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n-Fold Product and Deformation Mapping on $\mathbb{R}^n$ and Matlab Applications

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Abstract

This paper defines the $n-$fold product, which provides actions on R^n, which called deformation product. The set of deformation mapping of R^n rather than the deformation projection is a subset of all deformation mappings on R^n and has abelian group structures. This is a new approach to deformation products. If we replace the fixed point with a continuous curve, then we have a continuous deformation of $\mathbb{R}^n$.

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1 Introduction

Deformation of surfaces induced by adding one and two extra space variables to the motions of space curves in higher-dimensional similarity geometries is studied in [1]. The deformation can be applied either globally or locally. Local deformations can be imposed with any desired degree or derivative continuity [6]. Every compact C^∞-Riemannian manifold with at least 3 dimensions can be deformed comformally to a C^∞-Riemannian structure of constant scalar curvature [4].

Let $S = (M, r)$ be a submanifold of a Riemann manifold $\widetilde{M}$ and let $I = [-\delta, \delta]$ for some $\delta > 0$. A mapping $\gamma : I \times M \to \widetilde{M}$ is said to be a deformation of S if $\gamma_0 = r$ and if γ_t is an immersion for each $t \in I$.

A deformation of S is called *infinitesimal isometric deformation*, if g'(0)=0, where we denoted by g the Riemannian metric induced by γ_t [3].
Let M_1, M_2 be two immersed submanifolds of $\overline{M_1}, \overline{M_2}$ respectively; consider the immersions $r_i : M_i \longrightarrow \overline{M_i}, i = 1, 2.$

Let $\gamma^i : I \times (M_1 \times M_2) \longrightarrow \overline{M_1} \times \overline{M_2}$ where $\gamma(t, x_1, x_2) = (\gamma_t^1(x_1), \gamma_t^2(x_2))$, such that $\gamma_0 = r$, where r is an immersion of $M_1 \times M_2$ into $\overline{M_1} \times \overline{M_2}$ [5].

Deformation applied numerous fields, especially in algebraic geometry, plastic surgery, geology, animation technology, similarity geometry, etc [1],[2],[7]. While $M = \widetilde{M} = \mathbb{R}^n$ and the metric is Euclidean, a new approach to deformation theory can be provided.

The product $\mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$, $(c, X) = cX = (cx_1, cx_2, cx_n)$ is called *scalar multiplication*. If $M \subset \mathbb{R}^n$ is a subset, then cM is different from M but all changings along the axes have same ratio.

N-fold product, which will be defined in Section 2, gives different changes along the axes.

It can be controlled in its present form . Furthermore, it satisfies the definition of deformation in [3], [5] and its programme algorithm is functional for Matlab.

2 Deformation Product

First of all, we give n-fold product on $\mathbb{R}^n$ as follows.

Definition 2.1 $V^n : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$, $V^n(X, Y) = \sum_{i=1}^{n} x_i y_i e_i$ *is called* $n-$fold product*,*
where $X = \sum_{i=1}^{n} x_i e_i$, $Y = \sum_{i=1}^{n} y_i e_i$ *and* $\{e_1, ..., e_n\}$ *is standard base of* $\mathbb{R}^n$.

In $V^n(X, Y)$, if we choose $X = A$ then we can write $V_A^n(Y)$ for $V^n(X, Y)$
SPECIAL CASES and PROPERTIES OF V^n

1) $V_{e_i}^n$ is a $i-th$ projection mapping on $\mathbb{R}^n$,
2) $V_A^n = \sum a_i V_{e_i}^n$,
3) If $A = \sum c a_i$, c :constant, then V_A^n is scalar product with c,
4) For any $A = \sum a_i e_i$, $k-$items of $a_1, ..., a_n$ are 1 and the others are zero, then V_A^n is a projection $k-$dimensional subspace of $\mathbb{R}^n$,
5) $\forall A, B \in \mathbb{R}^n$, the product of V_A^n and V_B^n is shown V_{AB}^n and defined as

$$V_{AB}^n = V_A^n * V_B^n = V_{V_B^n}^n$$

Now we can define deformation in $\mathbb{R}^n$ by using an n-fold product.

