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Alternating Direction Implicit Method for Free Convection Simulation in a Cylindrical Enclosure

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Abstract

We suggest in this paper to study, by means of numerical simulations, the evolution of the stratification and the fluids dynamics. The mathematical models take into account the thermal convective exchanges between layers. The convection equations are solved by the finite differences methods using the following schemes: the Alternating Direction Implicit method (ADI) and the over-relaxation method. The cylindrical enclosure is laterally heated at a uniform heat flux density. The influence of Rayleigh number on the temperature and the velocity of the fluid are particularly studied. The study showed that the variations in temperature and velocity depend particularly much on the geometry of the enclosure.

Keywords: ADI, Free convection, Numerical simulation, stratification,

Nomenclature

D diameter

g acceleration of gravity

H height

P pressure

Pr Prandtl number

Ra Rayleigh number

r, z dimensionless cylindrical coordinates

u , v dimensionless velocities in r, z directions

Greek Symbols

β volumetric coefficient of expansion with temperature

θ dimensionless temperature

ν kinematic viscosity

τ dimensionless time

ψ stream function

Ω dimensionless modified vorticity

1 Introduction

Safety in the industry of gas liquefaction (Natural Gas for example) is a permanent concern of the industrialists. In fact, the stratification phenomenon or "Roll-Over" counts among the specific risks that can be generated, for the operating staff and for environment, by the use of some liquefied gas such LNG.

In a storage tank of liquefied gas, layers or cells in stratification of different densities can appear. Therefore, the tank heat inlet, heat and matter exchanges between cells and finally evaporation on the liquid surface lead to the equalization of the densities of the layers and finally their mixture. This mixing can be accompanied by a brutal increase in the evaporation flow. This brutal and important evaporation can, in some cases, generate an increase in pressure in the tank and bring to the opening of safety valves. If these latter are under-dimensioned, the internal tank can be damaged.

Different methods have been used to study the natural convective problem in cavities. Some authors studied the behavior of fluids in stationary regime by using a finite volume method [1]. Others used finite differences discretization procedures to solve the problem in the case of Cartesian coordinates [3]. The study has been extended to the cylindrical enclosure by using the Alternating

Direction methods [2]. In this paper, we propose an algorithm using the Alternating-Direction-Implicit (ADI) method to simulate the evolution of stratification and the dynamics of the fluid implied in the Roll-over phenomenon.

2 Mathematical formulation

The study deals with a cylindrical tank, which is laterally insulated by a uniform heat flux density. The thermal entries at the bottom are neglected. The lateral heat flux causes the evaporation of a quantity of fluid (liquefied gas) at the upper free surface. When the movements of the fluid are due to differences in temperatures, where the flow is produced by the fact that the variations in temperature involve variations of density, and under the effect of the gravity forces, densest layers go down with regards of the less dense layers. The natural convection is studied in a tank in two dimensions. The movement of the fluid is generated by a lateral heating with a constant and free flow due to evaporation.

2.1 Analytical development

For two-dimensional heat transfer in an incompressible fluid with constants physical properties (except for the density in the momentum equations), the dimensionless conservation equations are:

$$\frac{\partial u}{\partial r}+\frac{\partial v}{\partial z}+\frac{u}{r}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+v\frac{\partial u}{\partial z}=\frac{Pr}{Ra}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\right)-\frac{\partial P'}{\partial r} \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+v\frac{\partial v}{\partial z}=\theta+\sqrt{\frac{Pr}{Ra}}\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{\partial P'}{\partial z} \tag{3}$$

$$\frac{\partial \theta}{\partial \tau}+u\frac{\partial \theta}{\partial r}+v\frac{\partial \theta}{\partial z}=\frac{1}{\sqrt{RaPr}}\left(\frac{\partial^2 \theta}{\partial r^2}+\frac{1}{r}\frac{\partial \theta}{\partial r}+\frac{\partial^2 \theta}{\partial z^2}\right) \tag{4}$$

With:

