

Quantitative ARM Prediction for the J-PARC Muon g-2 Experiment

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Abstract

In this paper, we present an explicit derivation of the spin-precession frequency vector for relativistic charged particles moving in a planar orbit within a uniform magnetic field $\vec{B}$ under the framework of Alternative Relativistic Mechanics (ARM). Assuming that the kinematic Lorentz factor γ of muon orbiting in a macroscopic storage ring does not modify its static rest-frame spin magnetic dipole moment, we formulate the modified relativistic equation of motion and evaluate the combined effect of Larmor precession, Thomas precession, and cyclotron orbital motion. Unlike the Standard Model—where the anomalous precession frequency is γ -independent in a purely magnetic field $\vec{B}$ —ARM predicts an explicit Lorentz factor γ dependence proportional to the non-dimensional coupling parameter α_μ . We provide concrete quantitative predictions for the upcoming J-PARC muon g-2 Experiment ($E = 0$, $p = 300 \text{ MeV}/c$, $B = 3.0 \text{ T}$), demonstrating how J-PARC's distinct low-momentum regime serves as a definitive experimental test to distinguish ARM dynamics from standard Special Relativity dynamics.

Keywords: Alternative Relativistic Mechanics; Muon g-2; J-PARC E34; Fermilab E989; Spin Precession; Thomas Precession; Larmor Precession

1 Introduction

The anomalous magnetic momentum of the muon, $a_\mu = (g_\mu - 2)/2$, represents one of the most stringent tests of fundamental physics and quantum

electrodynamics. Experimental measurements executed at Fermilab (FNAL E989) [1] utilized “magic momentum” muons ($p \approx 3.094 \text{ GeV}/c$, $\gamma = 29.3$) in electrostatic focusing fields to eliminate electric fields contributions to spin precession.

The upcoming J-PARK E34 experiment [2] offers a complimentary, highly distinct experimental regime: an ultra-cold, low-momentum muon beam ($p \approx 300 \text{ MeV}/c$, $\gamma = 3.01$) confined in a high-precision, purely magnetic storage ring ($B = 3.0 \text{ T}$) with zero electric field ($E = 0$).

In standard Special Relativity (SR) governed by Bargmann-Michel-Telegdi (BMT) equation [3], the anomalous spin-precession frequency vector in a pure magnetic field $\vec{B}$ oriented along the z -axis reduced to:

$$\vec{\Omega}_a^{SM} = -a_\mu^{SM} \frac{qB}{M_0} \hat{z}, \quad (1)$$

which is strictly independent of the muon’s kinematic momentum or the Lorentz factor γ , and $a_\mu^{SM} = 11659181 \times 10^{-10}$ is the theoretical prediction (from the Standard Model) for a_μ .

In contrast, Alternative Relativistic Mechanics (ARM) [4] proposes a modified momentum-velocity relation, parameterized by a non-dimensional coupling constant $\alpha_\mu = 10^{-10}$. Under the core ARM postulate, the kinematic Lorentz factor γ of a muon orbiting in a macroscopic storage ring does not alter its static rest-frame spin magnetic dipole moment. This postulate makes a clean distinction between intrinsic structural properties and extrinsic kinematic inertia. In this paper, we derive the exact expression for the anomalous spin-precession frequency vector $\vec{\Omega}_a^{ARM}$ under ARM and evaluate the predicted effective anomaly $a_\mu^{ARM} = a_\mu(\text{eff}) = a_\mu^{SM} + \alpha_\mu(\gamma - 1)$ for both J-PARK E34 and FNAL E989.

2 Modified ARM Equation of Motion

In Alternative Relativistic Mechanics, the spatial momentum $\vec{p}$ of a muon with rest mass $M_0 = 105.66 \text{ MeV}/c^2$, and velocity $\vec{v}$ with kinematic Lorentz factor $\gamma = 1/\sqrt{1 - \frac{v^2}{c^2}}$ is defined as:

$$\vec{p} = M_0 \gamma (1 - \alpha_\mu + \alpha_\mu \gamma) \vec{v}.$$

For a positive muon ($q = +e$) in a uniform magnetic field $\vec{B} = B\hat{z}$ moving in a planar circular orbit ($\vec{v} \cdot \vec{B} = 0$), the orbital speed $v = |\vec{v}|$ and the Lorentz factor γ remain constant over time ($d\gamma/dt = 0$).