Definition 2.2 *For $A \in \mathbb{R}^n$, $\exists\ a_i \neq 0$, the mapping $V_A^n : \mathbb{R}^n \to \mathbb{R}^n$, $V_A^n(X) = V^n(A, X)$ is called* deformation *of $\mathbb{R}^n$ with A.*

Definition 2.3 *$k-$items of $a_1, ..., a_n$ are different than zero and $\exists\ a_i \neq 1$ and the others are zero, then V_A^n is called* deformation projection *on $k-$dimensional subspace.*

It can be seen deformative projection of S^2 to xy-plane in Figure 1.

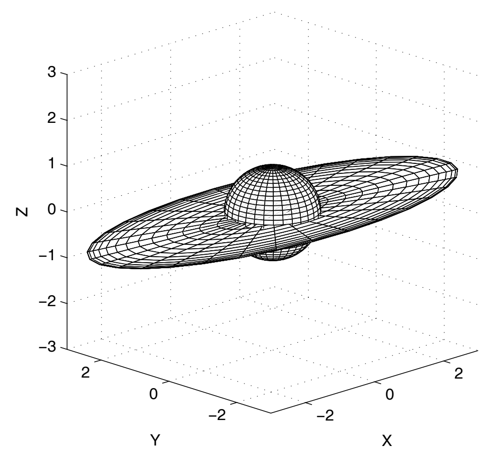

Figure 1: Deformative projection of unit sphere to xy-plane

Theorem 2.4 *Let the set of deformation mapping of $\mathbb{R}^n$ rather than the deformation projection be $D\mathbb{R}(^n)$. Then $(D(\mathbb{R}^n), *)$ is an abelian group.*

1) If $V_A^n, V_B^n \in D(\mathbb{R}^n)$, then $A = (a_1, ..., a_n), B = b_1, ..., b_n)$ $\forall i,\ a_i \neq 0, b_i \neq$ 0

$$V_A^n * V_B^n = V_{V_B^n}^n = \sum a_i b_i e_i$$

and

$$a_i b_i \neq 0.$$

So $V_A^n * V_B^n \in D(\mathbb{R}^n)$.
2) For $I = (1, ..., 1)$, V_I^n is a unit element.
3) $\forall V_A^n \in D(\mathbb{R}^n)$, $V_{\frac{n}{A}} = \sum \frac{1}{a_i} e_i$ is an inverse element.
4) $V_A^n * (V_B^n * V_C^n) = (V_A^n * V_B^n) * V_C^n$
5) $V_A^n * V_B^n = \sum a_i b_i e_i = \sum b_i a_i e_i = V_B^n * V_A^n$

Example 2.5 *Let triangle PQR be $P = (1, 0, 0)$, $Q = (0, 2, 0)$, $R = (0, 0, 2)$ and $A = (2, 3, 1/2)$. Then $V_A(PQR) = P'Q'R'$, where $P' = (2, 0, 0)$, $Q' = (0, 6, 0)$ and $R' = (0, 0, 1)$.*

Example 2.6 *It can be seen some fixed deformation of unit sphere in Figure 2; where, Case 1: Sphere* $A = (1, 1, 1), T = (0, 0, 0)$*; Case 2:* $A = (4, 3, 1), T = (5, 5, 5)$*; Case 3:* $A = (2, 3, 0, 5), T = (-5, 2, 0)$*; Case 4:* $A = A(2, 4, 1), T = (2, 2, 3)$*; Case 5:* $A = (-2, 1, 5), T = (2, 2, -3)$.

3 Continuous Deformation

Definition 3.1 *Suppose that* $\alpha(t) \subset \mathbb{R}^n$ *is a differentiable curve. The mapping* $V^n_{\alpha(t)} : \mathbb{R}^n \to \mathbb{R}^n$ *is called* continuous deformation *on* $\mathbb{R}^n$.