$Ra=\frac{g\beta\Delta T_0 H^3}{a\upsilon}$ Rayleigh number

$Pr=\frac{\nu}{a}$ Prandtl number

The dimensionless modified vorticity is :

$$\Omega=r\left(\frac{\partial u}{\partial z}-\frac{\partial v}{\partial r}\right) \tag{5}$$

and the dimensionless stream function such that:

$$u = \frac{1}{r}\frac{\partial \psi}{\partial z} \tag{6a}$$

$$v = -\frac{1}{r}\frac{\partial \psi}{\partial r} \tag{6b}$$

One thus has:

$$\Omega = -\frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial r^2} + \frac{\partial^2 \psi}{\partial z^2} \tag{7}$$

This equation can be regarded as a particular case (permanent) of the following general case (variable):

$$\frac{\partial \psi}{\partial \tau} = -\Omega - \frac{1}{r}\frac{\partial \psi}{\partial r} + \left(\frac{\partial^2 \psi}{\partial r^2} + \frac{\partial^2 \psi}{\partial z^2}\right) \tag{8}$$

If one derives the equation (2) with regard to z and the equation (3) with regard to r, while using continuity equation, the subtraction of the 2 equations obtained gives us:

$$\frac{\partial \Omega}{\partial \tau} = -r\frac{\partial \theta}{\partial r} + \frac{2}{r^2}\frac{\partial \psi}{\partial z}\Omega - \frac{1}{r}\left(\frac{\partial \psi}{\partial z} + \sqrt{\frac{Pr}{Ra}}\right)\frac{\partial \Omega}{\partial r} + \frac{1}{r}\frac{\partial \psi}{\partial r}\frac{\partial \Omega}{\partial z} + \sqrt{\frac{Pr}{Ra}}\left(\frac{\partial^2 \Omega}{\partial r^2} + \frac{\partial^2 \Omega}{\partial z^2}\right) \tag{9}$$

2.2 Initial and Boundary conditions

To complete a two-dimensional natural convection problem, the knowledge of initial and limit boundary conditions is needed. They are written in dimensionless forms using the stream function, the vorticity and the temperature:

Initial conditions ($\tau = 0$): $\psi = 0$, $\Omega = 0$, $\theta = 0$

Boundary condition:

i) The centerline axis ($r = 0$) :

$$\psi = \frac{\partial \psi}{\partial z} = 0 , \; \Omega = 0 , \; \frac{\partial \theta}{\partial r} = 0$$

ii) The lateral wall ($r = r_{max} = \frac{D}{2H}$) :

$$\psi = \frac{\partial \psi}{\partial z} = \frac{\partial \psi}{\partial r} = 0 , \; \Omega = \frac{\partial^2 \Psi}{\partial r^2} , \; \frac{\partial \theta}{\partial r} = 1$$

iii) The bottom (z = 0) :

$$\psi = \frac{\partial\psi}{\partial z} = \frac{\partial\psi}{\partial r} = 0\,,\ \Omega = \frac{\partial^2\Psi}{\partial z^2}\ ,\ \frac{\partial\theta}{\partial z} = 0$$

iv) The upper free surface (z = 1):

$$\psi = \frac{\partial\psi}{\partial z} = \frac{\partial\psi}{\partial r} = 0\,,\ \Omega = \frac{\partial^2\Psi}{\partial z^2}\ ,\ \frac{\partial\theta}{\partial z} = -4Fo\frac{H}{D}$$

F_0 is the heat flux fraction that is lost by evaporation at the upper free surface.

3 Numerical resolutions

We are interested in the discretization of partial derivative equations on uniform grid, in which the unknown variable is a dependent function on several variables of space and time, and to know the evolution of the unknown factor (temperature or velocity) in a finite and homogeneous field starting from known initial values.

The energy and vorticity equations are solved by the ADI method -Alternating Direction Implicit-. The ADI method is a process to reduce the two- or three-dimensional problems to a succession of two or three one-dimensional problems [5]. The tridiagonal linear systems obtained are solved by the Thomas algorithm [4].

