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Dual-Tone MFSK for Frequency-Hopped Spread-Spectrum Multiple Access Communications

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Abstract

This paper investigates a dual-tone MFSK signalling technique for a frequency-hopped spread-spectrum communication system. It allows for multiple access communications with less transmission bandwidth and thus greater number of users than conventional MFSK. Both uncoded and coded dual-tone MFSK systems are considered in multiple access interference and additive white Gaussian noise with and without Rayleigh fading.

Keywords: Spread-spectrum; Frequency-hopping; Multiple-access

1 Introduction

Frequency-hopped spread-spectrum (FH-SS) is used in communication systems to provide anti-jamming performance and to mitigate the effects of multiple access interference and fading. This paper considers a FH-SS multiple access (FH-SSMA) communication system with M-ary frequency-shift-keying (MFSK) orthogonal modulation. One M-ary symbol is transmitted per hop and noncoherent (NC) matched filter demodulation is employed at the receiver. In this system, K transmitter-receiver pairs communicate simultaneously over a channel of spread spectrum bandwidth W_{ss}. The data from each user may be encoded with an (n,k) Reed-Solomon (RS) code of rate $r = k/n$.

Each transmitter-receiver pair has a unique hopping pattern that selects the MFSK frequency band during each hop. The hopping patterns are modelled as independent, identically distributed (i.i.d.) random sequences uniformly distributed over a set of q frequency bands. The receiver input consists of the sum of K transmitter outputs plus additive white Gaussian noise (AWGN). The output of the matched filter demodulator is passed to an envelope detector, and a decision is made on the output as to which symbol was most likely to have been transmitted.

system. fading. In general, errors in multiple access systems are due primarily to multiple access interference. Given that there are q frequency bands available and the hopping is independent, the probability of at least one of the $K-1$ other users hopping to the same frequency band as a given user during that user's dwell time is [2]

$$P_{h,k} = 1 - (1 - P_h)^{K-1}, \tag{1}$$

where $P_h = 1/q$ for synchronous FH and $P_h = 2/q - 1/q^2$ for asynchronous FH. In this paper, only synchronous frequency hopping is considered. No side information is available as to whether there are multiple users transmitting in the same band.

With conventional MFSK FH-SSMA, each transmitter sends one of the M possible signals every T seconds on one of the q possible MFSK frequency bands. The transmission bandwidth is given by

$$W_M = \frac{W_{ss}}{q} = \frac{M}{T}, \tag{2}$$

with orthogonal signalling and noncoherent detection. The number of available MFSK frequency bands is restricted to

$$q = \frac{W_{ss}}{W_M} = \frac{W_{ss}T}{M}. \tag{3}$$

With coding, this is reduced to

$$q = \frac{rW_{ss}T}{M}. \tag{4}$$

¿From (1), it is seen that multiple access performance can be improved by having more frequency bands. In [3], it was found that increasing the number of frequency bands decreases the error rate. According to (3), this can be achieved by increasing W_{ss}, reducing M, or increasing T.

The solution considered here is dual-tone MFSK, which creates more frequency bands in a fixed channel bandwidth without modifying the above parameters. With dual-tone MFSK, two tones are transmitted simultaneously in the same band, rather than only one with conventional MFSK. The next section describes dual-tone MFSK. Section 3 considers the uncoded performance of a single user in AWGN and Rayleigh fading. Uncoded system performance with multiple access interference is presented in Section 4. In Section 5, coded system performance in multiple access interference is presented and compared to the results in [4]. Finally, Section 6 presents some conclusions.

2 Dual-Tone MFSK

With dual-tone MFSK, one of M possible dual-tone signals, each of which is composed of two of N possible tones, is transmitted. This results in a smaller bandwidth for each frequency band and thus more bands can be accommodated in a given channel bandwidth. Because there are two tones, the tone energy is half that of conventional MFSK, therefore noise performance is degraded. Thus dual-tone MFSK trades noise performance for multiple access performance.