Definition 3.2 *The function* $\|\alpha'(t)\|$ *is called* velocity function *of* $V^n_{\alpha(t)}$ *continuous deformation.*

If $\|\alpha'(t)\|$ is constant then we state that deformation is *stable*.

Definition 3.3 *Let* $\alpha(t)$ *and* $\beta(u)$ *are two continuous curve in* $\mathbb{R}^n$*. The mapping*

$$T_{\beta(u)} \circ V^n_{\alpha(t)} : \mathbb{R}^n \to \mathbb{R}^n$$

is called continuous deformation *on* $\mathbb{R}^n$*, where* $T_{\beta(u)}$ *is a translation with translation vector* $\beta(u)$.

Corollary 3.4 *Continuous deformation is a motion with two parameters.*

Let $M \subset \mathbb{R}^n$ *is a subset.* $(T_{\beta(u)} \circ V^n_{\alpha(t)})(M)$ *give the set of deformation of* M *along the curve* $\beta(t)$ *being deformed.*

At the end we provide the algorithm and applications of the deformation product, by using Matlab programme.

Example 3.5 *For* $(T_{\beta(u)} \circ V^n_{\alpha(t)})(S^2)$ *we take*
$\alpha(t) = (sin^2(t), cost, sin^2(t))$
$\beta(u) = (2 + cosu, 3 + sinu, u^2)$
$-\frac{\pi}{6} \le t \le 2\pi$, $-\frac{\pi}{6} \le u \le 2\pi$, *period is* $\frac{\pi}{6}$.

Matlab Programme for Example 3.2 is as follows.

```
close all, clear all, clc
        %deformative action of unit sphere along \alpha(t)
        %\alpha(t)=(cost, sint, 0,3.sint)
        %\beta(u)=(cosu, sint, sinu), t=u
   a=2
   axis([-a a -a a -a+1 a])
   hold on
```

