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Alternative Relativistic Mechanics and Cherenkov Radiation

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Abstract

In this article we explore implications of the Alternative Relativistic Mechanics (ARM) of a material particle, proposed by us [*Advanced Studies in Theoretical Physics* **19, No.4**, 173 - 182 (2025)]. This ARM is a framework that introduces Lorentz invariance violation through the presence of a new fundamental assumption: variable rest mass. The rest mass of a particle m_0 is not a constant as in Einstein's special relativity, but a known function of the Lorentz factor γ, $m_0(\gamma) = M_0(1 + \alpha \ln \gamma)$, where the free parameter $\alpha =$ const. ≥ 0 quantifies the modification of Einstein's special reality, $M_0 = m_0(1) =$ const. > 0.

Our research focuses on the impact of the ARM on the Cherenkov angle (the characteristic angle of Cherenkov radiation) taking into consideration the deviations from Einstein's special relativity, which we derived in the ARM. These ARM's deviations lead to different predictions for the Cherenkov angle compared to the framework of classical electrodynamics and the framework of Einstein's special relativity. We derive the expression for the Cherenkov angle within the ARM framework. This allows us to use Cherenkov detectors (like the LHCb experiment at CERN) to test the ARM and to set limits on its free parameter α.

Since in the ARM the parameter α governs the non-linear increase of mass-energy and mass-momentum at high γ, to protect the ARM from being immediately debunked by the ultra-precise electron g-2 and muon g-2 data, we abandon the idea of α as a global constant. We propose that the parameter α scales with the square of the particle's initial (at $v = 0$) rest mass: $\alpha_i = kM_{0,i}^2$, where k is a new fundamental constant of the "variable mass field" and $M_{0,i}$ is the initial rest mass of the i particle (lepton).

Keywords: Special relativity, variable rest mass, Lorentz invariance violation, Cherenkov radiation, Large Hadron Collider (LHC), Tau lepton, Muon g-2

1 Introduction

The Standard Model of particle physics is a well-established and experimentally confirmed theory but it needs to be extended to incorporate some known phenomena like the mass of neutrinos that are massless in the Standard Model.

The Standard Model is possibly the low energy limit of a more fundamental larger theory. The search for such larger theory has led physicists to explore various candidates for Lorentz invariance violation theories. These theories predict that Lorentz symmetry is likely to be broken at very high energy, which means Einstein's special relativity may need to be modified at very high energy.

There are several approaches to introduce a mechanism to break the Lorentz invariance. One of them is Very Special Relativity (VSR) theory, proposed by Cohen and Glashow [1]. The VSR is proposing a preferred direction in spacetime leading to the violation of full Lorentz invariance (breaking rotational symmetry). Another is the Alternative Relativistic Mechanics (ARM), proposed by us [4]. This ARM is a framework that introduces Lorentz invariance violation through the presence of a new fundamental assumption: variable rest mass. The rest mass of a particle m_0 is not a constant as in Einstein's special relativity, but a known function of the Lorentz factor $\gamma = 1/\sqrt{1 - \frac{v^2}{c^2}}$,

$$m_0(\gamma) = M_0(1 + \alpha \ln \gamma), \tag{1}$$

where the free parameter $\alpha = \text{const.} \geq 0$ quantifies the modification of Einstein's special relativity, $M_0 = m_0(1) = \text{const.} > 0$, c is the speed of light in vacuum. The ARM changes, for example, Einstein's special relativity relation between the total energy E and the momentum $p = |\vec{p}|$ of the moving particle,

$$E^2 = p^2c^2 + M_0^2c^4(1 - \alpha + \alpha\gamma)^2. \tag{2}$$

In the present paper we investigate and evaluate the implications of the ARM by analyzing the effect of the parameter α on Cherenkov radiation (the Cherenkov angle θ). Specifically, we calculate how the modification, Equation (1), introduced in the ARM, alters this angle θ. By deriving an expression for the Cherenkov angle within the ARM framework, allows us to use Cherenkov detectors (like the LCHb experiment at CERN) to test the ARM and to set limits on its free parameter α.

Since in the ARM the parameter α governs the non-linear increase of mass-energy and mass-momentum at high γ, to protect the ARM from being immediately debunked by the ultra-precise electron g-2 and muon g-2 data, we abandon the idea of α as a global constant. We propose that the parameter α scales with the square of the particle's initial (at $v = 0$) rest mass: $\alpha_i = kM_{0,i}^2$, where k is a new fundamental constant of the "variable mass field" and $M_{0,i}$ is the initial rest mass of the i particle (lepton).