For a given M, the value of N is the minimum such that

$$\frac{N(N-1)}{2} \geq M.$$

The values of N for M = 8, 16, 32, and 64 are

M	8	16	32	64
N	5	7	9	12
M/N	1.6	2.3	3.6	5.3

With conventional MFSK, each of the M symbols is represented uniquely by one of M tones. With dual-tone MFSK and a given M, the resulting value of $N(N-1)/2$ is usually greater than M, so there are more dual-tone patterns than symbols to be mapped. The dual-tone patterns should be selected so that the tones have nearly equal probabilities. Such a selection for $M = 8$, is shown below

Symbol	Bit Pattern	Tones
s_0	000	f_0, f_1
s_1	001	f_0, f_2
s_2	010	f_0, f_3
s_3	011	f_0, f_4
s_4	100	f_1, f_3
s_5	101	f_1, f_4
s_6	110	f_2, f_3
s_7	111	f_2, f_4

In this case, one tone (f_0), appears in four symbols, while the rest appear in only three.

Note that with $N = 5$ there are 10 distinct dual-tone patterns available. Since only eight patterns are required, two are unused. In the example above, these patterns are f_2, f_4 and f_3, f_4. When one of these two patterns is output by the receiver, the following algorithm is applied:

1. From the two selected tones, discard the one with the smallest amplitude.

2. The discarded tone is replaced with the one that has the next largest amplitude among the remainder, resulting in a new dual-tone pattern.

This process is repeated until a valid dual-tone pattern is obtained.

3 Uncoded Single User Performance

This section presents the uncoded dual-tone system performance in AWGN, and compares it with that of conventional MFSK. Performance with Rayleigh fading is also considered.

The table below shows the additional signal to noise ratio (SNR) required by dual-tone MFSK to achieve a given bit error rate (BER).

	M		
BER	8	16	32
10^{-3}	2.6 dB	2.6 dB	2.5 dB
10^{-4}	2.7 dB	2.7 dB	2.7 dB
10^{-5}	2.8 dB	2.8 dB	2.8 dB

The performance loss with dual-tone MFSK is less than 3 dB. This is the penalty for better multiple access performance, which is considered in the next section.

The performance with Rayleigh fading is now considered. The fading is assumed to be independent for every transmitted symbol. The following table gives the additional SNR required by dual-tone MFSK to achieve a given BER.

	M		
BER	8	16	32
10^{-3}	2.2 dB	2.7 dB	2.0 dB
10^{-4}	2.3 dB	2.6 dB	2.0 dB
10^{-5}	2.5 dB	2.6 dB	2.0 dB

Again, the dual-tone system has inferior performance when compared with the conventional system. Note that the performance difference is slightly less than in AWGN.

4 Multiple Access Performance

This section presents the multiple access performance of dual-tone MFSK FH-SSMA. Consider a synchronous multiple access system with K users transmitting. interference For a given channel bandwidth W_{ss} and transmission rate R, the parameters of interest are q, K and E_b/N_0. q for the conventional system is designated as q_1, and for dual-tone MFSK as q_2. A fixed channel bandwidth of $W_{ss} = 40.32$ MHz is assumed for all M to allow for comparisons. As given by (5), this results in a smaller number of MFSK bands for larger M. For $T = 50\mu$sec, the number of MFSK bands is given below

M	W_{ss} (MHz)	q_1	q_2
8	40.32	250	400
16	40.32	126	288
32	40.32	63	224

Fig. 1 gives the AWGN performance for MFSK with M = 8, 16, and 32 and $E_b/N_0 = 20$ dB. 1t denotes conventional MFSK and 2t denotes dual-tone MFSK. In the figure, the dashed lines represent the conventional system and the solid lines the dual-tone system. As expected, the multiple access performance of dual-tone MFSK is better than that of conventional MFSK for all M. This is because there are more frequency bands available with dual-tone MFSK, thus reducing the probability of a hit. The improvement is about 1.2 dB for M = 8, 2 dB for M = 16, and more than 4 dB for M = 32. In fact, the 32-ary dual-tone system is better than the 16-ary conventional system.

