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Review of the Conservation Law of Energy

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Abstract

In this paper, we examine the division of forces into internal and external components and explore their connection to the conservation of energy across various branches of physics, including classical mechanics, electromagnetism, quantum mechanics, and the theories of special and general relativity. While the partitioning of forces is a well-established concept in classical mechanics, it is often overlooked or inadequately addressed in other areas of physics. This oversight has significant implications for our understanding of fundamental physical principles and may influence the development of future technologies.

1 Introduction

This paper does not aim to provide a historical overview of the conservation of energy; that task is best left to historians of science [1]. The conservation of energy, often referred to as the first law of thermodynamics, is traditionally regarded as an absolute and universal law. It is widely presented in the literature as a principle that has never been violated, applying uniformly from the microscopic realm of elementary particles to the vast scale of the universe. Most sources assert that the energy of an isolated system remains constant; in other words, energy can neither be created nor destroyed without external influence. It is further assumed that the universe itself constitutes an isolated system, implying that the total energy in existence has remained unchanged since the beginning. Even the enigmatic dark energy, believed to be driving the accelerating expansion of the universe, is expected to conform to this principle. While scientists today possess a strong understanding of the various forms of energy and the processes by which energy can be transformed from one form to another, some of the foundational assertions made by physicists regarding energy conservation warrant re-examination. For example, it is commonly believed that the conservation of energy derives from the first law of thermodynamics. However, as shown in our previous work [2], this principle more fundamentally originates from Newtonian mechanics, specifically from Newton's third law [2-4]. This connection hinges on two key conditions: first, the system in question must be isolated; second, the internal forces within the system must satisfy Newton's third law. Surprisingly, despite Newton's laws being over three centuries old, a review of the scientific literature reveals that many physicists remain unaware that the force partitioning inherent in Newtonian mechanics implies energy conservation for internal forces, and violations of energy conservation when external forces are involved. This raises important questions: What form does the conservation law of energy take for open systems? How is it applied when forces do not satisfy Newton's third law? More concerning is the near-total absence of force partitioning in modern physical theories, as though it applies only to classical mechanics. These questions are far from trivial. A careful re-analysis not only deepens our understanding of fundamental physics but also opens the door to groundbreaking technological possibilities such as over-unity devices and advanced space propulsion systems.

2 The partition of forces

The principle of superposition of fields and forces in physics is a simple mathematical concept which states that the solution of a problem can be calculated by adding individual parts of the problem to obtain the total solution of the problem. This principle only applies to linear systems governed by linear differential equations that is the reason why the partition of forces between internal and external forces can be done. Let us quote a phrase found in internet

> Why do we need Newton's third law? Actually, it is almost a matter of common sense. Suppose that bodies a and b constitute an isolated system. If $\mathbf{f}_{ba} \neq -\mathbf{f}_{ab}$ then this system exerts a non-zero net force $\mathbf{f} = \mathbf{f}_{ab} + \mathbf{f}_{ba}$ on itself, without the aid of any external agency. It will, therefore, accelerate forever under its own steam. We know, from experience, that this sort of behavior does not occur in real life. For instance, I cannot grab hold of my shoelaces and, thereby, pick myself up off the ground. In other words, I cannot self-generate a force which will spontaneously lift me into the air: I need to exert forces on other objects around me in order to achieve this. Thus, Newton's third law essentially acts as a guarantee against the absurdity of self-generated forces.

We will of course contest in this paper the validity of the assertions made by the author. The partition of forces between internal forces which satisfy Newton's third law and external forces which violate Newton's third law has been examined for decades in the scientific literature including by the present author. However, most physicists do not draw the consequences of their analysis and worst as we shall show hereafter, they contradict their own work.

We must also note that the conservation law of energy is a misnomer, we must instead speak of conservation law of power. Only after integration with respect to time, that we can speak of conservation law of energy. Before proceeding, let us first recall some basic definitions concerning the classification of a system:

- An isolated system does not exchange energy or matter with its surroundings.
- A closed system is a system that exchanges only energy with its surroundings.
- An open system is a system that exchanges energy and matter with its surroundings.