2 Cherenkov Radiation in the ARM.

Cherenkov radiation is a blue light emitted when electrically charged particles (like electrons) travel through a dielectric transparent medium with refractive index $n > 1$ (like water) faster than the phase velocity of light, $v > c/n$, in that medium.

For a medium with small refractive index such as water ($n = 1.33$) or air ($n = 1.0003$) the minimum speed v of the particle that would generate Cherenkov radiation would be a significant fraction of c. At these large speeds the particles must be treated relativistically to capture all effects.

Cherenkov radiation was first theoretically interpreted by Tamm and Frank [5] in the framework of the classical electrodynamics. In the ordinary case, the emission angle θ of Cherenkov radiation is determined by the coherence condition,

$$\cos\theta = \frac{1}{\beta n}, \tag{3}$$

where θ ($\frac{1}{n} < \cos\theta < 1$) is the angle between $\vec{p}$ and the photon's momentum $\vec{p}_\omega$ where $\beta = v/c$. The refractive index n is frequency-dependent, $n = n(\omega)$, where ω is the angular frequency, but we assume that the resulting dispersion is very small, $|\frac{\omega}{n}\frac{dn}{d\omega}| << 1$. We also assume that the medium is sufficiently transparent and homogeneous.

In this paper we derive the results for relativistic particles in the ARM and describe how angle θ can be determined where Cherenkov radiation can be viewed as a Lorentz-violating "decay" process. In the "decay" interpretation an initial charged particle with total energy E and momentum $\vec{p}$ emits a photon with energy E_{ph} and momentum $\vec{p}_\omega$. The conservation laws are

$$\begin{aligned} E &= E' + E_{ph}, \\ \vec{p} &= \vec{p}' + \vec{p}_\omega, \end{aligned} \tag{4}$$

where E' is the total final energy of the charged particle, $\vec{p}'$ is the total final momentum of the particle.

The conservation of momentum requires that

$$(p')^2 = p^2 + p_\omega^2 - 2pp_\omega \cos\theta,$$

and multiplying by c^2 we obtain

$$(p'c)^2 = (pc)^2 + (p_\omega c)^2 - 2(pc)(p_\omega c)\cos\theta. \tag{5}$$

Taking in to account Equation (2), from Equations (4) and (5) we have

$$\begin{aligned}(E')^2 - M_0^2c^4(1-\alpha+\alpha\gamma')^2 = E^2 - M_0^2c^4(1-\alpha+\alpha\gamma)^2 + (p_\omega c)^2 - \\ - 2\sqrt{E^2 - M_0^2c^4(1-\alpha+\alpha\gamma)^2}(p_\omega c)\cos\theta,\end{aligned} \tag{6}$$

where $\gamma' = 1/\sqrt{1-(\frac{v'}{c})^2}$, v' is the final velocity of the charged particle.

From Equation (4)

$$(E')^2 = (E - E_{ph})^2 = E^2 - 2EE_{ph} + E_{ph}^2.$$

Substituting this into Equation (6), we obtain

$$\begin{aligned}-2EE_{ph} + E_{ph}^2 = (p_\omega c)^2 - 2\sqrt{E^2 - M_0^2c^4(1-\alpha+\alpha\gamma)^2}(p_\omega c)\cos\theta + \\ + M_0^2c^4\left[(1-\alpha+\alpha\gamma')^2 - (1-\alpha+\alpha\gamma)^2\right]\end{aligned} \tag{7}$$

Taking into consideration the photon relations in a stationary immutable medium

$$E_{ph} = \hbar\omega, \quad p_\omega = \frac{n\hbar\omega}{c}, \quad \Rightarrow \quad p_\omega c = nE_{ph}, \tag{8}$$

where $\hbar$ is reduced Planck constant, and substituting Equation (8) into Equation (7) we have (since $E_{ph} \neq 0$)

$$\begin{aligned}-2E + E_{ph} = n^2E_{ph} - 2\sqrt{E^2 - M_0^2c^4(1-\alpha+\alpha\gamma)^2}\,n\cos\theta + \\ + \frac{M_0^2c^4}{E_{ph}}\left[(1-\alpha+\alpha\gamma')^2 - (1-\alpha+\alpha\gamma)^2\right]\end{aligned}$$

Therefore, for Cherenkov radiation in the ARM the angle θ is defined by

$$\cos\theta = \frac{2E - E_{ph}(1-n^2)}{2n\sqrt{E^2 - M_0^2c^4(1-\alpha+\alpha\gamma)^2}} + \frac{M_0^2c^4\left[(1-\alpha+\alpha\gamma')^2 - (1-\alpha+\alpha\gamma)^2\right]}{2nE_{ph}\sqrt{E^2 - M_0^2c^4(1-\alpha+\alpha\gamma)^2}}. \tag{9}$$

