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A Numerical Study of Heat and Mass Transfer Effects on an Oscilaltory Flow of a Viscoelastic Fluid with Thermal Relaxation

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Abstract

We have studied numerically effect of heat and mass transfer on an oscillatory flow of a viscoelastic fluid with thermal relaxation. The dimensionless governing equations are solved using an explicit finite difference method. Numerical solutions for the velocity, the temperature, the concentration, the Nusselt number and Schmidt numbers are obtained and are shown graphically. The effects of cooling of a viscoelastic fluid compared to the Newtonian fluid have been discussed. The effect of the parameters like the Nusselt number, Schmidt number on the temperature and the concentration are studied. The main conclusions of the present study have been given.

Mathematics Subject Classification: 80A20

Keywords: Finite difference method, Oscillatory flow**,** Heat transfer and Mass transfer, Viscoelastic, Thermal relaxation

1. Introduction

A viscoelastic fluid which exhibits normal stress effect do not depend solely on fluid elasticity but this is generally the case and the most outstanding and important examples of systems showing normal stress effects are elastic in nature. The importance of the study of viscoelastic fluid increased during recent times. Applications involving viscoelastic fluid flow include such areas as microdispensing of bioactive fluids through high throughput injection devices, creation of cell attachment sites, scaffolds for tissue engineering, coatings and drug delivery systems for controlled drug release, and viscoelastic blood flow past valves.

Heat and mass transfer in convective flow of an incompressible rarefied visco-elastic fluid in presence of transverse magnetic field was studied by Agrawal et al [1]. Choubey [3] investigated the impulsive motion of a flat plate in a viscoelastic fluid in presence of a transverse magnetic field. Convection in a viscoelastic fluid through a porous layer heated from below was investigated by Rudraiah et al [10]. This problem was extended to Magneto hydrodynamic boundary layer by Ezzat and Sherief [7]. Elastic fluid flow of magneto hydrodynamic free convection through a porous medium has been studied by Ezzat et al [6].
Later magneto hydrodynamic flow and heat transfer in a rectangular duct with temperature dependent viscosity and Hall effects was investigated by Essia and Hazem [5]. Convective heat and mass transfer in a visco-elastic fluid flow through a porous medium over a stretching sheet investigated by Prasad et al [9]. Singh et al [11] studied heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. The problem of oscillatory convection with thermal relaxation has been investigated in more generally by Zakaria [13]. For perfectly conducting viscoelastic fluid free convection effects was studied by El-Bary [4]. Viscoelastic MHD flow and heat transfer over a stretching sheet with viscous and ohmic dissipations was investigated by Nandeppanavar et al [8].

A survey of literature reveals that the combined heat and mass transfer effects on a viscoelastic fluid flow has been studied by few researchers (Ref [1, 9]). Coming to the same effects on an oscillatory flow of a viscoelastic fluid with thermal relaxation has not been attracted by any researcher. This part of lacking on this particular problem motivated us to think in that direction. Consequently we tried to study the combined effects of heat and mass transfer on an oscillatory flow of a viscoelastic fluid with thermal relaxation.

2. Mathematical formulation

We consider a two-dimensional, unsteady free convective flow of a viscoelastic incompressible fluid which is bounded by a vertical infinite plane surface. We assume that the surface absorbs the fluid with a constant velocity and the velocity far away from the surface oscillates about a mean constant value with direction parallel to x' -axis. x' -axis is taken along the plane surface with direction opposite to the direction of the gravity and the y' -axis is taken to be normal to the plane surface. The heat due to viscous and joule dissipation are neglected for small velocities. All the fluid properties are assumed constant except that the influence of the density variation with temperature is considered only in the body force term. It is considered that the free stream velocity oscillates in magnitude but not in direction. Under the above stated assumptions and taking the usual Boussinesque approximation into account, the governing equations for the flow and temperature field in dimensionless form are given as under [Ref 12]