2.1 The partition of forces in classical mechanics

The misconception concerning the principle of conservation of power and energy is so great in the literature that it is necessary to review again with a new insight the point of view explained in our work since 1989 [5-18]. To do so, it is fundamental to recall some basic definitions in classical mechanics. Newton's second law of motion states that the equations of motion of two bodies or two point particles with masses m_1 and m_2 and positions $\mathbf{r}_1$ and $\mathbf{r}_2$ defined with respect to an arbitrary reference frame are described by the differential equations:

$$\frac{d\mathbf{P}_1}{dt} = \mathbf{F}_{12} + \mathbf{F}_{11} \qquad \frac{d\mathbf{P}_2}{dt} = \mathbf{F}_{21} + \mathbf{F}_{22} \tag{1}$$

with the definitions $\mathbf{P}_1 = m_1\mathbf{U}_1$ and $\mathbf{P}_2 = m_2\mathbf{U}_2$. In the preceding equations, $\mathbf{F}_{12}$ is the mutual force exerted by the particle 2 on the particle 1, in the same manner $\mathbf{F}_{21}$ is the mutual force exerted by the particle 1 on the particle 2. We must distinguish between the mutual forces $\mathbf{F}_{12}$ and $\mathbf{F}_{21}$ and the proper forces $\mathbf{F}_{11}$ and $\mathbf{F}_{22}$ acting on the particles due to sources which can be outside the system or not. We can speak of interaction between two particles only if the mutual forces follow Newton's third law if we have the condition $\mathbf{F}_{12} = -\mathbf{F}_{21}$. We call these forces internal forces, therefore, an external force is by definition a force that does not follow Newton's third law. The forces violating Newton's third law also violate the conservation law of energy as we shall see. These forces, the magnetic Lorentz force and the Coriolis force are well analyzed in our papers [2,5-18]. There is a well-known analogy between these two forces which have for expression:

$$\mathbf{F}_B = \frac{q}{c}\mathbf{V} \wedge \mathbf{B} \qquad \mathbf{F}_C = 2m_0\boldsymbol{\omega}_e \wedge \mathbf{V} \tag{2}$$

We must point out the fact that if there is in nature no force violating Newton's third law then there is no possibility and no need to partition the forces. In that case, the Lorentz and the Coriolis forces do not exist and we must use either the Ampere force or the Whittaker force to describe the interactions between particles. This is the point of view adopted by Assis in his book [19].

The partition of forces is obtained by considering two points: the center of mass of the system which is a point $\mathbf{r}$ where the entire mass $m = m_1 + m_2$ of the system is concentrated which is defined by the relation $m\mathbf{r} = m_1\mathbf{r}_1 + m_2\mathbf{r}_2$. The motion of this point is only determined by the effect of external forces since we have:

$$\frac{d}{dt}m\mathbf{U} = \frac{d\mathbf{P}_1}{dt} + \frac{d\mathbf{P}_2}{dt} = \mathbf{F}_{11} + \mathbf{F}_{22} = \mathbf{F}_e \tag{3}$$

For an isolated system, we have $\mathbf{F}_{11} = \mathbf{F}_{22} = \mathbf{F}_e = 0$, therefore, the velocity of the center of mass is a rectilinear uniform motion according to Newton's first law:

$$\mathbf{U} = \frac{d\mathbf{r}}{dt} = \mathbf{C}t \tag{4}$$

The second point is called the relative particle with a reduced mass $M = m_1m_2/(m_1+m_2)$. This single particle is located at the place occupied by either the first or the second particle depending on the choice of the rest position as shown in figure 1. The distance $\mathbf{R}$ is therefore $\mathbf{R}_{12} = \mathbf{r}_1 - \mathbf{r}_2$ if the particle 2 is located at the origin of a reference frame or $\mathbf{R}_{21} = \mathbf{r}_2 - \mathbf{r}_1$ if the particle 1 is now the origin of our reference frame. For each choice, we have an equation of motion:

$$\frac{d}{dt}M\mathbf{V}_{12} = \mathbf{F}_{i1} \qquad \frac{d}{dt}M\mathbf{V}_{21} = \mathbf{F}_{i2} \tag{5}$$

with the definitions

$$\mathbf{F}_{i1} = \mathbf{F}_{12} + \frac{1}{m}(m_2\mathbf{F}_{11} - m_1\mathbf{F}_{22}) \tag{6}$$

$$\mathbf{F}_{i2} = \mathbf{F}_{21} - \frac{1}{m}(m_2\mathbf{F}_{11} - m_1\mathbf{F}_{22}) \tag{7}$$

We note that the monopole forces $\mathbf{F}_{11}$ and $\mathbf{F}_{22}$ can be used to build dipolar internal forces.