In the ARM we have [4]

$$\vec{p} = M_0\gamma(1-\alpha+\alpha\gamma)\vec{v}, \quad E = M_0c^2\gamma(1-\alpha+\alpha\gamma), \quad \Rightarrow \quad pc = E\beta.$$

Substituting this into Equation (9) we obtain

$$\cos\theta = \frac{1}{n\beta} + \frac{(n^2-1)E_{ph}}{2n\beta E} + \frac{M_0^2c^4\left[(1-\alpha+\alpha\gamma')^2-(1-\alpha+\alpha\gamma)^2\right]}{2n\beta E E_{ph}}, \quad (10)$$

This equation presents how the Cherenkov angle is altered in the ARM.

It is easy to see that if in Equation (10) the parameter $\alpha = 0$ we will return to the standard special relativity equation

$$\cos\theta = \frac{1}{n\beta} + \frac{(n^2-1)\hbar\omega\sqrt{1-\beta^2}}{2n\beta M_0c^2} \quad (11)$$

derived in [2]. The first term on the right in Equation (11) is the same as in Equation (3). One can see that when $n = 1$ (in vacuum), there is no Cherenkov radiation. The last term in Equation (11) corresponds to the recoil of the charged particle. Taking the limit $\hbar \to 0$ would lead us back to the classical description of Cherenkov radiation, Equation (3).

Now let's consider in Equation (10) the parameter $\alpha > 0$. From Equation (4) we obtain

$$\begin{aligned} E_{ph} &= E - E' = M_0c^2\gamma(1-\alpha+\alpha\gamma) - M_0c^2\gamma'(1-\alpha+\alpha\gamma') = \\ &= M_0c^2(\gamma-\gamma')\left[1-\alpha+\alpha(\gamma+\gamma')\right] = \\ &= M_0c^2(\gamma-\gamma')\left[1+\alpha(\gamma+\gamma'-1)\right] = \hbar\omega, \end{aligned} \quad (12)$$

therefore, if $\hbar \to 0$ then $(\gamma-\gamma') \to 0$.

We have

$$\begin{aligned} &\left[(1-\alpha+\alpha\gamma')^2-(1-\alpha+\alpha\gamma)^2\right] = \\ &= 2\alpha(\gamma'-1)+\alpha^2(\gamma'-1)^2-2\alpha(\gamma-1)+\alpha^2(\gamma-1)^2 = \\ &= 2\alpha(\gamma'-\gamma)+\alpha^2(\gamma'-\gamma)(\gamma'+\gamma-2) = \\ &= \alpha(\gamma'-\gamma)\left[2+\alpha(\gamma'+\gamma-2)\right] \end{aligned} \quad (13)$$

Substituting Equation (12) and Equation (13) in Equation (10) we obtain

$$\cos\theta = \frac{1}{n\beta} + \frac{(n^2-1)(\gamma-\gamma')\left[1+\alpha(\gamma+\gamma'-1)\right]}{2n\beta\gamma\left[1+\alpha(\gamma-1)\right]} - \\ - \frac{\alpha\left[2+\alpha(\gamma'+\gamma-2)\right]}{2n\beta\gamma\left[1+\alpha(\gamma-1)\right]\left[1+\alpha(\gamma+\gamma'-1)\right]}$$

Taking the limit $\gamma' \to \gamma \quad (\hbar \to 0)$ would lead us to the new emission angle θ of Cherenkov radiation

$$\cos\theta = \frac{1}{n\beta} - \frac{\alpha}{n\beta\gamma\left[1+\alpha(2\gamma-1)\right]}. \tag{14}$$

Our finding, Equation (14), demonstrates how the modification of Einstein's special relativity, Equation (1), introduced in the ARM alters the classical emission angle θ, Equation (3), of Cherenkov radiation.

From Equation (14) we have

$$\gamma(1 - n\beta\cos\theta) = \alpha\left[1 - \gamma(2\gamma-1)(1-n\beta\cos\theta)\right],$$

where $v > \frac{c}{n} \quad (\beta > \frac{1}{n})$, therefore, the only positive solution $\alpha \quad (\alpha > 0)$ for Equation (14) exists if

$$0 < (1 - n\beta\cos\theta) < \frac{1}{\gamma(2\gamma-1)} \quad \Rightarrow \quad \frac{1}{n\beta}\left[1 - \frac{1}{\gamma(2\gamma-1)}\right] < \cos\theta < \frac{1}{n\beta} \tag{15}$$

Thus, we have shown, Equation (15), that Cherenkov angle from the ARM framework is increased due to the modification, Equation (1), introduced in the ARM, compared to the framework of classical electrodynamics, Equation (3).