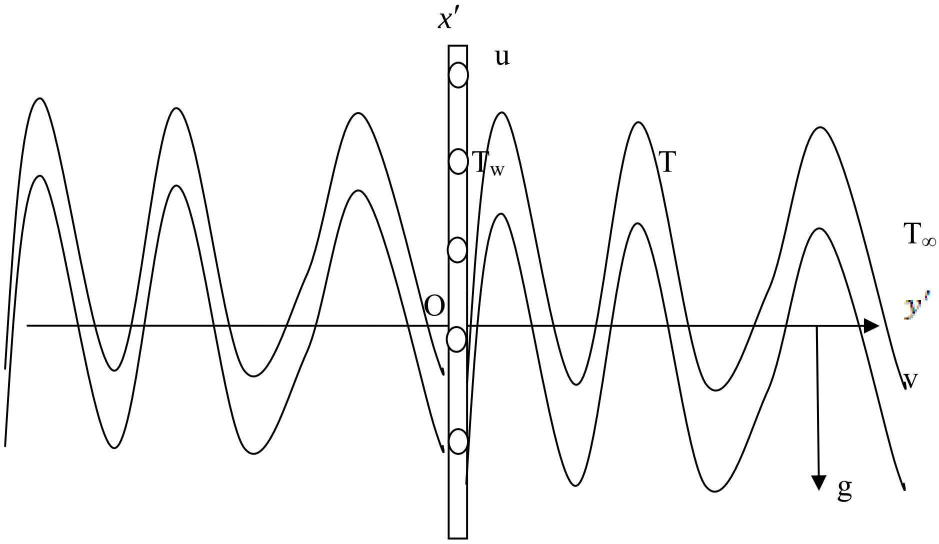

Fig .1. Geometry of the problem.

$$\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-k_0\left(\frac{\partial^3 u}{\partial y^2 \partial t}-\frac{\partial^3 u}{\partial y^3}\right)+G_r T+G_m C, \quad (1)$$

$$\frac{\partial T}{\partial t}-\frac{\partial T}{\partial y}=\frac{1}{P_r}\frac{\partial^2 T}{\partial y^2}-\tau_0\left(\frac{\partial^2 T}{\partial t^2}-\frac{\partial^2 T}{\partial t \partial y}\right), \quad (2)$$

$$\frac{\partial C}{\partial t}-\frac{\partial C}{\partial y}=\frac{1}{S_c}\frac{\partial^2 C}{\partial y^2}. \quad (3)$$

Initial condition has been neglected as the problem is in semi-infinite region. The relevant boundary conditions in dimensionless form are

$$\left.\begin{array}{l} u=-(1+\varepsilon e^{int}),\ T=(1+\varepsilon e^{int}),\ C=1 \text{ at } y=0, \\ u\to 0,\ T\to 0,\ C\to 0 \quad \text{as } y\to\infty. \end{array}\right\} \quad (4)$$

The dimensionless quantities introduced in the above equations are defined as

$$y=\frac{y' v_0}{\nu}, \quad u=\frac{u'}{v_0}, \quad t=\frac{t\ v_0^2}{\nu}, \quad n=\frac{n'\nu}{{v_0}^2}, \quad k_0=\frac{1}{\rho}k_0'\left(\frac{v_0^2}{\nu}\right), \quad (5)$$

$$P_r = \frac{\rho v c_p}{\lambda}, C = \frac{C' - C_\infty}{C_w - C_\infty}, T = \frac{T' - T_\infty}{T_w - T_\infty}, G_r = \frac{vg\beta(T_w - T_\infty)}{v_0^3}, G_m = \frac{vg\beta^*(C_w - C_\infty)}{v_0^3},$$