In the same manner as for the equations of motion, we can write the equations of power with the same partition:

$$\frac{d}{dt}\frac{1}{2}m_1\mathbf{U}_1^2 = (\mathbf{F}_{11} + \mathbf{F}_{12}) \cdot \mathbf{U}_1 \qquad \frac{d}{dt}\frac{1}{2}m_2\mathbf{U}_2^2 = (\mathbf{F}_{22} + \mathbf{F}_{21}) \cdot \mathbf{U}_2 \tag{8}$$

$$\frac{d}{dt}\frac{1}{2}m\mathbf{U}^2 = \mathbf{F}_e \cdot \mathbf{U} \qquad \frac{d}{dt}\frac{1}{2}M\mathbf{V}_{12}^2 = \mathbf{F}_i \cdot \mathbf{V}_{12} \tag{9}$$

it follows the identity:

$$\frac{d}{dt}\frac{1}{2}m_1\mathbf{U}_1^2 + \frac{d}{dt}\frac{1}{2}m_2\mathbf{U}_2^2 = \frac{d}{dt}\frac{1}{2}m\mathbf{U}^2 + \frac{d}{dt}\frac{1}{2}M\mathbf{V}_{12}^2 \tag{10}$$

The preceding equation provides a form of conservation law of power (and energy) which has nothing to do with the classical definition of the conservation law of energy. This equation specifies simply that the sum of powers is invariant during the mathematical process of splitting the forces, no kinetic energy is lost during the transformation of the equations. But it does not mean that the total power (and energy) of one side of the equation is conserved since the total power magnitude depends on space and time in the general case.

Newtonian mechanics defines two different kinds of quantities, a fact which is ignored in most textbooks in physics, namely the quantities $\mathbf{V}$, $\mathbf{R}$ and $\mathbf{F}_i$ are invariant in a change of reference frame while the numerical values of the quantities of $\mathbf{U}$, $\mathbf{r}$ and $\mathbf{F}_e$ depend on the choice of a reference frame. We can write the power equations in a form which will be very useful in the discussion hereafter about the resolution of the paradox concerning the superposition principle of fields.

$$\frac{d}{dt}\left(\frac{1}{2}m\mathbf{U}^2\right) = \mathbf{U} \cdot \mathbf{F}_e = P_{11} + P_{22} \tag{11}$$

$$\frac{d}{dt}\left(\frac{1}{2}M\mathbf{V}_{12}^2\right) = \mathbf{V}_{12} \cdot \mathbf{F}_{i1} = 2P_{12} \tag{12}$$

In the above equations, the P terms define powers related to the external and internal forces. Therefore, the equality $2P_{12} = \mathbf{V}_{12} \cdot \mathbf{F}_{i1} = 2P_{21} = \mathbf{V}_{21} \cdot \mathbf{F}_{i2}$ is satisfied in Newtonian dynamics because of the reciprocity concept $\mathbf{V}_{12} = -\mathbf{V}_{21}$ and Newton's third law $\mathbf{F}_{i1} = -\mathbf{F}_{i2}$.

It follows that the reciprocity of the rest reference frames is linked to the existence of Newton's third law as shown in figure 1 for the three possibilities. The reciprocity concept and Newton's third law are two faces of the same coin. We also note that the reference frame at rest is not an inertial reference frame