The iterative method of gauss is used to solve the equation of the stream function. In this case, the calculation stops when the maximum difference between iterations becomes lower than 10^{-5}.

The simulations allow describing the variable regime before reaching the steady state.

The convergence criterion is $\dfrac{\sum_i\sum_j\left|F_{i,j}^{n+1} - F_{i,j}^{n}\right|}{\sum_i\sum_j\left|F_{i,j}^{n+1}\right|}$, where F is the temperature or the vorticity. Variations by less than 10^{-7} are adopted for convergence at each time step.

4 Results and discussions

The study of the natural convection problem in a cylindrical tank is related to three parameters: the aspect ratio $\frac{H}{D}$ the Prandtl number Pr and the Rayleigh number Ra.

The simulations are done by a 21x21 regular mesh. The parameters used are: Pr = 2.2 (Prandtl number of LNG), $10^3 \le Ra \le 5x10^4$ and $0.125 \le \frac{H}{D} \le 0.5$

Figures 1 and 2 represent respectively the temperature and the vorticity changes with regard to the time at various positions in the tank.

One can notices that the fluid layer close to centreline axis undergoes a very weak variation of its temperature, then those variations become important as we move away from the lower wall tank to the lateral wall and the upper free surface, before reaching the steady state. A variable dynamic behaviour changes with time, towards a stationary asymptotic limit.

Figures 3 and 4 represents, for Ra = 10^4 and $\frac{H}{D} = 0.125$, the flow field and the isotherms in variable regime. One sees that initially the flow is one-cellular: a cell develops near the lateral wall of the tank. The cell moves more and more towards the center of the cylinder before a second cell grows close to the lateral wall. This displacement of the cell is accompanied by an increase of the temperature and the vertical velocity which reaches a value of 0.021 at time 23.5 .

Each layer absorbs a quantity of heat more or less important and transmits it to the higher layer until the thermal balance.

The heating of the liquid causes the formation of layers of different temperatures; these layers can undergo a rotational movement followed by an inversion (Roll-Over), and hence an evaporation.

To study the influence of the Rayleigh number, we plotted the isotherms with various Ra values. Figure 5 shows clearly that the Rayleigh value has a strong impact on thermal and dynamic behaviour of the fluid. In fact, for small values of Ra, the isotherms are slightly deformed, and this deformation becomes more and more important as Ra increases. The maximum value of T_{max} corresponds to the lowest value of Ra. Figure 6 shows the effect of the cylindrical tank aspect ratio H/D for Ra = 10^3 . In this case, we notice that the perturbation reaches more quickly the centre of the tank when it is less broad. It can be observed that the minimum temperature remains practically constant.

