$\{N_t = B\}$: "The safety unit is under progressive repair and the robot is in standby at time t."

$\{N_t = C\}$: "The robot is under progressive repair and the safety device is in standby at time t."

State A is called the safe state and state B is called the shut–down state. Note that the event : "The robot and the safety device are *both* under repair at time t" is a **P**–null set! For $K = A, B, C$ let

$$\wp_K(t) := \mathbf{P}\{N_t = K\}, \quad t \geq 0.$$

We recall that the **S**–system is only available (functioning) in state A. Therefore, the *point* availability of the **S**–system is given by $\wp_A(\cdot)$. Let

$$\wp_A(\infty) := \lim_{t\to\infty} \wp_A(t),$$

provided that the precious limit exists. $\wp_A(\infty)$ is called the *long–run* availability of the **S**–system. Observe that

$$\wp_A(\infty) = \lim_{T\to\infty} \frac{1}{T} \int_0^T \wp_A(t) dt.$$

Notations

- The Borel algebra on $[0,\infty)$ is denoted by $\mathcal{B}([0,\infty))$.
- The indicator (function) of a set $\{\cdot\} \in \mathcal{B}([0,\infty))$ is denoted by $\mathbf{1}\{\cdot\}$.
- The Laplace–transform of any function $\alpha(\cdot)$, locally integrable and bounded on $[0,\infty)$ is denoted by the corresponding character marked with an asterisk. For instance,

$$\alpha^*(z) := \int_0^\infty e^{-zt}\alpha(t)dt, \quad \text{Re } z > 0.$$

- If $\alpha(\cdot)$ is a right–continous function of bounded variation on $[0,\infty)$, then we define

$$\alpha^\vee(z) := \int_{0-}^\infty e^{-zt}d\alpha(t), \quad \text{Re } z \geq 0,$$

where

$$\int_{0-}^\infty e^{-zt}d\alpha(t) := \alpha(0) + \int_0^\infty e^{-zt}d\alpha(t).$$

Note that the product rule, e.g. Brémaud (1991, Appendix) implies that

$$\alpha^\vee(z) = z\alpha^*(z), \quad \text{Re } z > 0.$$

- The notation $X \sim Y$ signifies that the random variables X and Y are equally distributed.

3 Preliminary properties

For direct reference, we state the following properties. Let

$$p_s(t) := \int_{0-}^t e^{-\lambda_s(t-u)}d\sum_{n=0}^\infty \varphi_s^{n\star}(u),$$

where

$$\varphi_s(u) := \int_0^u (1 - e^{-\lambda_s(u-v)})dR_s(v).$$

$\varphi_s^{n\star}(\cdot)$ denotes the n–fold convolution of $\varphi_s(\cdot) \equiv \varphi_s^{1\star}(\cdot)$. For $n = 0$, $\varphi_s^{0\star}(u)$ represents the Heaviside unit–step function with the unit–jump at $u = 0$.
Property 3.1 Vanderperre (2001), Vanderperre et al. (2005).

- $p_s(0) = 1$, $0 < p_s(t) \leq 1$, $p_s(\infty) = (1 + \lambda_s \mathbf{E}r_s)^{-1}$.

- $p_s(\cdot)$ *is Lebesgue–absolutely continuous on* $(0,\infty)$ *and of bounded variation on* $[0,\infty)$.

- $$p_s^*(z) = \frac{1}{z+\lambda_s(1-\mathbf{E}e^{-zr_s})}\,, \quad \text{Re } z > 0.$$

- $$p_s^{\vee}(z) = \frac{1}{1+\lambda_s \mathbf{E}r_s\gamma(z)}\,, \quad \text{Re } z \geq 0, \tag{1}$$

  where
  $$\gamma(z) := \begin{cases} \frac{1-\mathbf{E}e^{-zr_s}}{z\mathbf{E}r_s} & \text{, if Re } z \geq 0, z \neq 0, \\ 1 & \text{, if } z = 0. \end{cases}$$

Note that
$$\gamma(z) = \frac{1}{\mathbf{E}r_s}\int_0^\infty e^{-zu}(1-R_s(u))du, \quad \text{Re } z \geq 0.$$

Theorem 3.1

$$F_v(t) = \int_0^t \sum_{k=0}^\infty R_s^{k\star}(t-u)e^{-\lambda_s u}\frac{(\lambda_s u)^k}{k!}dF(u)\,.$$

$$\mathbf{E}v_f = \mathbf{E}f(1+\lambda_s\mathbf{E}r_s)\,.$$

Proof.