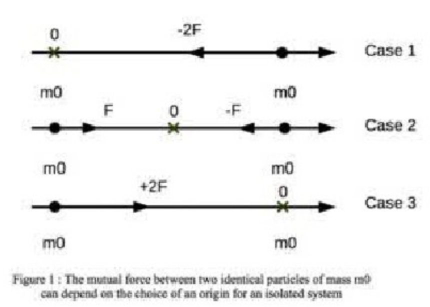

Figure 1: The mutual force between two identical particles of mass m0 can depend on the choice of an origin for an isolated system

which is obvious if we use the formulation 2 in figure 1. This fact can also be deduced from the existence of the recoil effect. The equations 11 and 12 show that the partition of forces satisfies a covariant principle but not in an invariant manner since equation 12 is invariant in a change of reference frame while the equation 11 is not. We can also point out that only two reference frames are needed to define the two velocities **U** and **V**. Therefore, there is no need to consider an infinite set of inertial reference frames as defined in the relativity principle that would be discussed hereafter.

The above partition of forces seems to be a mathematical procedure that is well-known and published including by the present author a long time ago. However, in writing this paper, we made again a search in the scientific literature in internet to see how this partition is defined and used in physics. To my own surprised, i discovered that the partition of forces was mainly used in mechanics and not in all branch of physics and worst even in classical mechanics this partition was used in a wrong manner. I advise the reader to check in the literature what i am saying since i will take three examples taken from the references [20,21,22] where a wrong use of this partition is taught to students. The authors in these references introduce the above partition quite correctly and from there, these authors [20, p.21], [21, p.2] and [22, p.341] consider the case where the external force is a gravitational force which derives from a potential not defined by the authors but we can guess that it has the form:

$$E_G = -G\frac{mm_3}{R} \tag{13}$$

Knowing that $m = m_1 + m_2$, they write the equation of motion:

$$\frac{d}{dt}m\mathbf{U} = \mathbf{F}_e = -\nabla E_G \tag{14}$$

The author (20,p.21) multiplies the preceding equation by **U** and defines an equation of conservation of energy for external force:

$$\frac{d}{dt}\left(\frac{1}{2}m\mathbf{U}^2 + E_G\right) = 0 \tag{15}$$

Worst, the same author goes on saying suppose that internal forces are also conservative and he defines a law of energy which is similar to the one previously defined for external force. In fact, the authors have in mind the motion of the center of mass with a gravitational interaction with a third particle m_3. The correct equation of motion must be similar to equation 12, namely:

$$\frac{d}{dt}M\mathbf{V} = \mathbf{F}_i = -\nabla E_G \tag{16}$$

with the definitions:

$$M = \frac{mm_3}{m + m_3} \qquad \mathbf{R} = \mathbf{r} - \mathbf{r}_3 \qquad \mathbf{V} = \frac{d\mathbf{R}}{dt} \tag{17}$$

We can use Jacobi coordinates [18,p.102] to describe the gravitational interaction between N particles or bodies. The equation 14 is obviously wrong because by definition an external force does not derive from a potential function while an internal force always derives from a potential function.

The partition of forces resulting from Newton's third law is not just a mathematical trick but a fundamental law of physics that applies to all branch of physics including quantum mechanics as we shall see. Therefore, the equations 2 and 4 describe more the physical reality than the two equations 1. The partition of forces helps to define the degree of interaction between bodies or systems. Consequently, the masses m and M are more fundamental masses than the masses m_1 and m_2. This point of view is proved by experimental facts such as the electrostatic pendulum experiment described hereafter and the Mossbauer effect [18,p.167] also called recoil-free gamma ray resonance absorption. The resonance absorption of gamma rays is made possible by fixing atomic nuclei in the lattice of solids so that energy is not lost in recoil during the emission and absorption of radiation.

Newton's third law implies the existence of the potential energy $E_P[\mathbf{R}(t)]$ which must be a function of $\mathbf{R}$ alone in order to satisfy the identity $\mathbf{F}_{i1} = -\mathbf{F}_{i2}$. The potential energy is usually defined in the literature as the energy stored in an object. Common types include the gravitational potential energy, the elastic potential energy of a spring, and the electric potential energy of an electric charge and so on. For example, in the free encyclopedia Wikipedia, we get the following definition:

> In physics, potential energy is the energy held by an object because of its position relative to other objects, stresses within itself, its electric charge, or other factors. Common types of potential energy include the gravitational potential energy of an object that depends on its mass and its distance from the center of mass of another object, the elastic potential energy of an extended spring, and the electric potential energy of an electric charge in an electric field.