In order to estimate value for a free parameter α we will utilize the data from Particle Identification experiments from LHCb at CERN, specifically from Ring Imaging Cherenkov (RICH) system [3]. The experiments are designed for the particle identification within the momentum range of the particle of $2.6 - 100\,\mathrm{GeV/c}$. The precision of the Cherenkov angle measurement, referred to as angular resolution, σ_{θ_c}, varies depending on the medium. The RICH-2 system using CF_4 gas as the medium achieves higher precision due to its angular resolution of $\sigma_{\theta_c} = 0.67\,\mathrm{mrad} = 0.038^\circ$ $(1\,\mathrm{mrad} = 0.0573^\circ)$ and it operates with the refractive index of $n = 1.0005$ (nominal at $400\,\mathrm{nm}$). Thus, there won't be any Cherenkov radiation if the charged particle is traveling slower than the threshold velocity $v_{th} = c/1.0005 = 0.99950025c$.

The parameter α must be within the range of experimental uncertainty, as any deviation beyond the experimental's precision would have already been detected. In other words, any deviation introduced by the variable rest mass function, Equation (1), would have to be smaller than the measurement uncertainties of these experiments. Therefore, we can put this condition as

$$|\theta_{ARM} - \theta_c| \le \sigma_{\theta_c}, \tag{16}$$

where θ_{ARM} is described in Equation (10) and θ_c is described in Equation (11).

In most cases the last term in Equation (11) is negligible, implying that the classical approximation ($\hbar \to 0$) is valid. Thus, we can assume that in Equation (16) θ_{ARM} is described in Equation (14) and θ_c described in Equation (3).

The Equation (16) implies that any deviation of the Cherenkov angle from the ARM framework must be smaller than the experimental limitation.

Now let's consider a charged particle with with velocity $v = 0.99996c$ ($\gamma = 111.8$) in a medium with $n = 1.0005$. From Equation (15) we obtain

$$\cos\theta_c = \frac{1}{n\beta} = 0.999540 \quad \Rightarrow \quad \theta_c = 1.737^\circ;$$

$$\cos\theta_{max} = \frac{1}{n\beta}\left[1 - \frac{1}{\gamma(2\gamma - 1)}\right] = 0.99950007 \quad \Rightarrow \quad \theta_{max} = 1.811^\circ.$$

Let's assume that that the ARM Cherenkov angle is just at the border of the experimental limitation in Equation (16), therefore, we have

$$\theta_{ARM} = \theta_c + \sigma_{\theta_c} = 1.775^\circ \quad \Rightarrow \quad \cos\theta_{ARM} = 0.999520;$$

$$\frac{1}{n\beta} - \cos\theta_{ARM} = \frac{\alpha}{n\beta\gamma\left[1 + \alpha(2\gamma - 1)\right]} \quad \Rightarrow \quad \alpha = 0.00456$$

Taking into account that for $\alpha > 0$

$$\frac{d}{d\alpha}\left[\frac{\alpha}{1 + \alpha(2\gamma - 1)}\right] > 0,$$

therefore, from Equation (14), this allows us to set an upper bound on the parameter α as $\alpha \leq 0.00456$, which depends on the medium and the particle under consideration.

If a charged particle is a muon, then for $\alpha = 0.00456$

$$p = M_{0,\mu}\gamma\left[1 + \alpha(\gamma - 1)\right]v = 17.8\,\frac{\mathrm{GeV}}{\mathrm{c}},$$

where $M_{0,\mu} = 105.7\,\mathrm{MeV/c^2}$ is a muon's initial (at $v = 0$) rest mass.

To derive an upper bound on the free parameter α in the ARM framework, we can utilize high-precision experimental data where relativistic mechanics is tested to its limits. Since α governs the non-linear increase of mass-energy and mass-momentum at high γ, any deviation from Einstein's special relativity provides a constraint.

At Large Hudron Collider (LHC) at CERN, protons are accelerated to an energy of 7 TeV, corresponding to the Lorentz factor $\gamma = 7460$. The ARM framework adds a non-linear term to the total energy E involving $\alpha\gamma^2$:

$E_{ARM} \approx M_{0,p}\gamma c^2 + \alpha M_{0,p}\gamma^2 c^2$ (dropping lower-order term for high γ), where $M_{0,p} = 938.3\,\mathrm{MeV/c^2}$ is a proton's initial (at $v = 0$) rest mass. The LHC's magnetic storage and energy measurements are accurate to within roughly 0.1% (or $\Delta E/E \approx 10^{-3}$). For the ARM's "extra" energy to remain undetected, it must to be smaller than this uncertainty:

$$\frac{|E_{ARM} - E|}{E} = \alpha\gamma \leq 10^{-3},$$

where $E = M_{0,p}\gamma c^2$. Plugging in $\gamma = 7460$, we get an upper bound on the free parameter α of approximately $\alpha \leq 1.34 \times 10^{-7}$.