$$\tau_0 = \frac{v_0^2 \tau_0{}'}{\nu} \qquad S_c = \frac{\nu}{D}.$$

Where u is the velocity along the x' -axis, v_0 is constant obtained after integration conservation of mass in pre non-dimensional form not mentioned, v is the velocity along y' -axis, ϑ is the kinematic viscosity, g is the acceleration due to gravity, T is the temperature of the fluid, C is the concentration of the fluid, β is the coefficient of volume thermal expansion, β^* is the coefficient of volume expansion with concentration, C_p is the specific heat at constant pressure, ϵ is a constant, T_w is the temperature of the surface, C_∞ is the concentration far away from the surface, T_∞ is the temperature far away from the surface, C_w is the concentration of the surface, τ_0 is the thermal relaxation, P_r is the Prandtl number, S_c is the Schmidt number, G_r is the Grashoff number, G_m is the mass Grashoff number, ρ is the density, t is the time, n is the frequency of oscillation of the fluid and k_0 is the elastic parameter and D is the chemical molecular diffusivity.

3. Numerical scheme

The governing momentum, energy and mass transport equations (1), (2) and (3) are being solved numerically by using the finite difference method explained in [2]. The rectangular region is divided into a grid of mesh points (T_i, Y_j). Δh and Δk represents the uniform step lengths in x' and Y direction respectively. Using the central differences for first, second and third order partial derivatives of the dependent variables u and first, second order partial derivatives for T and C as defined in equation(6) below,

$$\begin{aligned}
\left(\frac{\partial T}{\partial t}\right)_{i,j} &= \frac{T_{i+1,j} - T_{i-1,j}}{2(\Delta k)}; \left(\frac{\partial T}{\partial y}\right)_{i,j} = \frac{T_{i,j+1} - T_{i,j-1}}{2(\Delta h)} \\
\left(\frac{\partial C}{\partial t}\right)_{i,j} &= \frac{C_{i+1,j} - C_{i-1,j}}{2(\Delta k)}; \left(\frac{\partial C}{\partial y}\right)_{i,j} = \frac{C_{i,j+1} - C_{i,j-1}}{2(\Delta h)} \qquad (6) \\
\left(\frac{\partial u}{\partial t}\right)_{i,j} &= \frac{u_{i+1,j} - u_{i-1,j}}{2(\Delta k)}; \left(\frac{\partial u}{\partial y}\right)_{i,j} = \frac{u_{i,j+1} - u_{i,j-1}}{2(\Delta h)}
\end{aligned}$$

$$\left(\frac{\partial^2 T}{\partial t^2}\right)_{i,j} = \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta k)^2};\ \left(\frac{\partial^2 T}{\partial y^2}\right)_{i,j} = \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{(\Delta h)^2}$$

$$\left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{(\Delta h)^2}\quad \left(\frac{\partial^2 C}{\partial y^2}\right)_{i,j} = \frac{C_{i,j+1} - 2C_{i,j} + C_{i,j-1}}{(\Delta h)^2}$$

$$\left(\frac{\partial^2 T}{\partial t \partial y}\right)_{i,j} = \frac{T_{i+1,j+1} - T_{i-1,j+1} - T_{i+1,j-1} + T_{i-1,j-1}}{4(\Delta k)(\Delta h)}$$

$$\left(\frac{\partial^3 u}{\partial y^3}\right)_{i,j} = \frac{u_{i,j+2} - 2u_{i,j+1} + 2u_{i,j-1} - u_{i,j-2}}{2(\Delta h)^3}$$

$$\left(\frac{\partial^3 u}{\partial t \partial y^2}\right)_{i,j} = \frac{u_{i+1,j+1} - 2u_{i+1,j} + u_{i+1,j-1} - u_{i-1,j+1} - 2u_{i-1,j} + u_{i-1,j-1}}{2(\Delta k)\ (\Delta h)^2}$$