The modified Lorentz force equation of orbital motion is given by:

$$\frac{d\vec{p}}{dt} = M_0 \gamma (1 - \alpha_\mu + \alpha_\mu \gamma) \frac{d\vec{v}}{dt} = q(\vec{v} \times \vec{B})$$

Solving for the planar acceleration vector $\vec{a} = \frac{d\vec{v}}{dt}$:

$$\vec{a} = \frac{q}{M_0\gamma(1 - \alpha_\mu + \alpha_\mu\gamma)}(\vec{v} \times \vec{B})$$

The orbit cyclotron frequency vector $\vec{\Omega}_c$ defined via $\vec{a} = \vec{\Omega}_c \times \vec{v}$, is explicitly derived as:

$$\vec{\Omega}_c = -\frac{qB}{M_0\gamma(1 - \alpha_\mu + \alpha_\mu\gamma)}\hat{z} \quad (2)$$

3 Derivation of Total Precession Frequency Vector and Anomalous Frequency Vector in ARM.

The total laboratory-frame spin-precession frequency vector $\vec{\Omega}_s$ is the sum of the rest-frame Larmor precession vector $\vec{\Omega}_L$ transformed to the lab frame and the kinematic Thomas precession vector $\vec{\Omega}_T$:

$$\vec{\Omega}_s = \vec{\Omega}_L + \vec{\Omega}_T \quad (3)$$

The relativistic kinematic Thomas precession frequency vector $\vec{\Omega}_T$ is given by:

$$\vec{\Omega}_T = -\frac{\vec{v} \times \vec{a}}{c^2} \frac{\gamma^2}{\gamma + 1} \quad (4)$$

Evaluating the vector product $\vec{v} \times \vec{a}$ for planar circular motion ($\vec{v} \cdot \vec{B} = 0$):

$$\vec{v} \times \vec{a} = \frac{q}{M_0\gamma(1 - \alpha_\mu + \alpha_\mu\gamma)}\vec{v} \times (\vec{v} \times \vec{B}) \quad (5)$$

Applying the vector triple product expansion $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$, we obtain

$$\vec{v} \times (\vec{v} \times \vec{B}) = \vec{v}(\vec{v} \cdot \vec{B}) - \vec{B}v^2 = -v^2 B \hat{z} \quad (6)$$

Substituting Equation (5) and Equation (6) into Equation (4), we have

$$\vec{\Omega}_T = -\left[-\frac{qv^2B}{M_0\gamma(1 - \alpha_\mu + \alpha_\mu\gamma)}\hat{z}\right] \frac{\gamma^2}{(\gamma + 1)c^2}$$

Utilizing the algebraic identity $\frac{v^2}{c^2} \frac{\gamma^2}{\gamma + 1} = \frac{\gamma^2 - 1}{\gamma + 1} = \gamma - 1$, we obtain

$$\vec{\Omega}_T = +\frac{(\gamma - 1)qB}{M_0\gamma(1 - \alpha_\mu + \alpha_\mu\gamma)}\hat{z}$$

Under the fundamental ARM premise – that the kinematic Lorentz factor γ of a muon orbiting in a macroscopic storage ring does not alter its static rest-frame spin magnetic dipole moment – the conversion from proper time τ to laboratory time t ($dt = \gamma d\tau$) yields:

$$\vec{\Omega}_L = -g_\mu \frac{qB}{2M_0} \hat{z} = -(1 + \alpha_\mu^{SM}) \frac{qB}{M_0} \hat{z}$$

The anomalous precession frequency vector $\vec{\Omega}_a$ measures the rate of spin precession relative to the cyclotron vector $\vec{\Omega}_a = \vec{\Omega}_s - \vec{\Omega}_c = \vec{\Omega}_L + \vec{\Omega}_T - \vec{\Omega}_c$ (taking into account Equation (3)). We have

$$\vec{\Omega}_a = \left[-(1 + a_\mu^{SM}) \frac{qB}{M_0} + \frac{(\gamma - 1)qB}{M_0\gamma(1 - \alpha_\mu + \alpha_\mu\gamma)} \right] \hat{z} - \left[-\frac{qB}{M_0\gamma(1 - \alpha_\mu + \alpha_\mu\gamma)} \hat{z} \right]$$

Combining the kinematic terms in the brackets:

$$\begin{aligned} \vec{\Omega}_a &= \left[-(1 + a_\mu^{SM}) \frac{qB}{M_0} + \frac{\gamma qB}{M_0\gamma(1 - \alpha_\mu + \alpha_\mu\gamma)} \right] \hat{z} = \\ &= - \left[1 + a_\mu^{SM} - \frac{1}{(1 - \alpha_\mu + \alpha_\mu\gamma)} \right] \frac{qB}{M_0} \hat{z} \quad (7) \end{aligned}$$