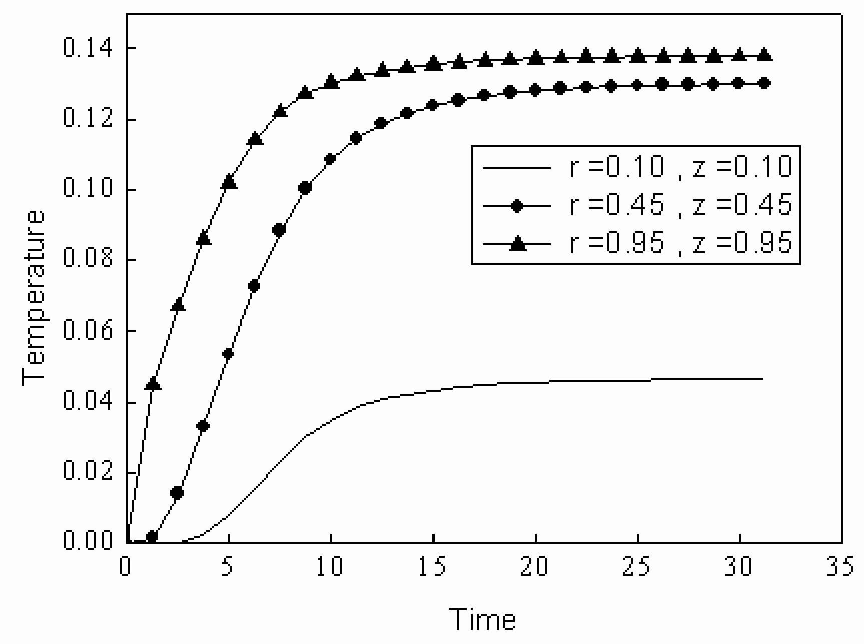

Figure 1. Temperature evolution versus time for H/D= 0.5 and Ra = 1E03

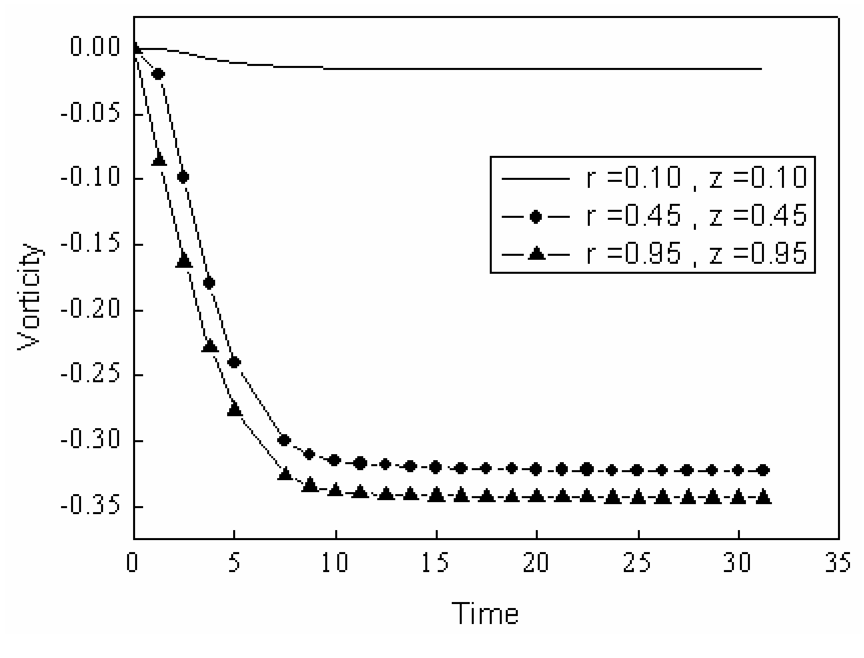

Figure 2. Vorticity evolution versus time for H/D= 0.5 and Ra = 1E03

