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Bistability between a Chaotic Bursting State and a Depolarized Steady State in a Mathematical Model of Snail RPa1 Neurons

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Abstract

This study presents a novel form of bistability in a mathematical model of snail RPa1 neurons. This model was previously reported as a system of nonlinear ordinary differential equations simulating the membrane potential oscillations of snail RPa1 neurons. Here, we perform a numerical simulation revealing that whether the model shows a chaotic bursting state or a depolarized steady state is dependent on the initial condition. Notably, a certain transient current pulse can change the dynamical state of the model from a chaotic bursting state to a depolarized steady state, or vice versa. Taken together, these results indicate that the snail RPa1 neuron model can show bistability between chaotic bursting and depolarized steady states.

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Keywords: bistability, mathematical model of snail RPa1 neurons, chaotic bursting state, depolarized steady state

1 Introduction

The mathematical model of snail RPa1 neurons is noteworthy because it shows rich dynamical behaviors. Specifically, this model shows two types of chaotic activity: a chaotic spiking state and a chaotic bursting state [1]. This model was originally developed as a system of nonlinear ordinary differential equations (ODEs) based on the Hodgkin–Huxley concept to simulate the dynamics of the membrane potential of snail RPa1 neurons [1]. In addition to showing chaotic activity, a previous study demonstrated that the model shows a certain type of bistability, namely, the coexistence of a chaotic spiking state and a periodic bursting state under a certain specific parameter condition [1]. Therefore, it is natural to ask whether the model also exhibits bistability that involves a chaotic bursting state and, if so, the type of bistability that the model shows. In other words, what kind of dynamical state can coexist with a chaotic bursting state? Answering this question will provide a deeper understanding of the relationship between bistability and chaotic activity in the snail RPa1 neuron model. However, this question remains unanswered. Previous studies report that a chaotic bursting state can change into different dynamical states due to variation in the various model parameters [2, 3]. For example, a chaotic bursting state can change into a depolarized steady state due to variation in the time constant of potassium conductance [2]. Given this previous finding, we hypothesized that the model may exhibit bistability between a chaotic bursting state and a depolarized steady state. To evaluate this hypothesis, the present study performs a numerical simulation of the snail RPa1 neuron model.

2 Mathematical Model

The snail RPa1 neuron model investigated in the present study is described by the following system of nonlinear ODEs (see [1] for further details):

$$\frac{dV}{dt} = \frac{1}{0.02}\Bigg(I_{app} - 0.13\left(\frac{1}{1+e^{-0.2(V+45)}}\right)(V-40) - 0.18 m_B h_B (V+58) \\ -0.02(V-40) - 0.25(V+70) \\ -400 m^3 h (V-40) - 10 n^4 (V+70) \\ -{m_{Ca}}^2 (V-150) - 0.01\left(\frac{1}{1+e^{-0.06(V+45)}}\right)\left(\frac{1}{1+e^{15000([\mathrm{Ca}]-0.00004)}}\right)(V-150)\Bigg) \quad (1)$$

$$\frac{dm_B}{dt} = \frac{1}{0.05}\left(\frac{1}{1+e^{0.4(V+34)}} - m_B\right) \quad (2)$$

$$\frac{dh_B}{dt} = \frac{1}{1.5}\left(\frac{1}{1+e^{-0.55(V+43)}} - h_B\right) \quad (3)$$

$$\frac{dm}{dt} = \frac{1}{0.0005}\left(\frac{1}{1+e^{-0.4(V+31)}} - m\right) \tag{4}$$

$$\frac{dh}{dt} = \frac{1}{0.01}\left(\frac{1}{1+e^{0.25(V+45)}} - h\right) \tag{5}$$

$$\frac{dn}{dt} = \frac{1}{0.015}\left(\frac{1}{1+e^{-0.18(V+25)}} - n\right) \tag{6}$$

$$\frac{dm_{Ca}}{dt} = \frac{1}{0.01}\left(\frac{1}{1+e^{-0.2V}} - m_{Ca}\right) \tag{7}$$

$$\frac{d[\mathrm{Ca}]}{dt} = 0.002\left(-\frac{{m_{Ca}}^2 (V-150)}{2F\left(\frac{4}{3}\pi 0.1^3\right)} - 50[\mathrm{Ca}]\right) \tag{8}$$

, in which V (mV) (the membrane potential of snail RPa1 neurons), m_{B}, h_{B}, m, h, n, and m_{Ca} (various gating variables), and [Ca] (mM) (the concentration of intracellular calcium) are state variables; t (s) is time; I_{app} (nA) (a transient current pulse) is a system parameter; and F is the Faraday constant. Equations (1)–(8) are numerically solved using the free and open source software Scilab (http://www.scilab.org/).

