

# Neuro-Sliding Mode Control of Piezo-Positioning Mechanism

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## Abstract

In this work, a Neuro-Sliding mode controller (NSMC) for piezo-positioning mechanism (PEA) was designed. This controller takes into account parameter uncertainties and unknown external disturbances. The LuGre model is used to establish the mathematical model of the system including the mechanical motion dynamics, the hysteresis friction, the load disturbance and parameter uncertainties. Using the developed model, a Neuro-Sliding mode controller based on one layered neural network with linear activation functions is proposed. The main controller goal is to minimize an error function determined from Lyapunov stability criteria and sliding-mode control theory. The validity of the proposed controller is presented using simulations results.

**Keywords:** Neuro-Sliding mode, Piezo-positioning mechanism, LuGre model, Lyapunov stability criteria

## 1. Introduction

The piezo-electric actuator (PEA) is a well-known device used for monitoring extremely small displacements. The PEA has many advantages including a nano-scale resolution, a large force developed, a high stiffness and a fast response. However, it also has some bad characteristics such as nonlinearities in its model (caused by the Hysteresis) and high sensitivity of the temperature. Since the Hysteresis characteristics of PEA are usually unknown, it is difficult to establish an exact dynamic model for systems including all effects. This increases the difficulties in the design of the servo control with high performance requirements. Modeling techniques for PEA have been presented in many references. In [7], the model is elaborated by including a first-order hysteresis effect and modeling the PEA as a distributed parameter system. In ([2] and [14]), the developed friction models including the characteristics of the Stribeck effect, the hysteresis, and the spring-like behaviors were assembled into the mechanical motion dynamics.

In another hand, the sliding-mode control is a well-known control nowadays which is suitable to control some nonlinear systems because it can deal with parameter uncertainties and external disturbance [3][4][11]. However, since it is necessary to know in prior the system dynamics, the calculation of the control input and disturbance is necessary. In most real-life engineering problems, this step cannot be found easily or can be obtained partially. The merging established control structures such sliding mode control based on intelligent algorithms appeared to be a good solution. In recent years, the concept of neuro-sliding mode control has been growing into large research topics [5][10][12], because of a major advantages of this control approach is that no priori knowledge of the system parameters, or of the initial estimates of the neural networks, is required in the physical realisation. The neuro-sliding mode controller adopted here consists of a one layered neural network with linear activation functions.

In this paper, a neuro-sliding mode controller for a piezoelectric actuator (PEA) is presented. The controller to be designed is determined by combining the requirements of the Lyapunov stability criteria and the sliding mode control theory. The performance of designed controller is tested using numerical simulation.

## 2- Neuro-sliding mode control

Consider the nonlinear system:

$$\dot{x} = f(x) + B(x)u + d \quad (1)$$

Where  $x \in \mathfrak{R}^n$  is the state vector,  $y \in \mathfrak{R}$  is the system output and  $u \in \mathfrak{R}$  is the control input.  $f(x) \in \mathfrak{R}^n$  is an unknown, continuous and bounded nonlinear

function,  $B(x) \in \mathfrak{R}^n$  is a known input vector where its elements are continuous and bounded and  $d \in \mathfrak{R}^n$  is an unknown, bounded external disturbance. The nonlinear function  $f(x)$  is unknown but the system order is known.

The main goal is to design a neuro-controller, for the uncertain system with partially known dynamics, which forces the system output  $y$  to follow a desired trajectory  $y_d$ . The trajectory error is given by  $e_i = [e, \dots, e^{(n-1)}]^T \in \mathfrak{R}^n$ , where  $e = y_d - y$ .

## 2.1 Controller design

The natural selection of the sliding manifold, for the system (1), can be in the following form:

$$S = Ge_i = 0 \quad (2)$$

Where  $S = \dot{e} + \lambda e$  and  $\lambda$  is a real and positive defined constant.