- By the law of total probability,

  $$\begin{aligned} \mathbf{P}\{v_f \leq t\} &= \int_0^\infty \mathbf{P}\{v_f \leq t | f = u\}dF(u) \\ &= \int_0^t \mathbf{P}\left\{\sum_{i=1}^{n_u} r_{s,i} \leq t-u\right\}dF(u) \\ &= \int_0^t \sum_{k=0}^\infty R_s^{k\star}(t-u)\mathbf{P}\{n_u = k\}dF(u). \end{aligned}$$

  But $\{n_u, u \geq 0\}$ is a homogeneous Poisson process with parameter λ_s, i.e.
  $$\mathbf{P}\{n_u = k\} = e^{-\lambda_s u}\frac{(\lambda_s u)^k}{k!} \quad ; k = 0, 1, \ldots$$

  Hence,

  $$\mathbf{P}\{v_f \leq t\} = \int_0^t \sum_{k=0}^\infty R_s^{k\star}(t-u)e^{-\lambda_s u}\frac{(\lambda_s u)^k}{k!}dF(u)\,.$$

- The Laplace–Stieltjes convolution theorem entails that

$$\mathbf{E}e^{-zv_f} = \mathbf{E}e^{-(z+\lambda_s(1-\mathbf{E}e^{-zr_s}))f}, \quad \mathrm{Re}\, z \geq 0. \tag{2}$$

  Thus, $\mathbf{E}v_f$ follows from the relation

$$\mathbf{E}v_f = -\left.\frac{\partial}{\partial z}\mathbf{E}e^{-zv_f}\right|_{z=0}.$$

Remarks 3.1

- Observe that the function $p_s(t), t \geq 0$ induces a finite *signed* measure on $\mathcal{B}([0,\infty))$ denoted by $\mu_s(\cdot)$. Clearly, by property 3.1,

$$\int_{[0,\infty)} \mu_s(dx) = \frac{1}{1+\lambda_s \mathbf{E}r_s}. \tag{3}$$

- It is fairly obvious that the state probability $p_s(\cdot)$ can be generalized for arbitrary distributions by means of Renewal Theory, e.g. Serfoso (1990).

Involving the functions $F_v(\cdot)$ and $R(\cdot)$, let

$$\varphi_R(u) := \int_0^u F_v(u-w)dR(w),$$

and

$$p_R(t) := \int_{0-}^t (1-F_v(t-u))d\sum_{n=0}^{\infty}\varphi_R^{n\star}(u). \tag{4}$$

The following properties are stated without proof.
Property 3.2

- *$p_R(0) = 1$, $0 < p_R(t) \leq 1$.*

- *If f is non–lattice, then*

$$p_R(\infty) = \frac{\mathbf{E}v_f}{\mathbf{E}r + \mathbf{E}v_f}. \tag{5}$$

-

$$p_R^*(z) = \frac{1}{z}\frac{1-\mathbf{E}e^{-zv_f}}{1-\mathbf{E}e^{-zr}\mathbf{E}e^{-zv_f}}, \quad \mathrm{Re}\, z > 0. \tag{6}$$

Remarks 3.2 Consider the decomposition of Eq. (4), i.e.

$$p_R(t) = \sum_{n=0}^{\infty} \varphi_R^{n\star}(t) - \int_{0-}^{t} F_v(t-u) d \sum_{n=0}^{\infty} \varphi_R^{n\star}(u).$$

Clearly, $p_R(\cdot)$ is a difference of two right–continuous increasing functions. Hence, $p_R(\cdot)$ is of bounded variation on any compact of $[0, \infty)$. However, since $p_R(\infty)$ exists, we may conclude that $p_R(\cdot)$ is a right–continuous function of bounded variation on $[0, \infty)$.
Consequently, $p_R(\cdot)$ is uniquely determined by $p_R^*(\cdot)$. Note that our remark is crucial to determine the point availability $\wp_A(\cdot)$ by inversion of the corresponding Laplace–transform $\wp_A^*(\cdot)$. See Chapter 5 for further details.