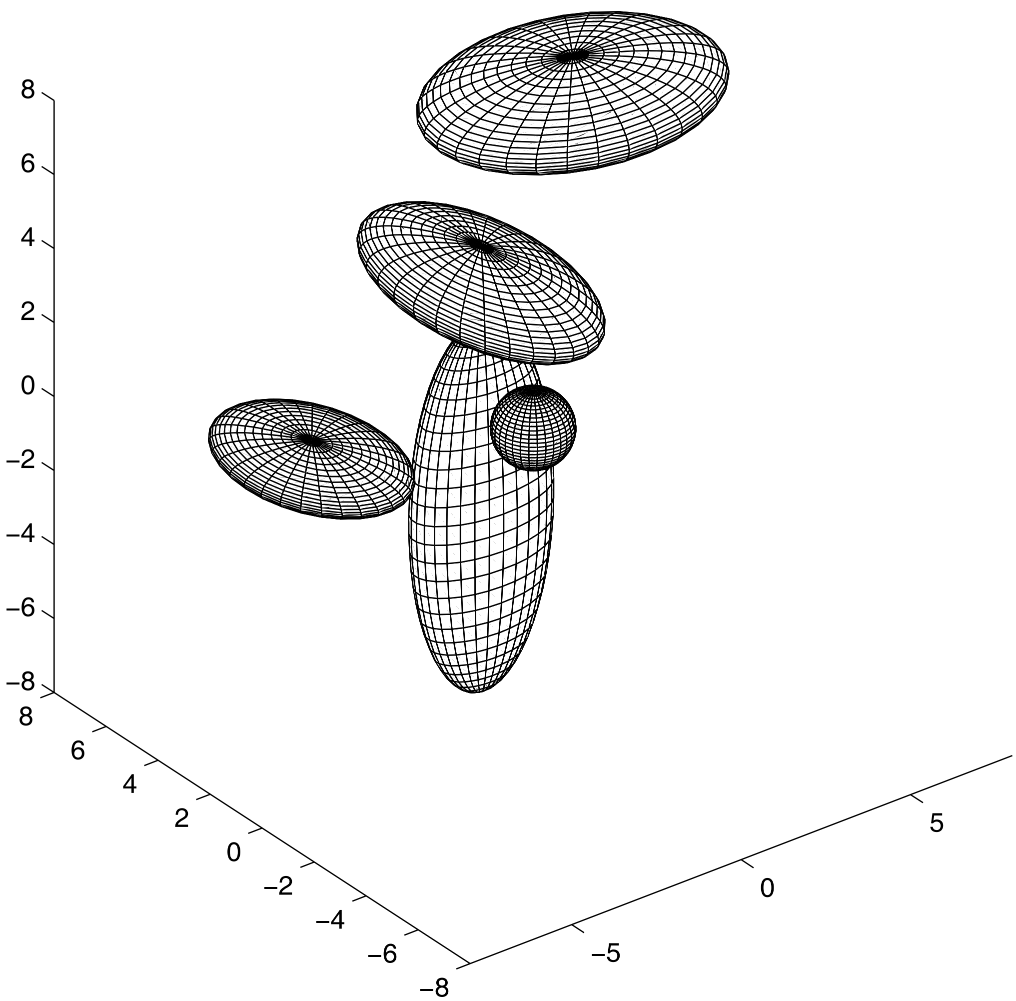

Figure 2: Some special deformations of unit sphere

```
   axis square
   for t=20:20:160

                     c=cosd(t)
                     d=sind(t)
                     e=sind(t)*sind(t)
k = 5;
n = 2^k-1;
theta = pi*(-n:2:n)/n;
phi = (pi/2)*(-n:2:n)'/n;
X =c+c*cos(phi)*cos(theta);
Y =d+d*cos(phi)*sin(theta);
Z =e-0.3*e*sin(phi)*ones(size(theta));
    colormap([1 1 1;1 1 1])
    C = hadamard(2^k);
                               surf(X,Y,Z,C)

camlight;
lighting gouraud;
pause(1)
end
```

S^2 $(T_{\beta(u)} \circ V^n_{\alpha(t)})(S^2)$ is illustrated in Figure 3.

4 Conclusion

In this article, $V^n : \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$ n-fold product is defined. This product gives us a scalar product and a deformation product if we choose X special case for $V^n(X, Y)$. Furthermore we define deformation product and deformation mapping from $V^n(X, Y)$. This form of deformation is effective for Matlab application in $\mathbb{R}^3$.

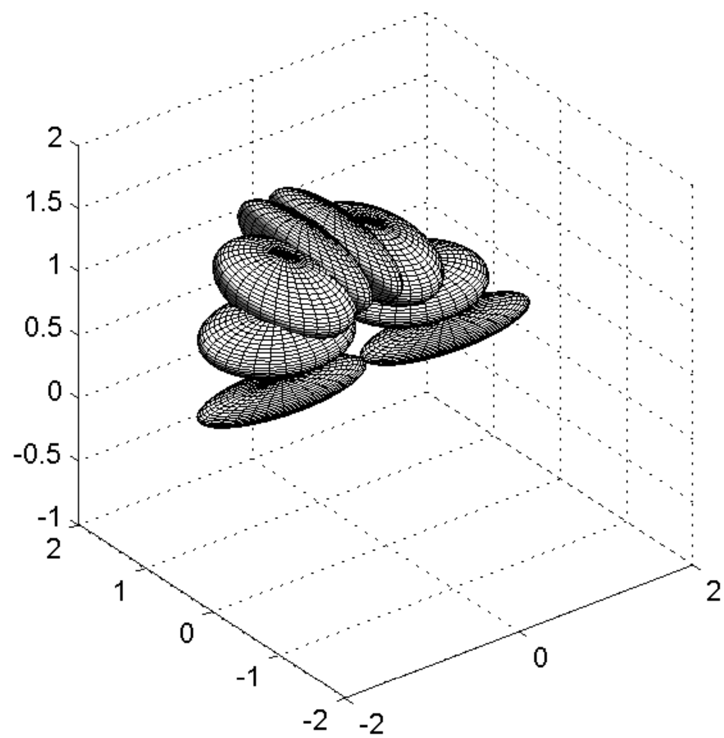

Figure 3: Deformation of S^2 along a curve

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