The Lyapunov Function candidate can be selected as :

$$V = \frac{1}{2} S^2 \quad (3)$$

Which is positive defined function. The time derivative of this Lyapunov Function candidate ( $\dot{V}$ ) should be negative defined. In order to use this condition in selection of the control, we selected the following form.

$$\dot{V} = -DS^2 \quad (4)$$

Where  $D$  is a positive defined constant. Equations (3) and (4) with necessary arrangements, give:

$$S(\dot{S} + DS) = 0 \quad (5)$$

Therefore, for  $S \neq 0$ , the control law can be calculated by satisfying the following equation:

$$\dot{S} + DS = 0 \quad (6)$$

Finally, by using (1) and (6), the control law becomes:

$$U = -GB(x)^{-1}(Gx_{ref} + Gf(x) + Gd + DS) \quad (7)$$

This controller, which uses the variable derivatives, the system becomes sensitive to parameters and disturbances variations during the convergence time. It may

causes also some oscillations in the output in presence of discontinuities. Several solutions have been proposed in the literature to solve these weaknesses. One of the solutions is the use of a neural network structure that minimizes the function (6) and eventually makes it equal zero. The neural network consists of only one layer of weights and uses linear a activation function in the output layer. The proposed structure of the neuro-controller is shown in figure 1.

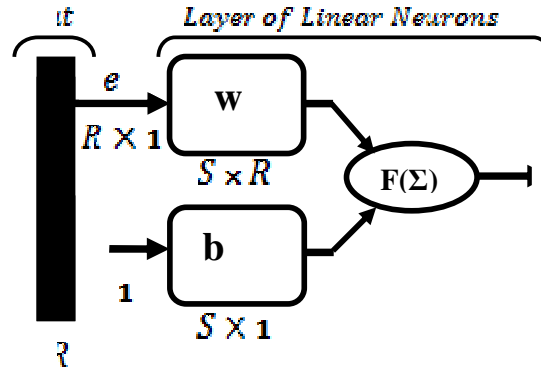


Figure 1. Structure of the neuro-controller

Where  $R$  and  $S$  are respectively the numbers of elements in input vector and the numbers of neurons in the layer. The input layer has a linear activation, an output node and a linear activation function. The detailed mathematical description of the neural network [1] is given by:

$$U = \sum_{i=0}^n w_i e^{(i-1)} + b \quad (8)$$

Where  $w_i$  are adaptable weights that are updated during the operation. As stated above, and in order to push the function to zero, the error function has to be:

$$E = \frac{1}{2} (\dot{S} + DS)^2 \quad (9)$$

The weights learning algorithm is derived based on the back propagation approach [13]. The tuning law is to give weight's increments to be proportional to the negative gradient of the performance criterion with respect to the weights. For updating of the weights between the hidden layer and the output layer, we define an increment as:

$$\dot{w}_i = -\eta \frac{\partial E}{\partial w_i} \quad (10)$$

Where  $\eta$  is the learning constant generally chosen between 0 and 1. Using the chain rule and noting that the weights are independent, the partial derivative of (10) can be written as:

$$\frac{\partial E}{\partial w_i} = \frac{\partial E}{\partial U} \frac{\partial U}{\partial w_i} \quad (11)$$

Substituting (9) into (11) and taking the derivatives, the following equation is obtained:

$$\frac{\partial E}{\partial w_i} = (\dot{S} + DS) \frac{\partial(S)}{\partial U} e^{(i-1)} \quad (12)$$