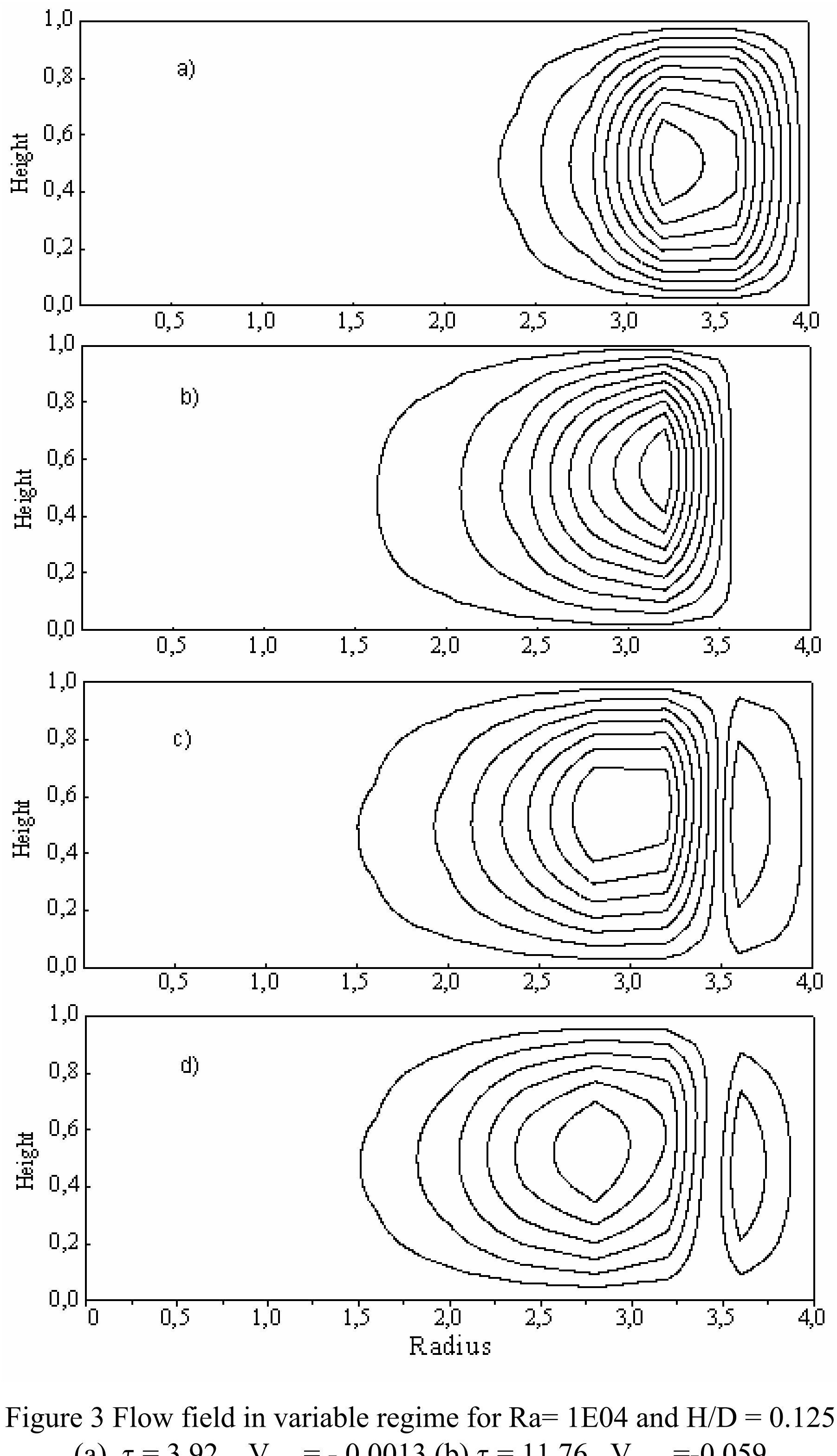

Figure 3 Flow field in variable regime for Ra= 1E04 and H/D = 0.125
(a) $\tau = 3.92$, V_{max}= - 0.0013 (b) $\tau = 11.76$,V_{max} =-0.059
(c) $\tau = 15.68$, V_{max}= 0.0099 (d) $\tau = 19.6$, V_{max} = 0.016