Expanding $\frac{1}{1 - \alpha_\mu + \alpha_\mu\gamma} = \frac{1}{1 + \alpha_\mu(\gamma - 1)}$ in a Taylor series for $\alpha_\mu \ll 1$:

$$\frac{1}{1 + \alpha_\mu(\gamma - 1)} \approx 1 - \alpha_\mu(\gamma - 1) + O(\alpha_\mu^2)$$

Substituting this back into Equation (7), we obtain

$$\vec{\Omega}_a^{ARM} \approx - [a_\mu^{SM} + \alpha_\mu(\gamma - 1)] \frac{qB}{M_0} \hat{z}$$

Thus, ARM predicts an effective anomalous magnetic moment parameter:

$$a_\mu^{ARM} = a_\mu(\text{eff}) = a_\mu^{SM} + \alpha_\mu(\gamma - 1)$$

Using the inputs $a_\mu^{SM} = 11659181 \times 10^{-10}$, $\alpha_\mu = 10^{-10}$, we evaluate a_μ^{ARM} and the resulting anomalous precession shift for J-PARK E34 and Fermilab E989:

1. J-PARK E34 ($\gamma = 3.01$): shift $\Delta a_\mu^{ARM} = \alpha_\mu(\gamma - 1) = +2.01 \times 10^{-10}$, $a_\mu^{ARM} \times 10^{10} = 11659183$.
2. FNAL E989 ($\gamma = 29.3$): shift $\Delta a_\mu^{ARM} = \alpha_\mu(\gamma - 1) = +28.3 \times 10^{-10}$, $a_\mu^{ARM} \times 10^{10} = 11659209.3$.

4 Discussion and Conclusion

The exact derivation of spin precession in Alternative Relativistic Mechanics (ARM) reveals a structural property of particle transport in magnetic fields:

1. Exact SR limit ($\alpha_\mu = 0$): When the ARM parameter $\alpha_\mu = 0$, the γ -dependent terms in the Thomas precession vector $\vec{\Omega}_T$ and cyclotron vector $\vec{\Omega}_c$, cancel identically, recovering the standard γ -independent anomaly frequency Equation (1).
2. Energy-Dependent ARM scaling ($\alpha_\mu = 10^{-10}$): When ARM momentum coupling parameter $\alpha_\mu = 10^{-10}$ is introduced, the effective anomaly acquires a linear dependence on the kinematic factor, $\Delta a_\mu^{ARM} = \alpha_\mu(\gamma - 1)$.

Because FNAL E989 operates at $\gamma \approx 29.3$ and J-PARC E34 operates at $\gamma \approx 3.01$, ARM predicts a systematic offset in the measured anomalous magnetic moment between the two experiments:

$$\Delta a_{\text{FNAL-J-PARC}}^{ARM} = \alpha_\mu(\gamma_{\text{FNAL}} - \gamma_{\text{J-PARC}}) \approx 26.3 \times 10^{-10}$$

The sub-ppm experimental precision targeted by J-PARC E34 (0.1 ppm corresponds to $\Delta a \approx 1.166 \times 10^{-10}$) makes J-PARC uniquely suited to detect this predicted shift.

Let's calculate Δa . The baseline anomalous magnetic moment of the muon is approximately:

$$a_\mu^{SM} \approx 1.1659181 \times 10^{-3} \quad (11659181 \times 10^{-10})$$

Multiplying the baseline anomaly a_μ^{SM} by the relative precision fraction of 0.1 ppm yields the absolute uncertainty Δa :

$$\Delta a = a_\mu^{SM} \times (0.1 \text{ ppm}) = (1.1659181 \times 10^{-3}) \times 10^{-7} = 1.166 \times 10^{-10}$$

Because the predicted ARM shift for J-PARC E34 is:

$$\Delta a_\mu^{ARM} = \alpha_\mu(\gamma - 1) = 2.01 \times 10^{-10}$$

a detector precision of 0.1 ppm ($\Delta a = 1.166 \times 10^{-10}$) provides sufficient resolution to detect the predicted 2.01×10^{-10} ARM shift at the significance level of nearly 2σ .

Therefore, comparing high-precision data across the distinct Lorentz factors of J-PARC and FNAL provides an empirical test to evaluate ARM momentum scaling against standard Special Relativity.

References

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