Substituting (2) into (12) gives:

$$\frac{\partial E}{\partial w_i} = (\dot{S} + DS) \frac{\partial(Gx_{ref} - Gx)}{\partial U} e^{(i-1)} \quad (13)$$

or

$$\frac{\partial E}{\partial w_i} = -(\dot{S} + DS)GB(x)e^{(i-1)} \quad (14)$$

Finally, the weight update algorithm can be stated as:

$$\dot{w} = -(\dot{S} + DS)\eta GB(x)e^{(i-1)} \quad (15)$$

### 3-Modeling of the piezo-actuated based positioning mechanism

The friction force model with hysteresis effect called LuGre model presented in [2][9], is summarized as follows:

$$\frac{dz}{dt} = v - \frac{\sigma_0}{g(v)} z|v| \quad (16)$$

$$F_H = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v \quad (17)$$

Where  $z$  is the vector of the state variables which represents the average deflexion of the contact force and  $v$  denotes the relative velocity between the two contact surfaces. The positive constants  $\sigma_0, \sigma_1, \sigma_2$  are typically unknown and difficult to identify. They represent the bristle stiffness, bristle damping and viscous-damping coefficient, respectively. The friction between two rigid

bodies described by (16) and (17) is illustrated in fig (2).

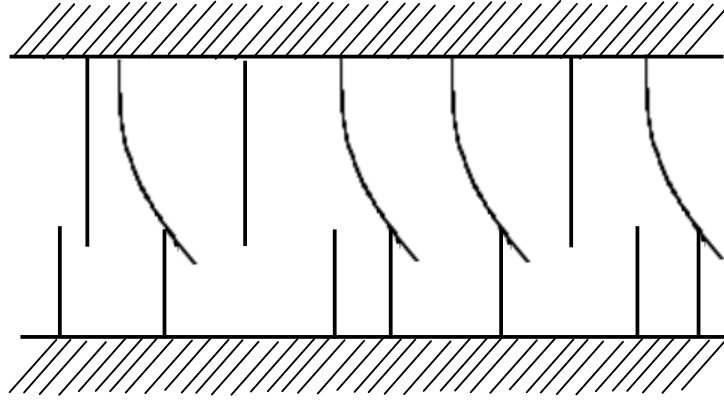


Fig 2: Representation of friction between two rigid blocks in contact

The function  $g(v)$  which is the Stribeck effect curve is given by:

$$\sigma_0 g(v) = F_c + (F_s - F_c)e^{-(v/v_s)} \quad (18)$$

Here,  $F_c$  is the coulomb friction which is independent of the velocity,  $F_s$  is the level of the stiction force and  $v_s$  is the stribeck velocity.

Combining equations (16) and (17), the hysteresis friction model (17) can be rewritten as:

$$F_H = \sigma_0 z - \frac{\sigma_0}{g(v)} \sigma_1 z |v| + (\sigma_1 + \sigma_2)v \quad (19)$$

which is functionally related to the system velocity, is used to represent the behavior of the hysteresis introduced by the piezo-actuator. The overall dynamics of the piezo-actuated positioning mechanism with one degree of freedom can be expressed as [6][8]:

$$M\ddot{x} + D\dot{x} + F_H + F_L = k_e U \quad (20)$$

Where  $M$  is the equivalent mass of the controlled mechanism,  $x$  is the displacement,  $F_L$  is The external load,  $k_e$  is the voltage to force coefficient,  $F_H$  is the hysteresis friction,  $U$  is the applied voltage to the piezo-actuator and  $D$  is the linear friction coefficient.  $E$  is named the uncertainty lumped, defined by  $E = \frac{F_H + F_L}{M}$ . Defining the state variables as  $(x = x_1, \dot{x}_1 = x_2)$ , the system dynamics shown in (20) can be rewritten as:

$$\ddot{x} = \frac{k_e}{M}U - \frac{1}{M}[D\dot{x} + F_H + F_L] \tag{21}$$

From (21) it can be seen that the dynamics of piezo-positioning mechanism are apparently nonlinear with respect to  $x_1$  and  $x_2$ . In order to validate the feasibility of the investigated hysteresis friction model shown in (21), a sinusoidal input voltage (5V, 0.5 Hz) is applied to the hysteresis friction model. From the figure 3, it is evident that the system dynamics including hysteresis can be successfully represented by (21).