The equations (1), (2) and (3) at each grid point (i,j) take respectively the following form of finite difference equations

$$-\left(\frac{1}{2(\Delta h)} + \frac{1}{(\Delta h)^2} + \frac{1}{(\Delta h)^3}\right)u_{i,j+1} + \left(\frac{1}{2(\Delta k)} - \frac{k_0}{(\Delta k)(\Delta h)^2}\right)u_{i-1,j} + \left(\frac{1}{2(\Delta k)} - \frac{k_0}{(\Delta k)(\Delta h)^2}\right)u_{i+1,j}) =$$
$$+\frac{-2}{(\Delta h)^2}u_{i,j} + \left(\frac{1}{2(\Delta h)} + \frac{1}{(\Delta h)^2} + \frac{k_0}{(\Delta h)^3}\right)u_{i,j-1} + \frac{k_0}{(\Delta h)^3}(u_{i,j+2} - u_{i,j-2}) + \frac{k_0}{2(\Delta k)(\Delta h)}(u_{i-1,j+1} - u_{i+1,j-1} -$$
$$u_{i-1,j-1}) + G_r T_{i,j} + G_m C_{i,j},$$

(7)

$$-\left(\frac{1}{2(\Delta h)} + \frac{1}{P_r(\Delta h)^2}\right)T_{i,j+1} + \left(\frac{1}{4(\Delta h)(\Delta k)}\right)(T_{i-1,j+1} - T_{i+1,j+1}) = \left(\frac{2}{(\Delta k)^2} - \frac{2}{P_r(\Delta h)^2}\right)T_{i,j} +$$
$$\left(\frac{\tau_0}{(\Delta k)^2} + \frac{1}{2(\Delta k)}\right)T_{i+1,j} + \left(\frac{1}{2(\Delta k)} - \frac{\tau_0}{(\Delta k)^2}\right)T_{i-1,j} + \left(\frac{1}{P_r(\Delta h)^2} - \frac{1}{2(\Delta h)}\right)T_{i,j-1} + \frac{\tau_0}{4(\Delta h)(\Delta k)}($$
$$T_{i-1,j-1} - T_{i+1,j-1}),$$

(8)

$$\left(1 + \frac{1}{S_C}\right)C_{i,j+1} + \left(\frac{\Delta t}{4\Delta y} - \frac{1}{2}\right)C_{i-1,j+1} - \left(\frac{\Delta t}{4\Delta y} + \frac{1}{2}\right)C_{i+1,j+1} = \left(\frac{\Delta t}{4\Delta y} + \frac{1}{2}\right)C_{i+1,j} + \left(-\frac{\Delta t}{4\Delta y} + \frac{1}{2}\right)C_{i-1,j}.$$

(9)

Since equations (1), (2) and (3) are coupled type, therefore they need to be solved in an iterative manner. Equations (1), (2) and (3) subject to the conditions given in equation (4) have been solved using block SLOR method. The resulting algebraic equations are solved by the successive over relaxation by using multi grid method [6] to calculate the unknown temperature and concentration values. Making use of temperature and concentration values obtained from (8) and (9) into equation (7), we obtain the numerical values of velocity. This process is repeated till the convergence criteria

$$\left|\frac{u_{i,j}^{new} - u_{i,j}^{old}}{u_{i,j}^{new}}\right| \le 10^{-12} \text{ and } \left|\frac{T_{i,j}^{new} - T_{i,j}^{old}}{T_{i,j}^{new}}\right| \le 10^{-12}.$$

is satisfied. The convergence is achieved by taking 64×64 mesh points for both hydrodynamic, thermal and diffusion parts.

4. Results and Discussion

The numerical computations are carried out for various parameters like P_r, G_r, G_m, S_c, k_0, n, τ_0, ε whose values have been chosen suitably. The numerical values of temperature have been obtained for different parameters like P_r and τ_0 as represented in the governing equation (2). Similarly the numerical values of velocity have been obtained for different parameters like G_r, G_m and k_0 as taken in the governing equation (1). Next we are interested from technological point of view to know the rate of heat transfer and mass transfer between the plate and the fluid, molecular concentration respectively. This can be found by using the non-dimensional quantities, the Nusselt number N_u and the Schmidt number S_c. The Nusselt number is defined as negative gradient of the temperature. The Schmidt number is negative gradient of concentration. The numerical values of the Nusselt number against time t are tabulated in table 4. Therefore table 4 shows the heat transfer for single time. Similar is for mass transfer.From the physical point of view, $G_r < 0$ corresponds to an externally heated plate as free convection currents are carried towards the plate. $G_r > 0$ corresponds to an externally cooled plate and $G_r = 0$ corresponds to the absence of free convection currents.