The performance in Rayleigh fading with $W_{ss} = 40.32$MHz and $E_b/N_0 = 30$ dB is given in Fig. 2. Although a higher SNR is now required to attain adequate performance, the dual-tone system performance is still superior to the conventional system. In terms of performance gain, the 32-ary system has the highest, followed by the 16-ary and 8-ary systems. The dual-tone 8-ary FSK system has virtually the same performance as that of the corresponding conventional system for small K, i.e., $K \leq 3$. Given the same channel bandwidth, the dual-tone system performance for small K is significantly improved by increasing E_b/N_0. This is because for small values of K, the background noise is the predominant cause of error. Conversely, for larger values of K, the performance is mainly determined by the interference from other users, which is better mitigated by the dual-tone system because there are more frequency bands.

5 Coded Multiple Access Performance

In this section, the performance of dual-tone FH-SSMA with Reed-Solomon (RS) coding is presented. In [4], the coded performance of conventional FH-

SSMA is given, and this is considered here as the benchmark for comparison. Accordingly, the same assumptions and system parameters are used to produce performance results for dual-tone MFSK.

The performance measure is the normalised throughput, which is defined as the average number of successfully transmitted information bits per channel use [4], for a given probability of error. The design parameters of interest are n, k, q, and M. As in [4], the background noise is assumed to be very low, and so is ignored in all simulations. Therefore, symbol errors are caused only by interference from other users.

The probability of correct decoding for an (n, k) RS code given K users is given by [5]

$$P_c(K,q) = \sum_{j=0}^{(n-k)/2} \binom{n}{j} P_s^j (1-P_s)^{n-j}, \tag{5}$$

where P_s is the symbol error probability given K users. The normalised throughput $W(K,q)$ in bits/channel use is [4]

$$W(K,q) = \frac{k \log_2 M}{n} K P_c(K,q) = r \log_2 M K P_c(K,q). \tag{6}$$

Fig. 3 shows the normalised throughput W for dual-tone NC 32-FSK FH-SSMA versus the number of users K for a (32,11) RS code. The number of available NC 32-FSK frequency bands for the conventional system is $q_1 = 10$ from [4] ($q_2 = 35$), and $n = 32$ is considered in all cases, as in [4]. The results for the conventional system are from [4], and are denoted by dotted lines. The dual-tone system results are represented by solid lines. In the figure, dual-tone 32-FSK has a much higher throughput than conventional 32-FSK because the former requires a smaller transmission bandwidth, and thus more frequency bands are available for a given bandwidth and code rate. Similar results were obtained for other values of k.

For M = 8 and 16, RS codes over GF(64) and GF(256), respectively, were employed so that two MFSK symbols make one RS code symbol. Figs. 4 and 5 give the coded multiple access performance for $M = 8$ and 16, respectively, with $k = 16$. ¿From these figures, it is clear that dual-tone MFSK is superior to conventional MFSK. These figures also show that the 32-ary system has the highest performance gain, followed by the 16-ary and 8-ary systems.

Finally, the normalised throughput, as given by (6), is optimised over the code rate. For $M = 8, 16$, and 32, the optimal code rate was found to be approximately 0.625, i.e., $n = 32$ and $k = 20$. The corresponding performance is shown in Figs. 6, 7, and 8 for $M = 8$, 16, and 32, respectively.

6 Conclusions

Dual-tone MFSK signaling has been evaluated in multiple access interference and additive white Gaussian noise (AWGN), with and without Rayleigh fading. The results indicate that using dual-tone MFSK improves the multiple access performance over that with conventional MFSK. This is due to the increased number of frequency bands in a fixed channel bandwidth. The cost is a loss of less than 3 dB in noise performance. The performance gain in multiple access environments is greater with higher M. Multiple access performance with Reed-Solomon coding has also been compared. The results show a significant improvement in data throughput. In addition, the optimum code rate which maximises the throughput has been determined for various M.