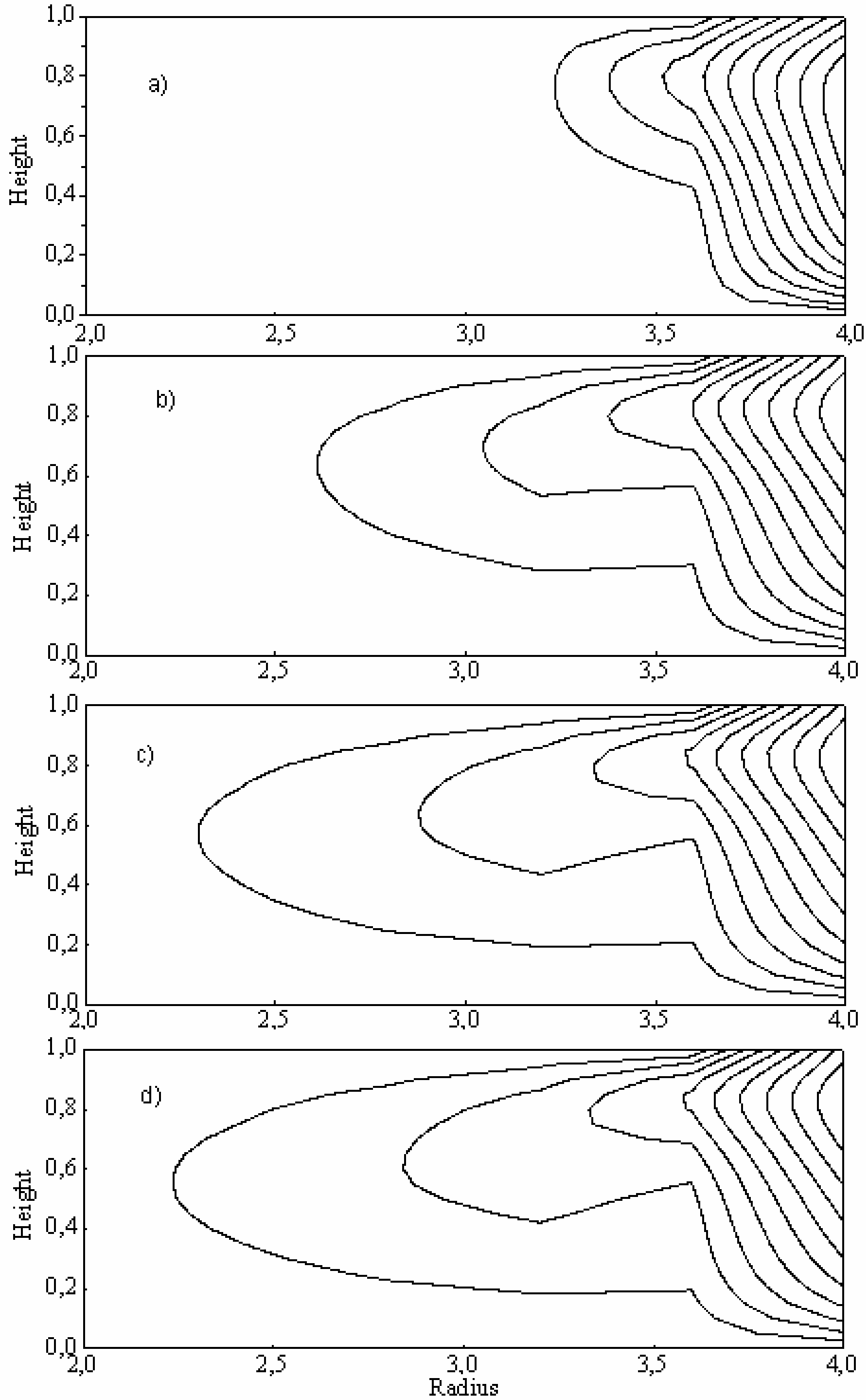

Figure 4 Isotherms in variable flow for Ra = 1E04 and H/D = 0.125 ;
(a) $\tau = 12$, $T_{max} = 0.23$; (b) $\tau = 29$, $T_{max} = 0.31$
(c) $\tau = 39$, $T_{max} = 0.31$; (d) $\tau = 88$, $T_{max} = 0.32$