The numerical solutions of temperature, concentration and velocity, have been tabulated in tables 1, 2, and 3 respectively. The temperature, concentration and velocity profiles have been shown in figs. 2, 3 and 4. While computing the numerical values, suitable choices of above mentioned parameters are taken into account. For the Newtonian fluids, the temperature profiles can be easily found from the numerical solutions of the temperature which have been given in Table 1 as well as Fig. 2 which shows the temperature profiles decreases with increasing the value of displacement y. It is to be noted that the value of the Prandtl number chosen as 6.7 while calculating numerical values temperature which corresponds to water which is a well known Newtonian fluid.

Similarly the concentration profiles for the case of hydrogen (S_c=0.22) decreases as is shown from the fig. 3. Similarly velocity is observed to be decreased for the case of cooling from Fig. 4 as well as Table 3. It may be noted that the Prandtl number P_r =0.73 has been chosen while finding velocity because this value corresponds to air which is again known to be the Newtonian fluid. The rate of heat transfer is also decreases at different times and for different P_r which can't be seen from Table 4 but in general it is true which has been verified in my earlier studies. In table 4, the values are given at single time. We know that the rate of heat transfer is the negative gradient of temperature; the skin- friction is the negative gradient of velocity. One can easily found the skin –friction and observe how it behaves at a given time as the numerical values of velocity are available with us in table 3 which we are avoided here. Finally, we observe from table 3 that the velocity decreases in case of cooling for a viscoelastic fluid. Therefore the effect of cooling on the velocity of a viscoelastic compared to Newtonian fluid has been observed from table 3 as mentioned just above. The numerical values of the mass transfer have been given in table 5. From this table it is observed that the numerical solutions of the mass transfer decreases at a given time.

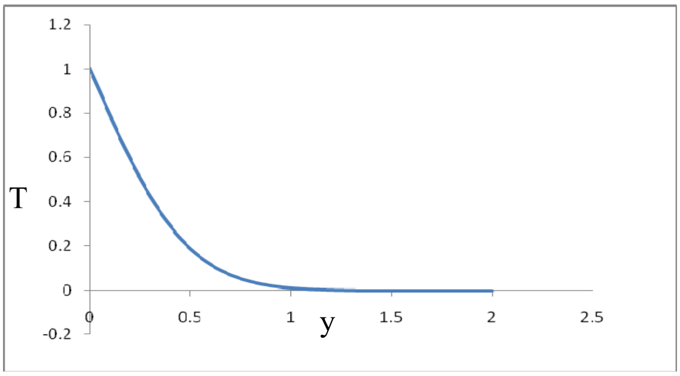

Fig.2. temperature profiles ($P_r = 6.7$, $\tau_0 = 2$)

t	y	Numerical solutions of T
0.1	0	1
0.1	0.1	0.786854
0.1	0.2	0.591183
0.1	0.3	0.423393
0.1	0.4	0.288843
0.1	0.5	0.187749
0.1	0.6	0.116394
0.1	0.7	0.06893
0.1	0.8	0.039068
0.1	0.9	0.021221
0.1	1	0.011034
0.1	1.1	0.005427
0.1	1.2	0.002386
0.1	1.3	0.00067
0.1	1.4	-0.00045
0.1	1.5	-0.00014
0.1	1.6	-0.00043
0.1	1.7	-0.00012
0.1	1.8	-0.00003
0.1	1.9	-0.00001
0.1	2	0

t	y	Numerical solutions of C
0.01	0	1
0.01	0.05	0.801452
0.01	0.1	0.776926
0.01	0.15	0.632318
0.01	0.2	0.505821
0.01	0.25	0.404772
0.01	0.3	0.320201
0.01	0.35	0.247618
0.01	0.4	0.186248
0.01	0.45	0.136168
0.01	0.5	0.096923
0.01	0.55	0.067328
0.01	0.6	0.045759
0.01	0.65	0.030496
0.01	0.7	0.019961
0.01	0.75	0.012833
0.01	0.8	0.008079
0.01	0.85	0.004921
0.01	0.9	0.002786
0.01	0.95	0.001254
0.01	1	0