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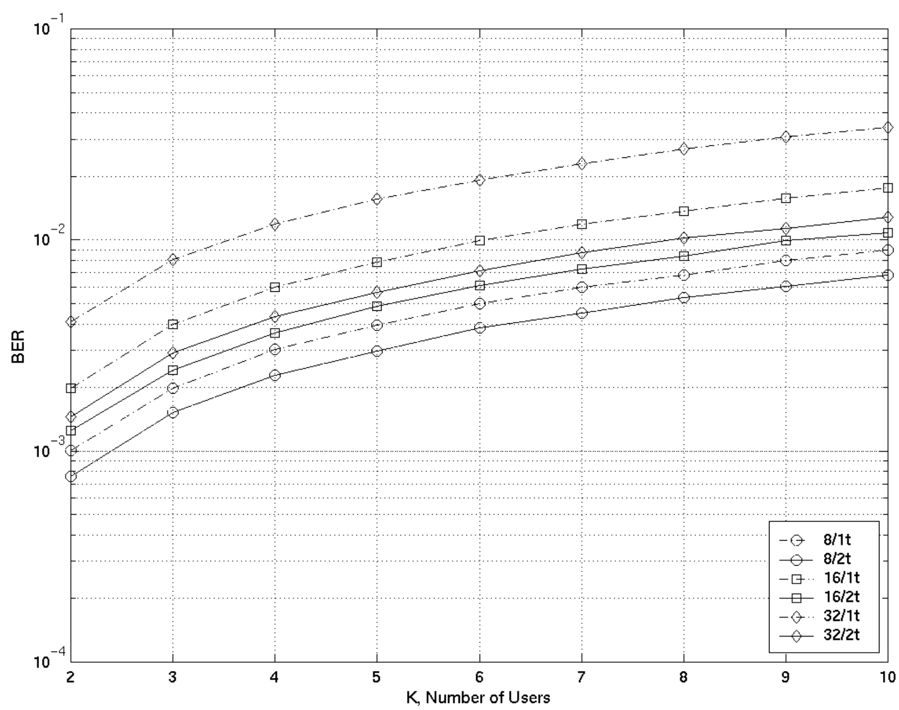

Figure 1: Performance of of FH-SSMA with $W_{ss} = 40.32$MHZ & $E_b/N_0 = 20$ dB in AWGN.

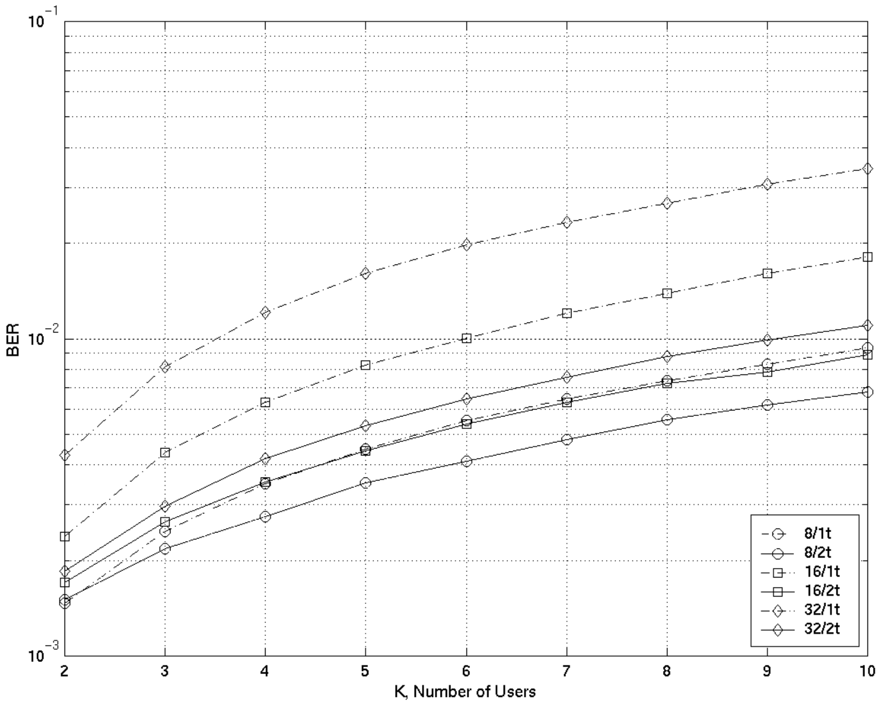

Figure 2: Performance of of FH-SSMA with $W_{ss} = 40.32$MHZ & $E_b/N_0 = 30$ dB in Rayleigh fading.