The model reference controller will be designed based on the estimated values. The damping ratio of reference model is chosen as 0.7 and the natural frequency of the reference model is chosen as 30.66 rad /s. Based on these two values, the reference model is chosen as:

$$G(s) = \frac{940}{s^2 + 42.92s + 940} \tag{22}$$

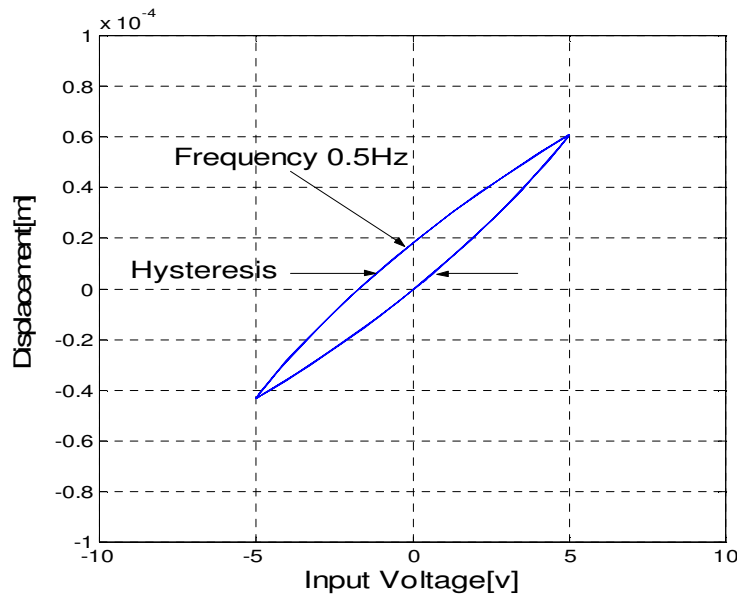


Figure 3: The hysteresis friction model

The block diagram of the corresponding neuro-sliding mode controller is given by figure 4.

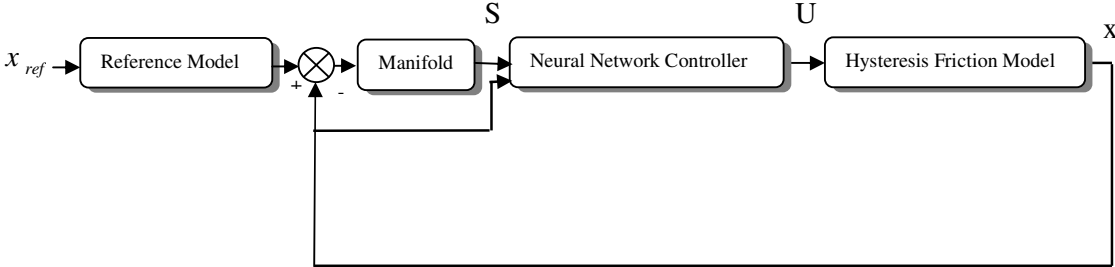


Figure 4: Block diagram of the controlled system

4-Simulation results

The simulation results of the NSMC system due to periodic sinusoidal command with amplitude 10 micron metre and frequency 0.5 Hz are shown in Figure 5.