This definition is incorrect since the potential energy does not belong to one object but to two objects in interaction. Moreover, the distance is not defined with respect to the center of mass but to the second object. A more correct definition is given by the Encyclopedia Britannica:

> Potential energy, stored energy that depends upon the relative position of various parts of a system. A spring has more potential energy when it is compressed or stretched. A steel ball has more

> potential energy raised above the ground than it has after falling to the Earth. In the raised position it is capable of doing more work. Potential energy is a property of a system and not of an individual body or particle, the system composed of the Earth and the raised ball, for example, has more potential energy as the two are farther separated. Potential energy arises in systems with parts that exert forces on each other of a magnitude dependent on the configuration, or relative position, of the parts. In the case of the Earth-ball system, the force of gravity between the two depends only on the distance separating them. The work done in separating them farther, or in raising the ball, transfers additional energy to the system, where it is stored as gravitational potential energy.

By definition, the internal force $\mathbf{F}_i$ derives from a potential function $E_P(\mathbf{R}, t)$, then the equation of motion for the reduced mass M becomes:

$$M\frac{d\mathbf{V}}{dt} = -\nabla E_P \tag{18}$$

One can multiply both sides of the above equation by $\mathbf{V}$, it follows

$$M\frac{d\mathbf{V}}{dt} \cdot \mathbf{V} = -\nabla E_P \cdot \mathbf{V} \tag{19}$$

The preceding equation can be rewritten in the following manner:

$$\frac{d}{dt}(\frac{1}{2}M\mathbf{V}^2 + E_P) = \frac{\partial E_P}{\partial t} \tag{20}$$

In the case where the potential energy is not an explicit function of time, the integration of the above equation gives the well-known conservation law of mechanical energy for the particle of mass M:

$$E_M = \frac{1}{2}M\mathbf{V}^2 + E_P = \text{Ct} \tag{21}$$

This equation defines an equation of transformation of one form of energy into another one. Most of our technology: motors and generators comply with the energy conservation principle because of Newton's third law. It is the reason why the efficiency of motors and generators can never be higher than 100% because they work as isolated systems. Therefore, we have conservation of the mechanical energy of a system only when the internal forces satisfy Newton's third law. There is no conservation law of energy if the forces do not follow Newton's third law or if the system is not isolated. In that case, the center of mass is accelerated since we have:

$$m\frac{d^2\mathbf{r}}{dt^2} = \mathbf{F}_e \neq 0 \tag{22}$$

The preceding equation must not be confused with the equation 16.

External forces which do not satisfy Newton's third law deserve special attention since one must recognize from the above calculation that there is no energy conservation principle for that kind of force from a classical point of view. Therefore, the existence of external forces implies the existence of closed and open systems where the energy is provided by other particles located outside the system or by the medium. The existence of external forces has an important consequence namely the possibility to do space propulsion using such a force as we shall see later.

The above partition of forces for rectilinear motion can be generalized to rotational motion when the Newton's forces are not central as in the case 2 of figure 1. We can decompose the angular momentum of a particle system in a sum of angular momentums of spin $\mathbf{L}_{ik} = \mathbf{R}_{ik} \wedge m_{ik}\mathbf{V}_{ik}$ and an orbital angular momentum associated to the center of mass of the system $\mathbf{L} = \mathbf{r} \wedge m\mathbf{U}$. These two different angular momentums can be associated with the corresponding angular momentum of the forces as follows:

$$\frac{d\mathbf{L}}{dt} = \mathbf{r} \wedge \mathbf{F} \qquad \frac{d\mathbf{L}_{ik}}{dt} = \mathbf{R}_{ik} \wedge [\mathbf{F}_{ik} + \frac{1}{m}(m_k\mathbf{F}_{ii} - m_i\mathbf{F}_{kk})] \tag{23}$$

The non central form of the Newton's third law is fundamental to justify the partition of rotational motion in two different kinds of rotation as for the rectilinear motion. The necessity to partition the forces is more obvious for rotation.