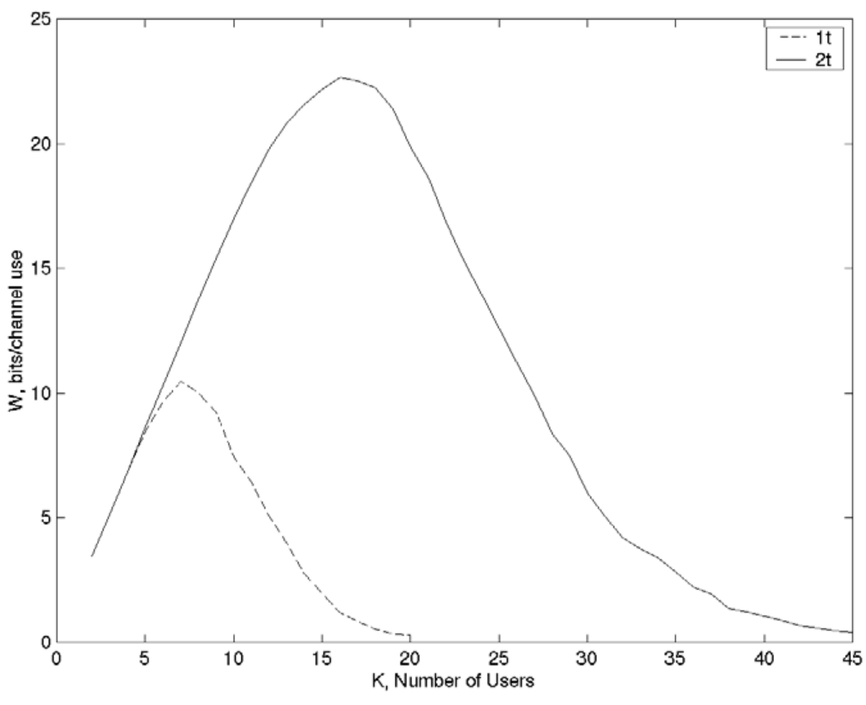

Figure 3: Normalised throughput for 32-FSK FH-SSMA with a (32,11) RS code.

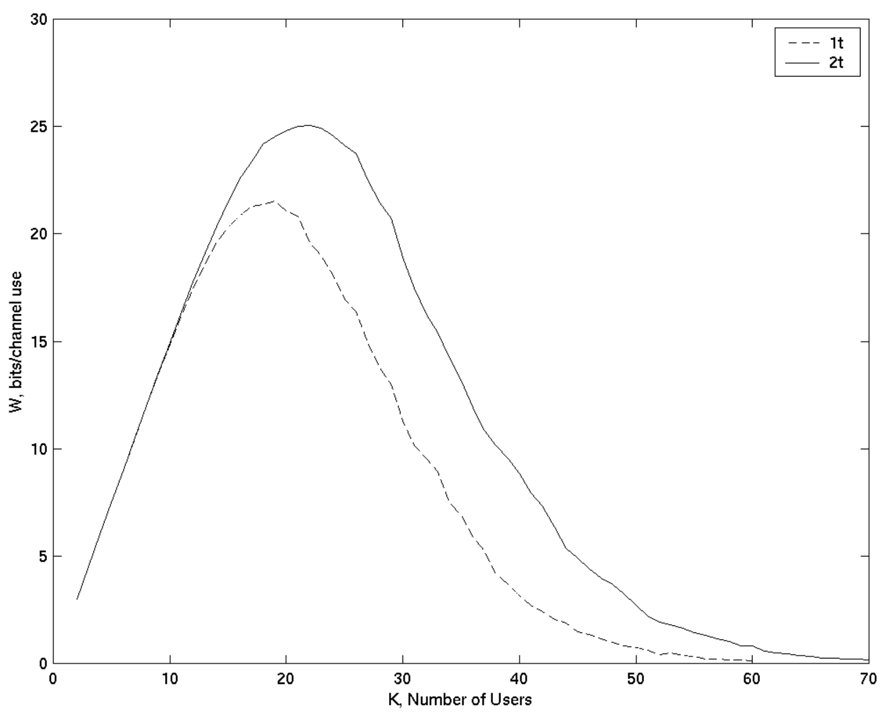

Figure 4: Normalised throughput for 8-FSK FH-SSMA with a (32,16) RS code.