4 Integral equation

In order to derive $\wp_A(\cdot)$, let

$$\varphi_v(t) := \int_0^t (1 - R(t-w)) dF_v(w) \,.$$

We recall that

$$\varphi_R(t) := \int_0^t F_v(t-w) dR(w) \,.$$

Theorem 4.1
The point availability of the **S**–*system satisfies the integral equation*

$$\wp_A(t) + \lambda_s \int_0^t \wp_A(t-u)(1 - R_s(u)) du = 1 - \int_{0-}^{t} \varphi_v(t-u) d \sum_{n=0}^{\infty} \varphi_R^{n\star}(u) \,. \quad (7)$$

Proof. The conservation law of probability entails that

$$\wp_A(t) + \wp_B(t) + \wp_C(t) = 1 \,. \quad (8)$$

On the other hand, the shut–down rule implies that

$$\begin{aligned} \wp_B(t) &= \int_0^t \wp_A(u) \mathbf{P}\{r_s > t-u, f_s \in du \mid f_s > u\} \\ &= \int_0^t \wp_A(u) \mathbf{P}\{r_s > t-u\} \mathbf{P}\{f_s \in du \mid f_s > u\} \,. \end{aligned}$$

A straightforward application of the law $\mathbf{P}\{\mathcal{E}_1 \cap \mathcal{E}_2\} = \mathbf{P}\{\mathcal{E}_1 \mid \mathcal{E}_2\} \mathbf{P}\{\mathcal{E}_2\}$ and the Markov property of the exponential distribution, reveals that

$$\mathbf{P}\{f_s \in du \mid f_s > u\} = \lambda_s du \,.$$

Hence,

$$\wp_B(t) = \lambda_s \int_0^t \wp_A(t-u)(1-R_s(u))du \,. \tag{9}$$

In order to determine $\wp_C(t)$, we consider a (fictitious) robot *without* safety device, with lifetime $l_R \sim v_f$ and attended by a single repair facility.
Along with the system, we consider an *alternating* renewal process $\{\mathcal{N}(t), t \geq 0\}$, generated by $\{F_v(\cdot), R(\cdot)\}$, with state space $\{\mathcal{A}, \mathfrak{C}\} \subset [0,\infty)$, characterized by the following mutually exclusive events :

$\{\mathcal{N}(t) = \mathcal{A}\}$: "The robot is operating at time t."

$\{\mathcal{N}(t) = \mathfrak{C}\}$: "The robot is under repair at time t."

Let $\wp_{\mathfrak{C}}(t) := \mathbf{P}\{\mathcal{N}(t) = \mathfrak{C} \,|\, \mathcal{N}(0) = \mathcal{A}\}$. Clearly, state $\mathfrak{C}$ is regenerative for the process $\{\mathcal{N}(t)\}$. Hence, by Renewal Theory

$$\wp_{\mathfrak{C}}(t) = \int_{0-}^t \varphi_v(t-u) d \sum_{n=0}^{\infty} \varphi_R^{n\star}(u) \,. \tag{10}$$

On the other hand, the Markov property of the exponential distribution implies that state C of the $\mathbf{S}$–system is regenerative for the process $\{N(t)\}$ as well. Moreover, due to the assumption of cold standby for the safety device, we may identify state $\mathfrak{C}$ with state C, i.e. $\wp_{\mathfrak{C}}(t) = \wp_C(t), t \geq 0$. Hence, by Eqs. (8), (9) and (10),

$$\wp_A(t) + \lambda_s \int_0^t \wp_A(t-u)(1-R_s(u))du + \int_{0-}^t \varphi_v(t-u) d \sum_{n=0}^{\infty} \varphi_R^{n\star}(u) = 1 \,.$$