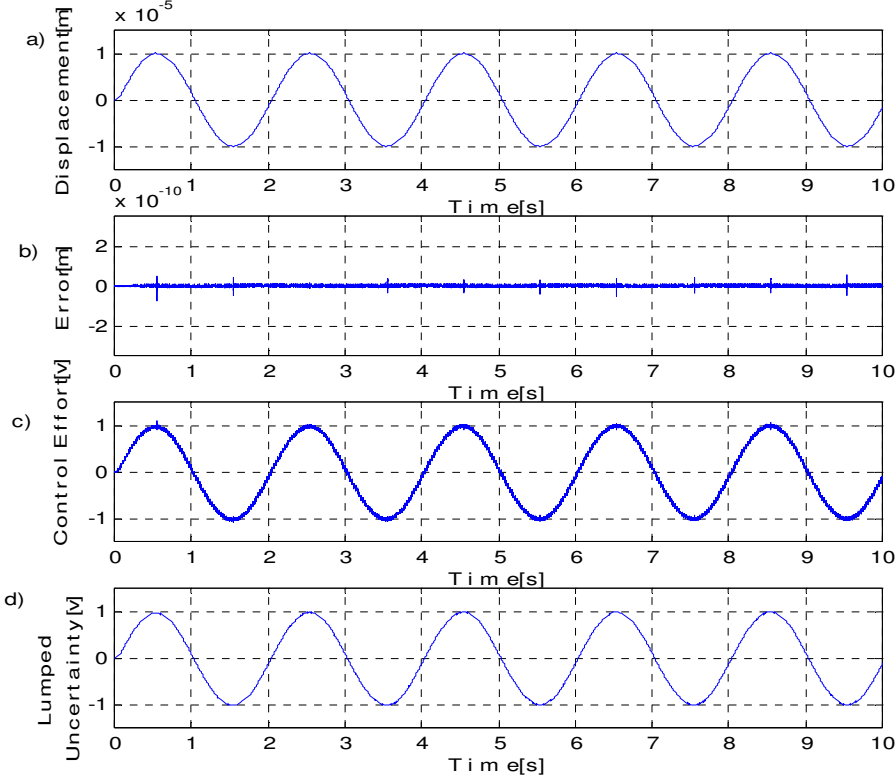


Figure 5: Simulation results of NSM control system for periodic sinusoidal command with conditions at 10µm, 0.5Hz: a) Tracking response; b) Tracking error; c) Control effort; d) lumped uncertainty.



The tracking response, tracking error, effort control and lumped uncertainty are shown in figures.6 (a),(b), (c), (d). From the simulation results, the tracking is excellent. As it is seen, the error bound is around  $0.32 \times 10^{-11}$  m, which indicates the high performance of the controller. Fig.9 shows the time evolution of the  $w_3$  and as it is seen there is no built up problem. In addition, to investigate the robustness of the proposed NSMC system, fig.7 a loading mass with 10 N is attached to investigate the mover of the controlled positioning mechanism. Fig.10. shows the phase plane. It is seen that a quick reaching line is achieved.