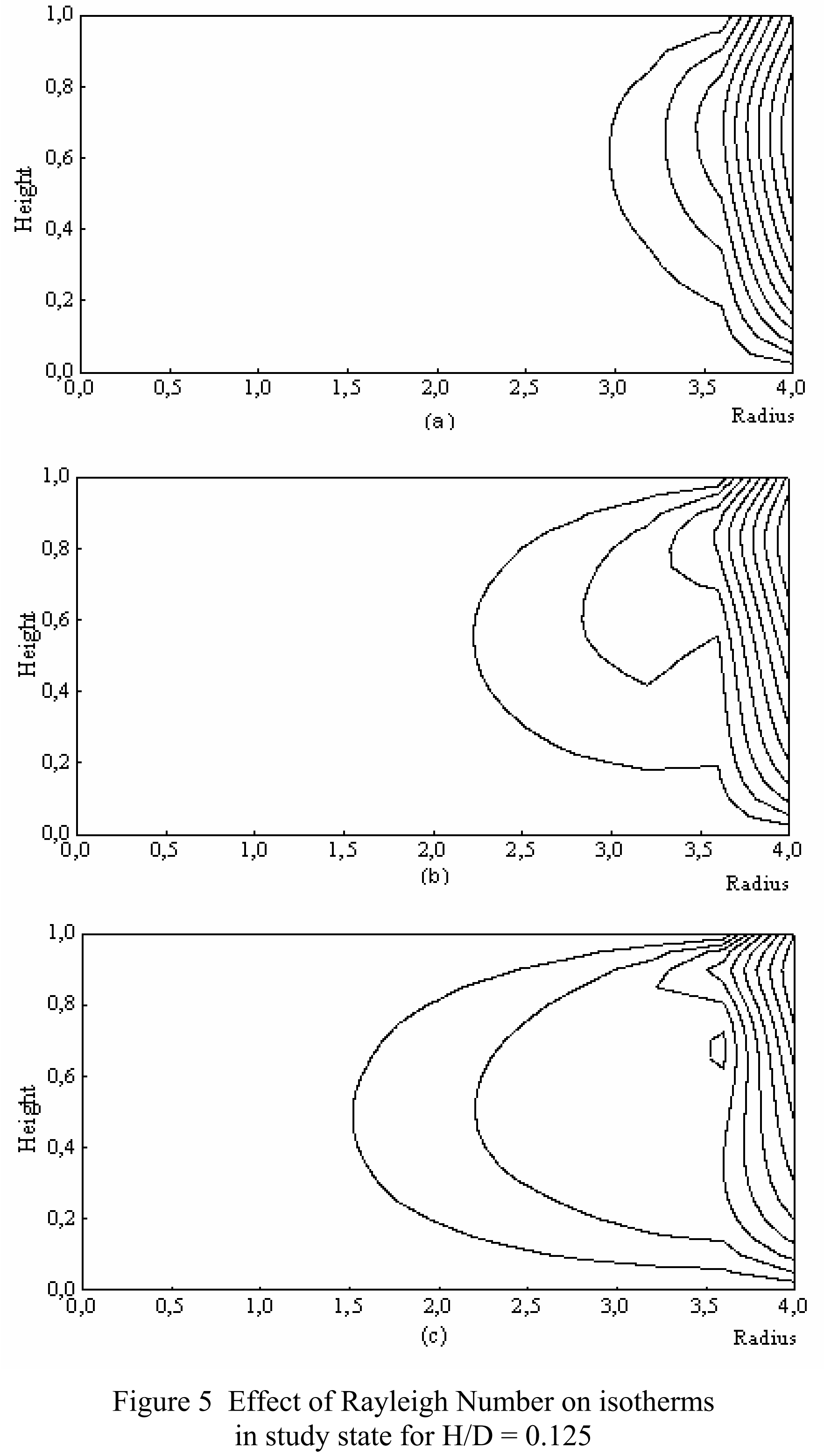

Figure 5 Effect of Rayleigh Number on isotherms
in study state for H/D = 0.125
(a) Ra=1E03 ; (b) Ra=1E04 ,(c) Ra = 5E04