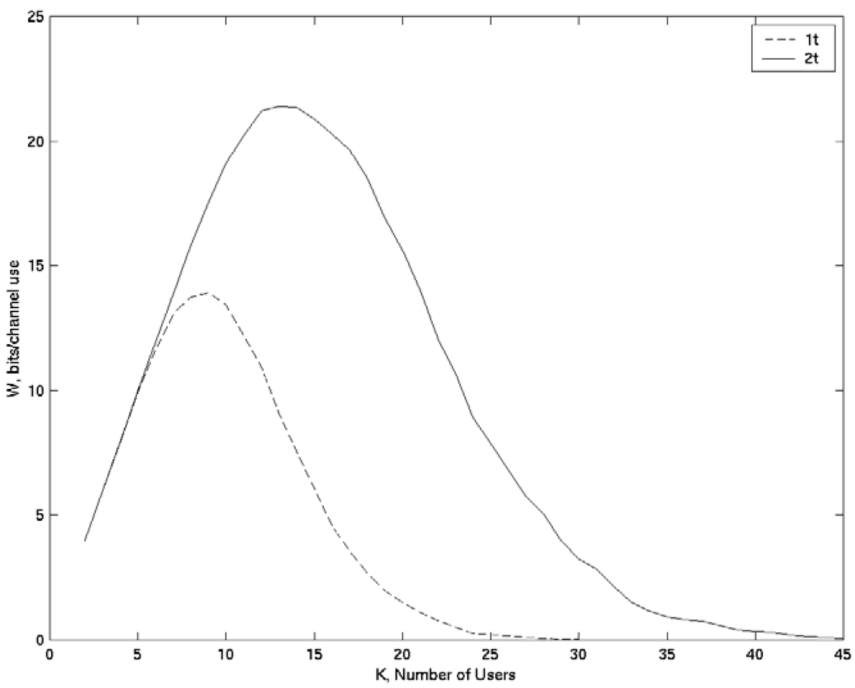

Figure 5: Normalised throughput for 16-FSK FH-SSMA with a (32,16) RS code.

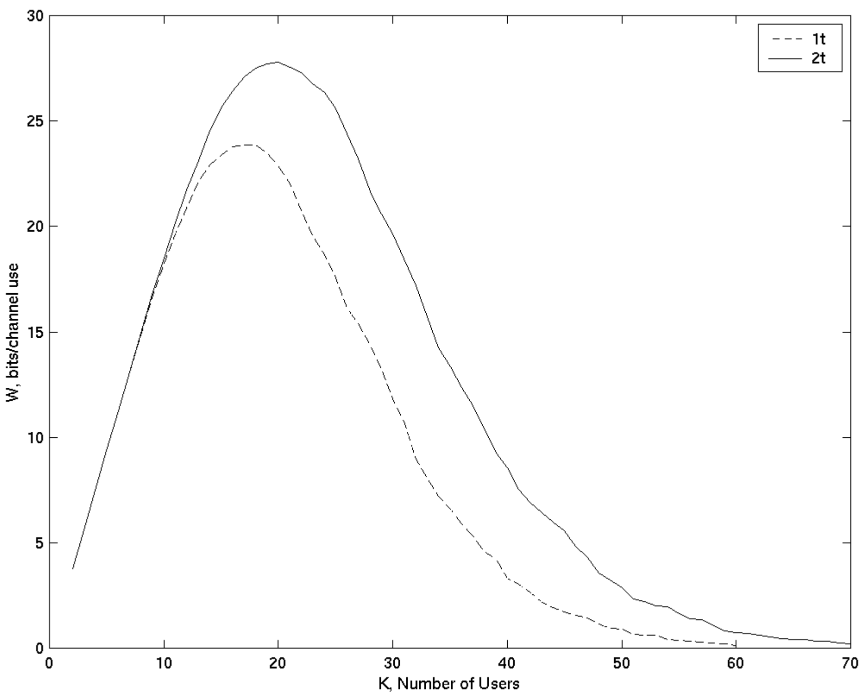

Figure 6: Normalised throughput for 8-FSK FH-SSMA with a (32,20) RS code.