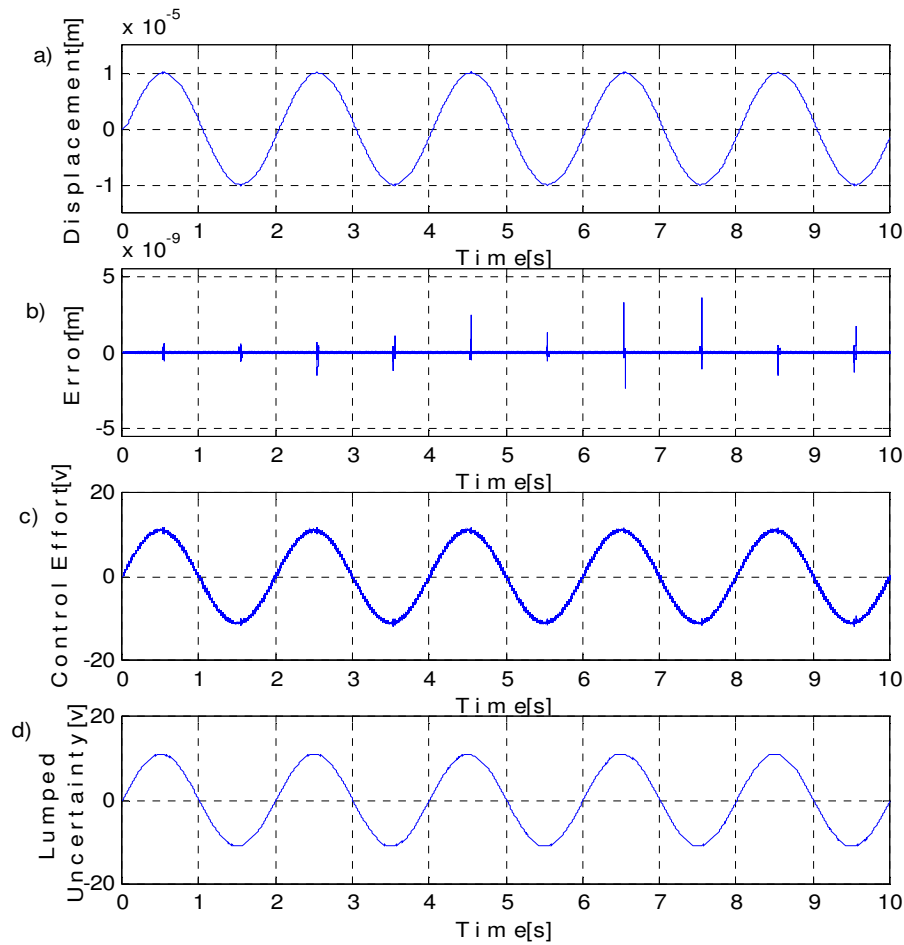


Fig.6: Simulation results of NSMC control system without external load of 10 N for periodic sinusoidal command with conditions at  $10\mu\text{m}$ , 0.5Hz: a) Tracking response; b) Tracking error; c) Control effort d) lumped uncertainty.

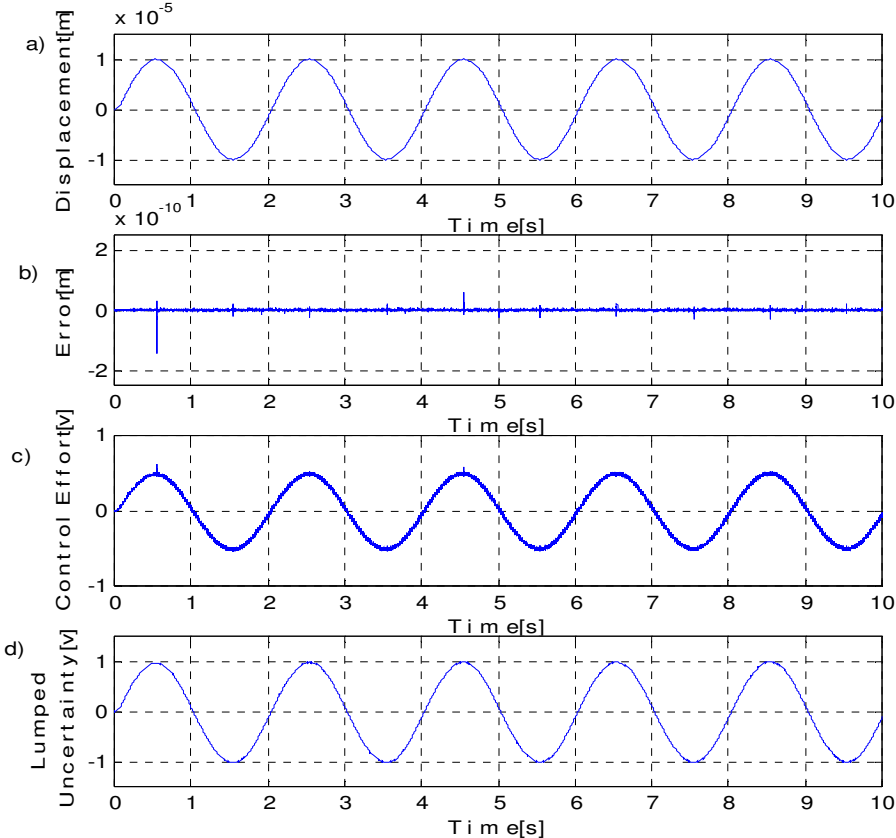


Figure 7: Simulation results of NSMC control system for periodic sinusoidal command without variation of parameter  $k_e = 2 * k_e$  with conditions at  $10\mu\text{m}$ ,  $0.5\text{Hz}$ : a) Tracking response; b) Tracking error; c) Control effort, d) lumped uncertainty