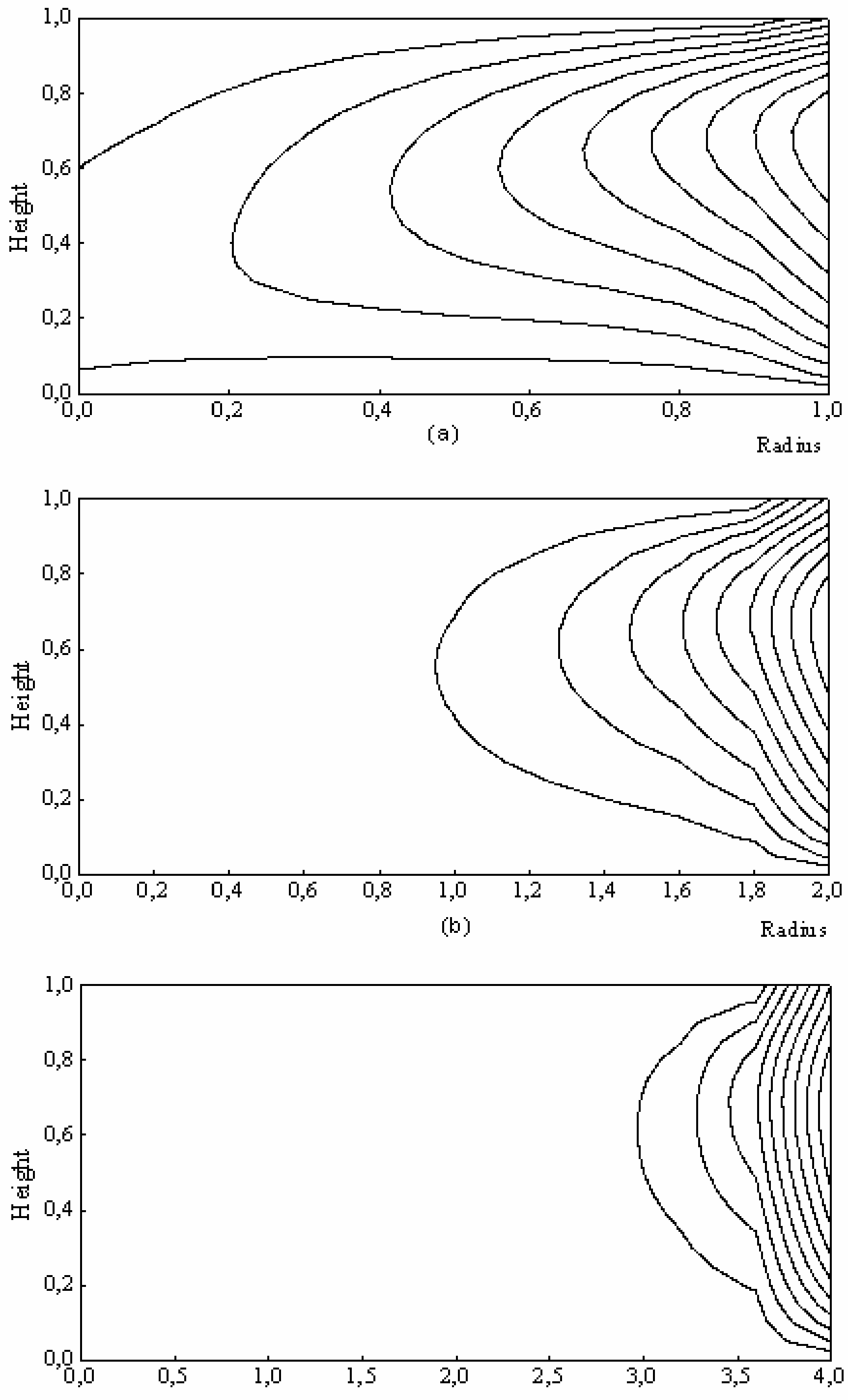

Fig. 6 Effect of tank geometry on isotherms in study state for Ra = 1E03
(a) H/D = 0.5 ; (b) H/D= 0.25 ; (c) H/D= 0.125

5 Conclusion

In this work we have presented a numerical simulation based on the Alternating Direction Implicit - ADI - method, in order to study the dynamic behaviour of liquefied natural gas - LNG – in a cylindrical storage tank.

The obtained results show that the temperature and the velocity of the LNG can be predicted in a two-dimensional system, when the LNG storage tank is laterally heated at a uniform flux density. A growth of a cell is observed near the lateral wall of the tank. Then this cell moves more and more towards the center of the cylinder before a second cell develops close to the lateral wall.

From the parametric study, we demonstrate that the maximum of LNG temperature decreases when the Rayleigh number increases, and the minimum temperature remains practically constant, while the maximum stream function increases as the aspect ratio decreases.

The practical consequences of this study appear interesting, since it makes it possible to envisage the thermal behavior of the LNG, and thereafter the phenomenon of Roll-Over which can be expected to be among the specific risks which can be generated, for the operating staff and for the environment.

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