5 Solution procedure

It should be noted that the integral equation is well–adapted to a Laplace transformation. In fact, the functions $\wp_K(\cdot)$ are locally integrable and bounded on $[0,\infty)$. Hence, the Laplace–transform $\wp_K^*(\cdot)$ exists for Re $z > 0$. Applying a Laplace transformation to Eq. (7) reveals that

$$\wp_A^*(z)\big(1 + \lambda_s \frac{1-\mathbf{E}e^{-zr_s}}{z}\big) = \frac{1}{z} - \frac{1}{z}\frac{(1-\mathbf{E}e^{-zr})\mathbf{E}e^{-zv_f}}{1-\mathbf{E}e^{-zr}\mathbf{E}e^{-zv_f}} \,.$$

Whence,

$$\wp_A^*(z) = \frac{1-\mathbf{E}e^{-zv_f}}{1-\mathbf{E}e^{-zv_f}\mathbf{E}e^{-zr}} \, \frac{1}{z+\lambda_s(1-\mathbf{E}e^{-r_s})} \,. \tag{11}$$

By Eqs.(1), (6) we have

$$\wp_A^*(z) = p_R^*(z)p_s^\vee(z).$$

Finally, taking Remark 3.2 into account, we obtain by inversion

$$\wp_A(t) = \int_{[0,\infty)} p_R(t-x)\mathbf{1}\{0 \le x < t\}\mu_s(dx).$$

Next, we deal with the long–run availability $\wp_A(\infty)$ of the **S**–system. It is plain that the existence of $p_R(\infty)$ implies the existence of $\wp_A(\infty)$. Applying the bounded convergence theorem, e.g. Doob (1994), entails that

$$\wp_A(\infty) = p_R(\infty) \int_{[0,\infty)} \mu_s(dx) \,.$$

Hence, by Eqs.(3),(5) and Theorem 3.1

$$\wp_A(\infty) = \frac{\mathbf{E}f}{\mathbf{E}f(1+\lambda_s\mathbf{E}r_s)+\mathbf{E}r} \,.$$

Theorem 5.1 *Let F, R, R_s be general distributions with finite mean. Suppose that f is non–lattice, then*

$$\wp_A(t) = \int_{[0,t)} p_R(t-x)\mu_s(dx) \,,$$

$$\wp_A(\infty) = \frac{\mathbf{E}f}{\mathbf{E}f(1+\lambda_s\mathbf{E}r_s)+\mathbf{E}r} \,.$$

6 Numerical example

Let $F(x) = p_1(1-e^{-\lambda_1 x}) + p_2(1-e^{-\lambda_2 x})$, where $p_1 > 0, p_2 < 0$, $p_1 + p_2 = 1, p_1\lambda_1 + p_2\lambda_2 = 0$ and without loss of generality, $0 < \lambda_1 < \lambda_2$. Note that $(1-F)^{-1}$ is log–convex. As a matter of fact, since $p_2 < 0$, we have

$$\frac{d^2}{dx^2}\log(1-F)^{-1} = -\frac{p_1p_2(\lambda_1-\lambda_2)^2e^{-(\lambda_1+\lambda_2)x}}{(p_1e^{-\lambda_1 x}+p_2e^{-\lambda_2 x})^2} > 0 \,, \quad x \ge 0.$$