2.2 The partition of forces in thermodynamics

The first law of thermodynamics is a statement on the energy conservation. In an interesting paper, Erlichson [23] and more recently Guemez [24] note that the first law of thermodynamics in most textbooks is incorrectly enounced. The confusion concerning the first law of thermodynamics results from the fact that this law was not formulated directly from Newton's law as we shall see.

The splitting of the forces has been investigating in our book [18, p.100], the most general equation we can write for a system of N particles interacting with its surrounding is given by the equation

$$\frac{d}{dt}(E_K + E_I) = \frac{\partial E_{pm}}{\partial t} + \frac{dW}{dt} + \frac{dQ}{dt} \tag{24}$$

with the definitions:

$$E_K = \frac{1}{2}m\mathbf{U}^2 \qquad E_I = E_{km} + E_{pm} + E_m \tag{25}$$

where we have included the rest energies of the particles $E_m = m_i c^2$ in the definition of the internal energy E_I which is also called mechanical energy. The definitions of the microscopic kinetic and potential energies have for expression:

$$E_{km} = \frac{1}{2}\sum_{i=1}^{N}\sum_{i\neq k=1}^{N}\frac{1}{2}m_{ik}\mathbf{V}_{ik}^2 \qquad E_{pm} = \frac{1}{2}\sum_{i=1}^{N}\sum_{i\neq k=1}^{N}E_{ik}(R_{ik},t) \tag{26}$$

where the mutual forces $\mathbf{F}_{ik} = -\nabla_R E_{ik}$ derive from a potential function $E_{ik}[R_{ik}(t),t]$. The work and heat power terms are defined by the equations:

$$\frac{dW}{dt} = \mathbf{U}\cdot\sum_{i=1}^{N}\mathbf{F}_{ii} \qquad \frac{dQ}{dt} = \frac{dE_d}{dt} + \sum_{i=1}^{N}\sum_{i\neq k=1}^{N}\frac{1}{m}(m_k\mathbf{F}_{ii} - m_i\mathbf{F}_{kk})\cdot\mathbf{V}_{ik} \tag{27}$$

The dissipative forces such as friction and drag forces are internal to the system and do not depend on the reference frame considered. The dissipative forces are not conservative, they have for expression $\mathbf{F}_{ik} = -F(V_{ik})\mathbf{V}_{ik}$ where $\mathbf{V}_{ik} = \mathbf{U}_i - \mathbf{U}_k$ is the relative velocity and F a positive function which usually depends on the velocity $\mathbf{V}_{ik}$. The power term is always negative and has for expression:

$$\frac{dE_d}{dt} = -\frac{1}{2}\sum_{i=1}^{N}\sum_{i\neq k=1}^{N}F(V_{ik})\mathbf{V}_{ik}^2 \tag{28}$$

Equation 24 is the exact formulation of the first law of thermodynamics for a N particles system providing a balance equation for the power terms relating the rates of dissipative work, heat and proper work to the rates of the macroscopic kinetic energy and the microscopic internal energy. Some physicists add an external potential power term in equation 24 which is not correct as shown in the above discussion. When the system is isolated, we have $\mathbf{F}_{ii} = 0$ and the equation 24 becomes:

$$\frac{dE_M}{dt} = \frac{\partial E_{pm}}{\partial t} + \frac{dE_d}{dt} \tag{29}$$

where E_M is the mechanical energy which is the internal energy without the rest energies of the particles. If the potential energy does not depend explicitly on time and in the absence of dissipation, we get the famous conservation law of energy:

$$\frac{dE_M}{dt} = 0 \quad \Rightarrow \quad E_M = E_K + E_P = \mathrm{Ct} \tag{30}$$

The conservation law of mechanical energy simply states a reversible transformation of one form of energy in another one in the case of an isolated system.

This law is presented in the literature as the most important law of physics which has never been violated. However, no conservation law is defined in the literature for open and closed systems. We might think that equation 24 is such law but it is not the case as we shall see. There are lots of confusion and misleading claims in thermodynamics books that begin when we want to integrate equation 24. Note that this power equation is a balance equation since both members of equation 24 depend on space and time and therefore are not constant terms.