3 Numerical Results

First, we perform a numerical simulation of the snail RPa1 neuron model varying the initial conditions in which I_{app} is set to 0. As V and [Ca] are experimentally interesting state variables [1], we illustrate the time courses of V and [Ca] in Figure 1. When the initial condition is set that $V = -42$ mV, $m_B = 0.95$, h_B, = 0.77, $m = 0.14$, h, = 0.1, $n = 0.048$, $m_{Ca} = 0.0002$, and [Ca] = 6.5×10^{-5} mM, the model shows a chaotic bursting state: bursting with various burst duration lengths is observed (Figure 1a, top panel). In contrast, when the initial condition is set that $V = -22$ mV, $m_B = 0.01$, h_B, = 0.999, $m = 0.98$, h, = 0.003, $n = 0.62$, $m_{Ca} =$ 0.01, and [Ca] = 1.0×10^{-6} mM, the model shows a depolarized steady state: no oscillatory activity is observed (Figure 1b top).

Next, we perform a numerical simulation varying I_{app}, and the time courses of V and [Ca] (Figure 2). When the simulation starts with same initial conditions as in Figure 1a, the model shows a chaotic bursting state when t is at 50 s or earlier (Figure 2a, top panel). Under such conditions, if a transient current pulse is injected into the model (i.e., t is between 50 and 60 s, I_{app} is 10 nA; otherwise, I_{app} is 0 nA), the dynamical state of the model changes to a depolarized steady state during the transient current pulse (see the horizontal bar in Figure 2a, top panel).

Even if the current pulse is terminated (i.e., t is 60 s and longer), the dynamical state of the model does not recover to a chaotic bursting state, i.e., the depolarized steady state continues (Figure 2a, top). Finally, we investigate the opposite direction (i.e., a depolarized steady state → a chaotic bursting state) where the simulation starts with the initial condition shown in Figure 1(b) and a transient current pulse is injected (i.e., t is 50 to 60 s, I_{app} is −40 nA; otherwise, I_{app} is 0 nA) (Figure 2b). In this case, we observe that the dynamical state of the model changes from a depolarized steady state to a chaotic bursting state (Figure 2b, top).

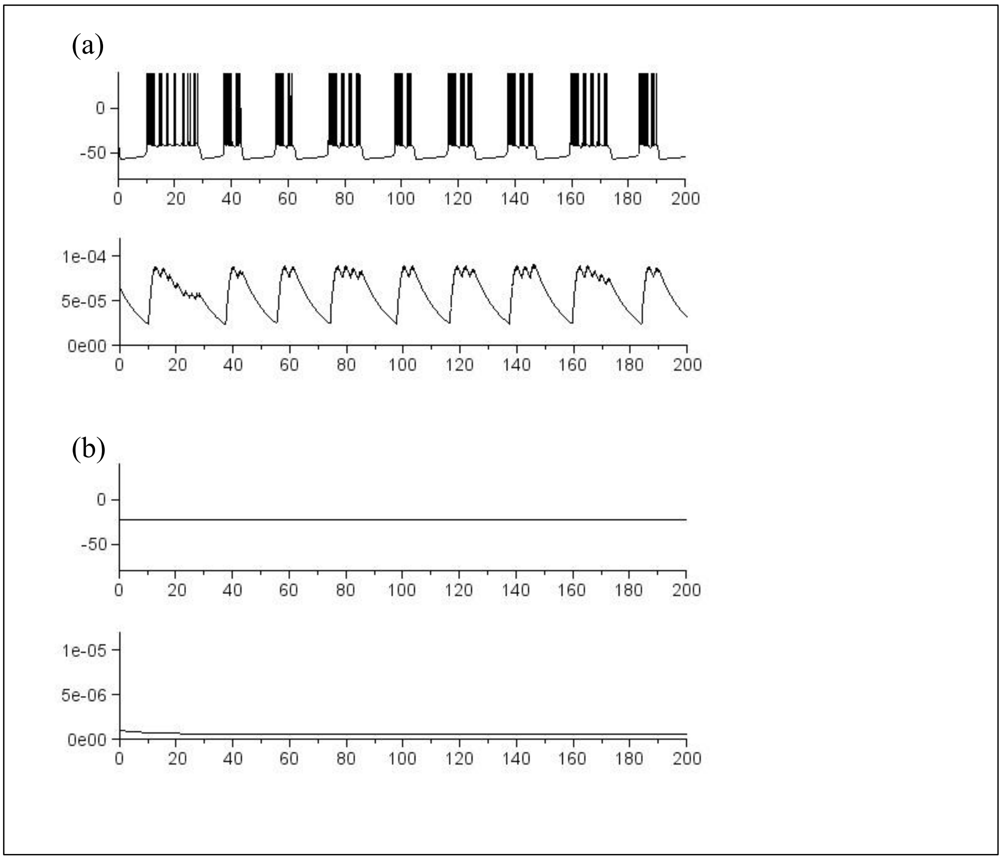

Figure 1. Time courses of V and [Ca] of the snail RPa1 neuron model under different initial conditions. The initial conditions are as follows: in (a): V = −42 mV, m_B = 0.95, h_B, = 0.77, m = 0.14, h, = 0.1, n = 0.048, m_{Ca} = 0.0002, and [Ca] = 6.5×10^{-5} mM; and in (b) V = −22 mV, m_B = 0.01, h_B, = 0.999, m = 0.98, h, = 0.003, n = 0.62, m_{Ca} = 0.01, and [Ca] = 1.0×10^{-6} mM. In both cases, I_{app} is 0. In (a) and (b), the horizontal axis indicates t (s) in both the top and bottom panels, while the vertical axis indicates V (mV) in the top panel and [Ca] (mM) in the bottom panel.