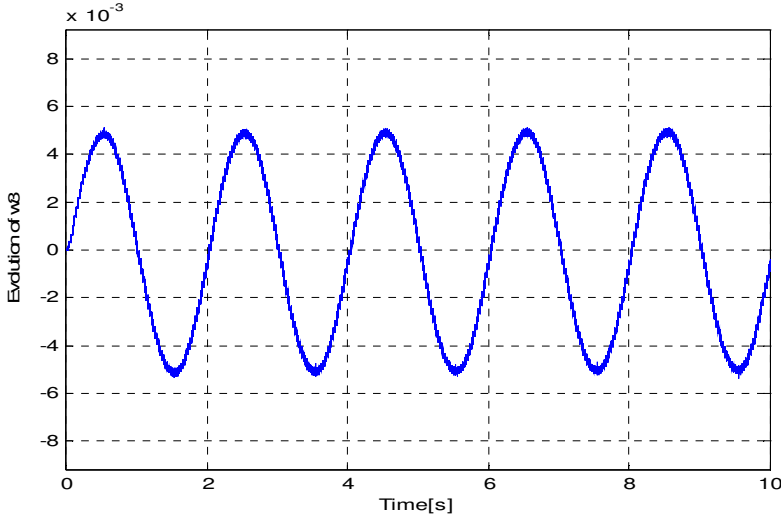


Figure 8. Time evolution of w3

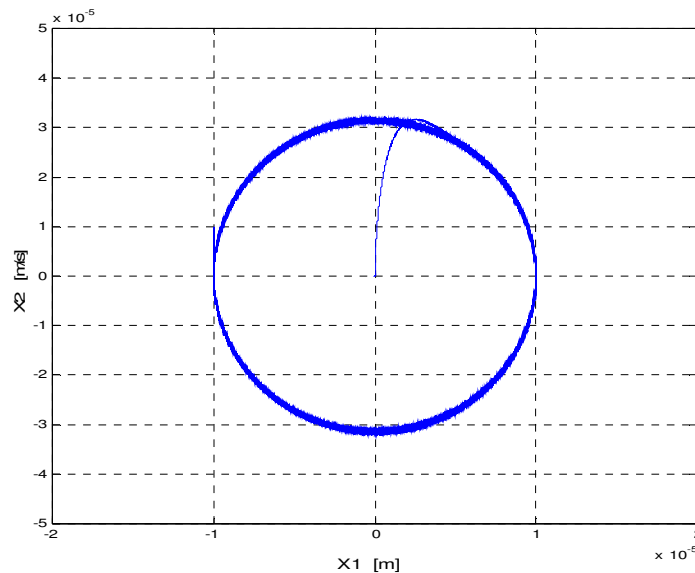


Fig 9: The phase plane

## 5. Conclusion

In order to control uncertain dynamical systems, the bound of the uncertainty is an important factor but may not be easily obtained by several reasons. A neuro-sliding mode with hysteresis estimation and compensation using NSMC for a piezo-actuator is proposed in this paper. The proposed methods included a hysteresis function model with the addition of a disturbance load and has been integrated into mechanical motion dynamics to effectively represent the overall system dynamics of a PEA with hysteresis. Using this control design, the asymptotic stability of the displacement tracking control is achieved. The results show the excellent performance of the proposed NSMC. As a conclusion, this controller is promising for the control of uncertain systems and is good candidate for the industry applications that needs intelligent controllers.

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