There are some ambiguities concerning the meaning of a potential function in mathematics and physics that is never stated in the literature. Let us consider a scalar function $E(\mathbf{r}, t)$ of several independent variables which has continuous first partial derivatives in a simply connected domain, in mathematics, the differential form of this function is given by the relation:

$$dE = \frac{\partial E}{\partial t} dt + \nabla E \cdot d\mathbf{r} \tag{31}$$

where the scalar function E is called a potential function.

In physics, we are interested to define the binary mutual interaction between two particles with the concept of work $dW = \mathbf{F}_{ik} \cdot d\mathbf{r}_i + \mathbf{F}_{ki} \cdot d\mathbf{r}_k$ for $i \neq k$. The first constraint is to impose the reciprocity of forces using Newton's third law which gives $\mathbf{F}_{ik} = -\mathbf{F}_{ki}$ and therefore $dW = \mathbf{F}_{ik} \cdot d\mathbf{R}_{ik}$ with the definition $\mathbf{R}_{ik} = \mathbf{r}_i - \mathbf{r}_k$. The question is to know when the mutual force is conservative or not. The reciprocity of forces is a necessary condition to define a conservative force but not a sufficient condition since we can build a reciprocal force from non reciprocal forces by taking into account the masses of the two particles. Potential energy in physics is defined from the position or configuration of the particles in the system. If a mutual force acting between two particles is a function only of the relative position of two particles then this function is said to be a conservative force if this force is also the gradient of the potential energy function $\mathbf{F}_{ik} = -\nabla E_{ki}$ where the potential function $E_{ki}[\mathbf{R}_{ik}, t]$ depends only upon the relative position of the particles.

Let us now define when a differential form is exact or not. The usual definition given in the literature concerns quantities that are function of the Eulerian coordinates $\mathbf{r}, t$. We have to extend the definition to functions of the path coordinates $\mathbf{R}(t), t$. The differential form is given by the expression:

$$dE = \frac{\partial E}{\partial t} dt + \nabla E \cdot d\mathbf{R} \tag{32}$$

This equation can be integrated in the following form:

$$E[\mathbf{R}(t), t] - E[\mathbf{R}_0(t_0), t_0] = \int_{t_0}^{t} a[\mathbf{R}(t), t']dt' + \int_{\mathbf{R}_0(t_0)}^{\mathbf{R}(t)} b[\mathbf{R}(t), t_0] \cdot d\mathbf{R} \tag{33}$$

with the conditions:

$$\frac{\partial E}{\partial t} = a[\mathbf{R}(t), t] \qquad \nabla E = \mathbf{b}[\mathbf{R}(t), t] \qquad \frac{\partial \mathbf{b}}{\partial t} = \nabla a \tag{34}$$

For a closed curve where $\mathbf{R}(t_0 + \Delta t) = \mathbf{R}_0(t_0)$, the integral equation above becomes

$$E[\mathbf{R}_0(t_0), t_0 + \Delta t] - E[\mathbf{R}_0(t_0), t_0] = \int_{t_0}^{t_0+\Delta t} a[\mathbf{R}(t), t']dt' \tag{35}$$

Even if the energy derives from a potential function, the energy is not conserved when the energy depends explicitly on time since the left hand side of the preceding equation is not zero unless the potential energy is a periodic function with the period Δt.

Let us give a few examples concerning the differential forms:

For a potential energy of the form $E[\mathbf{R}(t), t] = k(t)\mathbf{R}^2/2$, the differential form is exact since we verify the condition:

$$\nabla \frac{\partial E}{\partial t} = \frac{\partial}{\partial t} \nabla E = \frac{dk}{dt} \mathbf{R} \tag{36}$$

For a kinetic energy of the form $E[\mathbf{R}(t), t] = m(t)\mathbf{V}^2(\mathbf{R})/2$, the differential form is exact since we verify the condition:

$$\nabla \frac{\partial E}{\partial t} = \frac{\partial}{\partial t} \nabla E = \frac{1}{2} \frac{dm}{dt} \nabla(\mathbf{V}^2) \tag{37}$$

For a kinetic energy of the form $E[\mathbf{R}(t), t] = m\mathbf{V}^2[\mathbf{R}(t), t]/2$, the differential form is exact since we verify the condition:

$$\nabla \frac{\partial E}{\partial t} = \frac{\partial}{\partial t} \nabla E \tag{38}$$