Table 1: Numerical solutions of the temperature ($P_r = 6.7$, $\tau_0 = 2$)

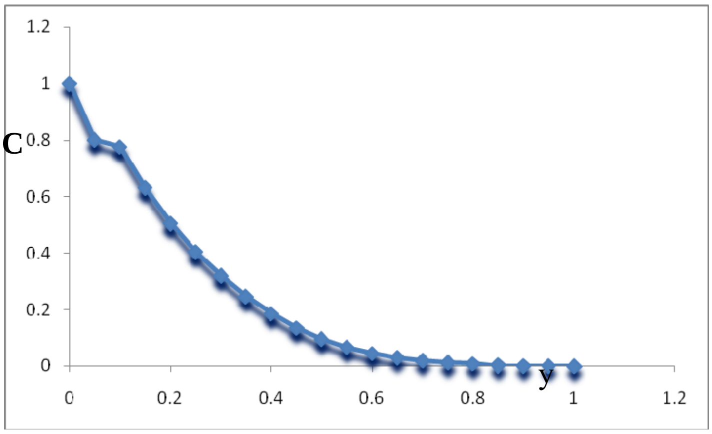

Fig. 3: concentration profiles for hydrogen. ($S_c = 0.22$)

Table 2: Numerical solutions of the concentration for hydrogen gas.

($S_c = 0.22$)

t	y	Numerical solutions of u
0.1	0	1
0.1	0.1	0.812202
0.1	0.2	0.625204
0.1	0.3	0.433124
0.1	0.4	0.345969
0.1	0.5	0.244258
0.1	0.6	0.166285
0.1	0.7	0.109048
0.1	0.8	0.068863
0.1	0.9	0.041873
0.1	1	0.024527
0.1	1.1	0.013851
0.1	1.2	0.007550
0.1	1.3	0.0039795
0.1	1.4	0.0020313
0.1	1.5	0.0010061
0.1	1.6	0.000484
0.1	1.7	0.000226
0.1	1.8	0.000016
0.1	1.9	0.000001
0.1	2	0

Table 3: Numerical solutions for velocity (P_r =0.733, G_r = 2, G_m =4,k_0=2)

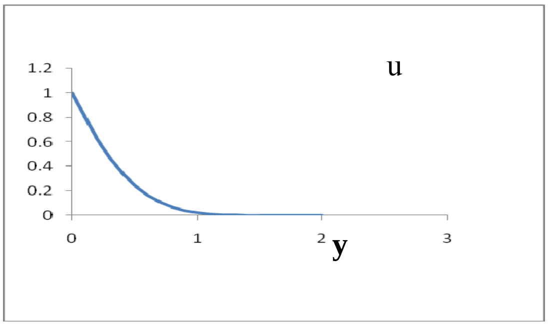

Fig.4. velocity profiles (P_r =0.733, G_r = 2, G_m =4,k_0=2)

t	Numerical solutions of N_u
0.1	1
0.1	0.879103
0.1	0.771932
0.1	0.679651
0.1	0.583622
0.1	0.495446
0.1	0.415113
0.1	0.343309
0.1	0.280138
0.1	0.225423
0.1	0.178752
0.1	0.139527
0.1	0.107016
0.1	0.0804106
0.1	0.0588658
0.1	0.0415422
0.1	0.027632
0.1	0.0163752
0.1	0.0070671
0.1	-0.000943
0.1	0