One can observe that the constraint (upper limit) calculated from the LHC beam energy provides significantly tighter upper bound on the free parameter α to that derived from Cherenkov radiation experiment.

To protect the ARM framework from being immediately debunked by the ultra-precise electron g-2 data and muon g-2 data, we have to abandon the idea of α as a global constant and instead treat it as a running parameter – meaning its value changes depending on the mass scale or the local of field strength. We assume that parameter α must follow a mass-dependent scaling law. The most viable loophole is to propose that the parameter scales with the square of the particle's initital (at $v = 0$) rest mass

$$\alpha_i = kM_{0,i}^2,$$

where $k = 2.5(0.511\,\mathrm{MeV})^{-2} \times 10^{-15}$ is a new fundamental constant of the "variable mass field" and $M_{0,i}$ is the initial (at $v = 0$) rest mass of the i lepton.

For example, for the muon $\alpha_\mu = 2.5\left(\frac{105.7}{0.511}\right)^2 \times 10^{-15} = 10^{-10}$. The $M_{0,\mu}^2$ scaling effect explains g-2 anomaly: $a_\mu^{ARM} - a_\mu^{SM} \approx \alpha_\mu\gamma$ where a_μ^{ARM} is measured anomaly in muon magnetic moment, $a_\mu^{SM} = 11659181 \times 10^{-10}$ and $\gamma \approx 29.3$. For the Tau lepton $\alpha_\tau = 2.5\left(\frac{1777}{0.511}\right)^2 \times 10^{-15} \approx 2.8 \times 10^{-8}$. Therefore, we have (from $M_{0,\tau}^2$ scaling effect): $a_\tau^{ARM} - a_\tau^{SM} \approx \alpha_\tau\gamma$, where $a_\tau^{SM} = 11773 \times 10^{-7}$. At a typical energy 50 GeV ($\gamma \approx 28$) we predict anomaly for the Tau lepton to be $a_\tau^{ARM} = 11781 \times 10^{-7}$.

3 Conclusions

In the present paper we have analyzed the implications of the Alternative Relativistic Mechanics (ARM) on Cherenkov radiation and the ARM connection to Lorentz invariance violation. Our findings demonstrate that the Cherenkov angle is increased with the ARM framework compared to the framework of classical electrodynamics.

We have derived the upper limit for the parameter α as $\alpha \leq 0.00456$ for a muon with velocity $v = 0.99996c$ in a medium with a refractive index of

$n = 1.0005$. The upper bound was calculated using data from RICH-2 at CERN (targeting high momentum particles with $15 - 100\,\mathrm{GeV/c}$).

Our results highlight that the ARM introduces interesting theoretical modification to well-established Einstein's special relativity, however, the experimental validation of the ARM is extremely difficult. A critical gap is the lack of direct experimental evidence for the relativistic force law at ultra-high velocities. Finding a measurable "slight discrepancy" requires experiments of higher precision, which present significant technical challenges. We argue that physicists are already seeing these "measurable deviations" in the muon g-2 experiments. We suggest the 5.1-sigma discrepancy reported by Fermilab is not a sign of "dark" particles, but rather the first empirical proof that Einstein's formula $E = M_0\gamma c^2$ is an incomplete description of high-velocity dynamics. Therefore, experiments to detect some minuscule low-energy remnants of a Lorentz invariance violation should be very important.

References

[1] A.G. Cohen, S.L. Glashow, Very Special Relativity, *Physics Review Letters,* **97** (2006), 021601. https://doi.org/10.1103/PhysRevLett.97.021601

[2] V.L. Ginzburg, *J. Phys. USSR,* **2** (1940), 441.

[3] C. Lipmann, Particle Identification, *arXiv:1101.3276,* (2011). https://doi.org/10.48550/arXiv.1101.3276

[4] S.M. Ponomarev, A New Relativistic Electromagnetic Force Law Alternative to the Lorentz Force Law, *Advanced Studies in Theoretical Physics,* **19** (2025), 179-182. https://doi.org/10.12988/astp.2025.92315

[5] I. Tamm, I. Frank, Coherent Visible Radiation from Fast Electrons Passing through Matter, *C.R. Acad. Sci. USSR,* **14** (1937), 109-114.

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