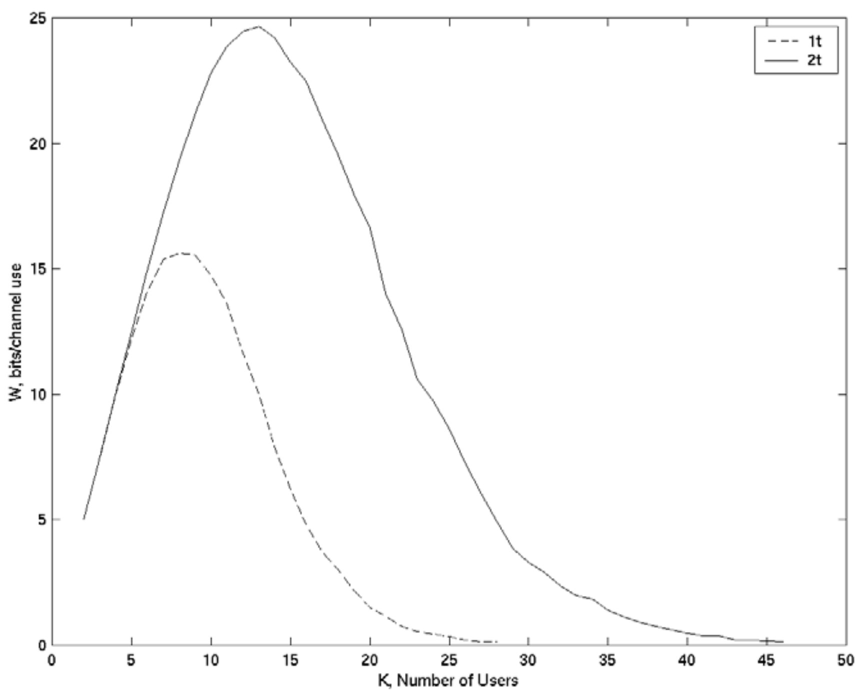

Figure 7: Normalised throughput for 16-FSK FH-SSMA with a (32,20) RS code.

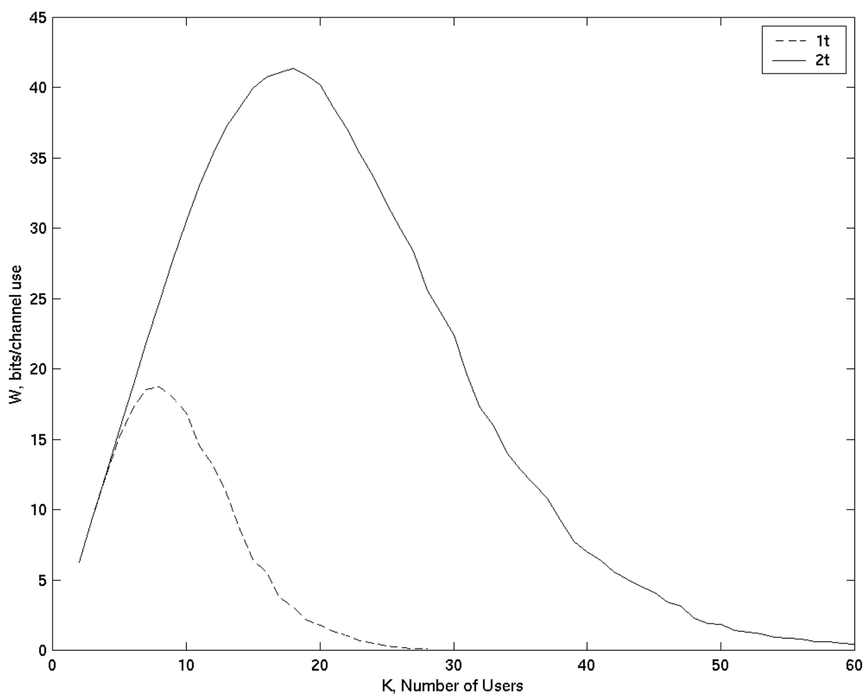

Figure 8: Normalised throughput for 32-FSK FH-SSMA with a (32,20) RS code.