Table 4: Numerical solutions of Heat transfer

t	Numerical solutions of S_c
0.1	1
0.1	0.948873
0.1	0.897782
0.1	0.846733
0.1	0.795777
0.1	0.744916
0.1	0.694185
0.1	0.643595
0.1	0.593163
0.1	0.5429
0.1	0.492811
0.1	0.4429
0.1	0.393163
0.1	0.343595
0.1	0.294184
0.1	0.244917
0.1	0.195775
0.1	0.146736
0.1	0.097779
0.1	0.048875
0.1	0

Table 5: Numerical solutions of Mass transfer

4. Conclusions

In this paper, we studied the effect of combined heat and mass transfer on an oscillatory flow of a viscoelastic fluid with thermal relaxation. Numerical solutions for governing coupled partial differential equations for various combinations of parameters obtained by applying an efficient numerical technique based on finite differences. We solved by this numerical method as solving by traditional methods is tedious and very complicated. The effects of these parameters examined on the temperature, concentration, velocity profiles, heat and mass transfer rate. The numerical solutions of the temperature, concentration velocity and heat and mass transfer rate are tabulated.

We observe that the temperature decreases when thermal radiation parameter increases for the case of Newtonian fluid. Also we found that the wall heat transfer rate decreases with increase of y and also similar behavior for mass transfer. It was found that the velocity decrease in case of cooling for a viscoelastic fluid compared to Newtonian fluid.

References

[1] H.L.Agrawal, P.C.Ram, R.Nath, Heat and mass transfer in convective flow of an incompressible rarefied visco-elastic fluid in presence of transverse magnetic field, Astro .Phy.Spac.Sci, 95 (1983), 439-451.

[2] W.F.Ames, Numerical solutions of partial differential equations, Academic press, New York, 1977.

[3] K.R. Choubey, The impulsive motion of a flat plate in a viscoelastic fluid in presence of a transverse magnetic field, Indian. J. Pure. appl. Maths, 19(8)(1985), 931-937.

[4] A.A. El-Bary, Computational treatment of free convection effects on perfectly conducting viscoelastic fluid, Applied Mathematics and Computation,170(2),(2005) 801-820.

[5] M.Eissa Sayed-Ahmad and Hazem Ali Attia, MHD flow and heat transfer in a rectangular duct with temperature dependent viscosity and Hall effects, Int.Comm.Heat Mass Transfer, 27(8), (2000), 1177-1187.

[6] M. Ezzat, M. Zakaria, O.Shaker,and F. Barakat, State space formulation to elastic fluid flow of magnetohydrodynamic free convection through a porous medium. Acta Mech. 119(1996), 147–164.

[7] Ezzat, M, Sherief. H A problem of a viscoelastic magneto-hydrodynamic fluctuating boundary layer flow past an infinite porous plate. Can. J. Phys, 71 (1994), 97–105.

[8]M.Nandeppanavar, M.SubhasAbelEmmanuel, SanjayanandandMahantesh, Viscoelastic MHD flow and heat transfer over a stretching sheet with viscous and ohmic dissipations, 13(9)(2008), 1808-1821.

[9] K.V. Prasad, M. Subhas Abel, Sujit Kumar Khan, Convective heat and mass transfer in a visco-elastic fluid flow through a porous medium over a stretching sheet, Int.J.Numerical. Meth.Heat .Fluid flow, 11(8), (2001), 779-793.

[10] N. Rudraiah, P. N. Kaloni, and P. V. Radhadevi, Oscillatory convection in a viscoelastic fluid through a porous layer heated from below, Rheological Acta, Vol. 28, No. 1,PP 48-53,1989 .
[11] A.K.Singh, A.K.Singh, and N.P.Singh, Heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity, Indian. J.pure appl. Math, 34(3), (2003), 429-442.
[12] K. Walters, Second-order effects in elasticity, plasticity and fluid dynamics, Pergamon Press, Oxford, 1964.
[13] M.Zakaria, Free convection effects on the oscillatory flow of a viscoelastic fluid with thermal relaxation in the presence of a transverse magnetic field, Applied mathematics and Computation, 139(2-3), (2003), 265-286.

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