Hence, f has an increasing failure rate. Moreover, F' has a single max at $x = (\log\lambda_2 - \log\lambda_1)/(\lambda_2-\lambda_1)$. Lastly, $F'(0) = 0$ and F' strongly decreases in a neighbourhood of infinity. Consequently, F belongs to an important family of Coxian distributions with tractable engineering applications. For instance, F is suitable to model wear–out. Furthermore, let $E_M(x) := 1 - e^{-x}\sum_{k=0}^{M-1} x^k/k!$, $M \ge 1$. As an example, let $\lambda_s = 0.5; \lambda_1 = 1; \lambda_2 = 2$. Observe that $p_1 = 2$ and $p_2 = -1$. Finally, let $R_s(\cdot) = E_1(\cdot)$, $R(\cdot) = E_2(\cdot)$. Note that $\mathbf{E}r_s = 1$, $\mathbf{E}r = 2$ and that $\mathbf{E}f = 3/2$. Hence, by Theorem 5.1, $\wp_A(\infty) = 6/17$. From Eq. (11) we obtain

$$\wp_A^*(z) = \frac{2N(z)}{zD(z)} \,, \quad \mathrm{Re}\; z > 0,$$

where $N(z) := (6+9z+2z^2)(z+1)$ and $D(z) := 34+47z+24z^2+4z^3$. The equation $D(z)=0$ has a real root $\rho=-3.083$ and 2 complex conjugate roots $\omega = a+ib$, $\bar{\omega}=a-ib$, where $a=-1.485$ and $b=0.794$. Hence,

$$\wp_A^*(z) = \frac{1}{2}\frac{N(z)}{z(z-\rho)(z-\omega)(z-\bar{\omega})}\,, \quad \text{Re } z > 0.$$

Clearly, $\wp_A(t)$ is continuous on $(0,\infty)$ and of bounded variation on $[0,\infty)$. Hence, by the inversion theorem,

$$\wp_A(t) = \frac{1}{2}\lim_{T\to\infty}\frac{1}{2\pi i}\int_{-iT+\delta}^{iT+\delta} e^{zt}\frac{N(z)\,dz}{z(z-\rho)(z-\omega)(z-\bar{\omega})}\,, \quad t>0.$$

Applying the residue theorem, entails that

$$\wp_A(t) = \wp_A(\infty) + \frac{1}{2}e^{\rho t}\frac{N(\rho)}{\rho(\rho-\omega)(\rho-\bar{\omega})} + \frac{1}{2}e^{\omega t}\frac{N(\omega)}{\omega(\omega-\rho)(\omega-\bar{\omega})}$$
$$+\frac{1}{2}e^{\bar{\omega}t}\frac{N(\bar{\omega})}{\bar{\omega}(\bar{\omega}-\rho)(\bar{\omega}-\omega)}\,.$$

Some numerical calculus reveals that

$$\wp_A(t) = \frac{6}{17} - \alpha e^{-3.083\,t} + e^{-1.485\,t}(\beta\cos 0.794\,t - \gamma\sin 0.794\,t),$$

where $\alpha = 0.28268, \beta = 0.92967, \gamma = 0.05249$. Figure 1 displays the graph of $\wp_A(t), 0 \le t \le 5$, $\wp_A(\infty) = 0.352941$. The value $\inf_{t\ge 0}\wp_A(t) = \wp_A(2.674430) = 0.344047$ clearly indicates that the overall availability (i.e. the point availability as well as the long–run availability) is not worse than 0.344047.

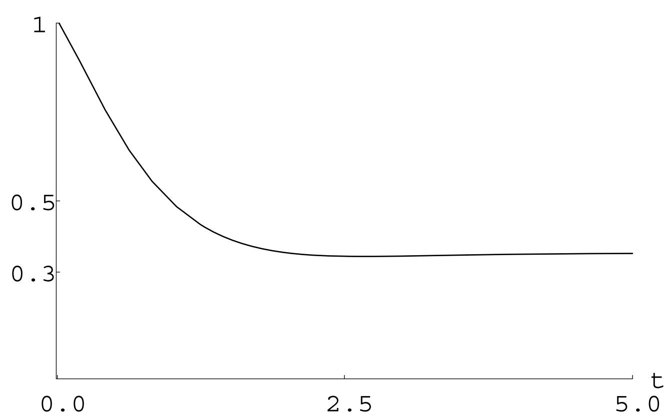

Figure 1: Graph of $\wp_A(\cdot)$, $\lambda_s = 0.5, \lambda_1 = 1, \lambda_2 = 2$

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