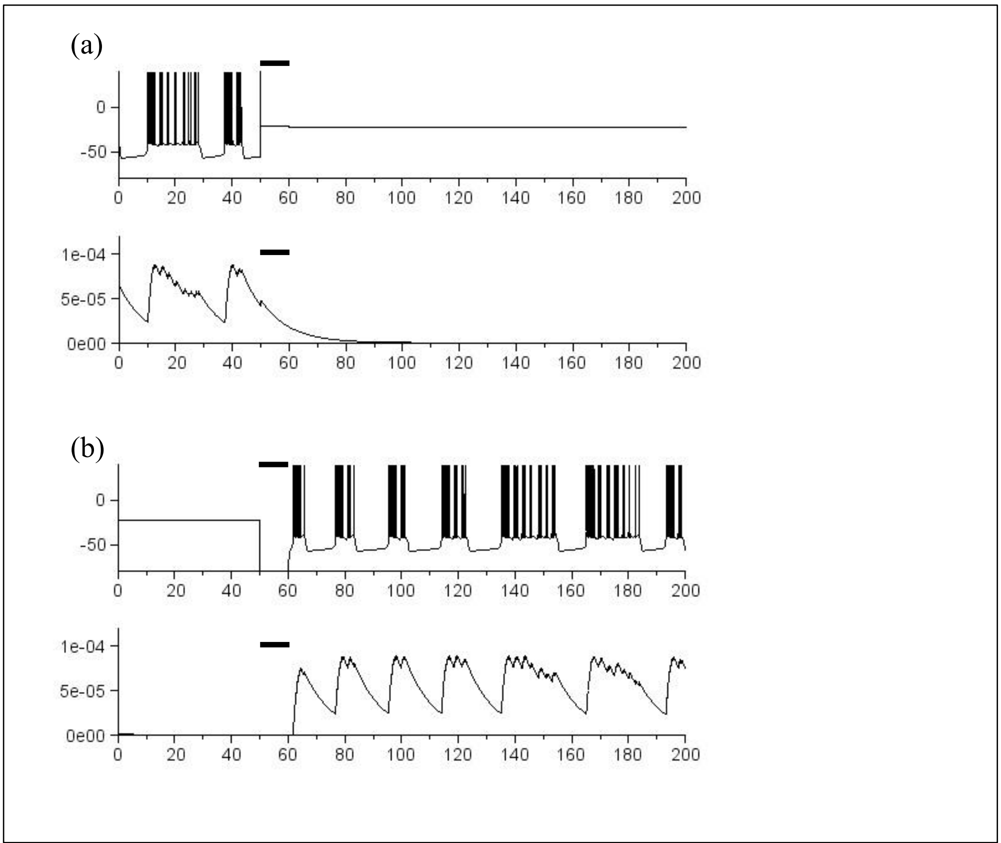

Figure 2. The transition from a chaotic bursting state to a depolarized steady state in (a) and vice versa in (b) due to the transient current pulse. (a) A current pulse with an amplitude of 10 nA and a duration of 10 s is injected at 50 s. The initial conditions are the same as Figure 1(a). (b) A current pulse with an amplitude of −40 nA and a duration of 10 s is injected at 50 s. The initial conditions are the same as Figure 1(b). In (a) and (b), the horizontal axis indicates t (s) in both the top and bottom panels, while the vertical axis indicates V (mV) in the top panel and [Ca] (mM) in the bottom panel. Horizontal bars between 50 and 60 s denote the duration of the transient current pulse.

4 Conclusion

This study focuses on the bistability of a mathematical model of snail RPa1 neurons, verifying a novel type of bistability between a chaotic bursting state and a depolarized steady state. This type of bistability differs from the previously reported bistability between a chaotic spiking state and a periodic bursting state [1]. Notably, bistability has been reported in previous studies of mathematical models of neurons different from the snail RPa1 neuron model (e.g., a leech interneuron model [4], a circadian pacemaker neuron model [5]). Again, the type of bistability reported in previous studies differs from that observed in the present study: the leech interneuron model shows bistability between a periodic bursting state and a hyperpolarized steady state [4], while the circadian pacemaker neuron model shows bistability of a periodic spiking state and a depolarized steady state [5]. In conclusion, the present study contributes to a deeper understanding of the relationship between bistability and chaotic activity in the snail RPa1 neuron model.

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