This is easily proved if we use the identity:

$$\nabla(\mathbf{A} \cdot \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} + \mathbf{B} \wedge \nabla \wedge \mathbf{A} + (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A} \wedge \nabla \wedge \mathbf{B}$$

For a dissipative energy of the form $E[\mathbf{R}(t), t] = -a\mathbf{V}^2[\mathbf{R}(t), t]$, the differential form is exact since as with the preceding case we verify the condition:

$$\nabla \frac{\partial E}{\partial t} = \frac{\partial}{\partial t} \nabla E \tag{39}$$

Therefore, the dissipative energy is irreversible only if it does not depend explicitly on time.

If k and m and the velocities $\mathbf{V}[\mathbf{R}(t)]$ do not explicitly depend on time, then from the above definitions the microscopic kinetic and potential energies are not exact differential forms contrary to the statement made in the literature. However, in that case, the problem depends only on one space variable $\mathbf{R}(t)$], therefore, it is always possible to define an exact differential form for the microscopic kinetic and potential energies which is used to calculate the particular time derivative.

We note that the power terms for the work W and the heat Q in equations 27 are not conservative, therefore, dW and dQ are not exact differential forms which seem to contradict the fact that the mechanical energy differential dE_M is an exact differential form. If we do not take into account the kinetic power term as done in the literature, how is it possible that simply adding two inexact differential forms results in an exact one. The first answer to that question is to take into account the kinetic term but there is a second answer. Let us consider the relation $E = E_1 + E_2$ where we impose that the differential dE is an exact differential form, then we have the condition:

$$\nabla \frac{\partial E_1}{\partial t} - \frac{\partial}{\partial t} \nabla E_1 = - \left(\nabla \frac{\partial E_2}{\partial t} - \frac{\partial}{\partial t} \nabla E_2 \right) \tag{40}$$

If the two members of the above equation are zero then both E_1 and E_2 are exact differential forms, if not they are both inexact differential. The above condition tell us that both energies E_1 and E_2 are either reversible or irreversible quantities but both at the same time.

An interesting property of exact differential is that the integration depends only on the initial and final states of the system and is independent of the path followed by the particles. This is not the case for inexact differential terms where we have to solve a system of N line integrals on specific trajectories calculated simultaneously with the motion equations. This an impossible task to accomplish for a system of N particles, therefore, we have to define average quantities to solve the problem.

The first quantity is the temperature which is defined by taking the average kinetic energy of atoms or molecules in a system. The transfer of heat energy dQ caused by a temperature difference dT is defined by the relation $dQ = mCdT$ where m and C are the mass and specific heat of the system. It is important to note that C depends on the process carried out. Another example is the heat dQ dissipated in a time dt by a current I crossing a resistance R which is given by the relation $dQ = RI^2dt$. For the work W, we meet often the famous relation $dW = -PdV$. There are different definitions of Q and W in the literature that are currently subject of debate, in particular when irreversible processes are under study.

2.3 Case of the dissipative force

If dissipative forces are present, such as friction or dragging forces, we have to take into account the force usually of the form $\mathbf{F}_d = -a\mathbf{V}$ with $a > 0$ in the equation of motion:

$$\frac{d}{dt}\left(\frac{1}{2}MV^2\right) = (\mathbf{F}_i + \mathbf{F}_d) \cdot \mathbf{V} \tag{41}$$

We recover the first law of thermodynamics which provides the equivalence between dissipative work and heat, established long ago after the famous Joule's wheel paddle experiment. Therefore, the conservation law for the total energy becomes:

$$E_T = E_K + E_P + E_D = \mathrm{Ct} \tag{42}$$

where the term $E_D = a\mathbf{V}^2$ is the dissipative energy which results from the work of a non conservative force. Therefore, the mechanical energy is no more a constant if a dissipative force is present. Some authors consider the dissipative force as an external force which can decrease the mechanical energy of the system. The physical situation examined in the conservation law of mechanical energy is a reversible transformation of one form of energy in another one if the dissipative force is zero.

This equation defines what we call an equation of energy transfer between two independent systems. The fact that we have an equation with a sign equal is considered by some physicists as a conservation law of energy with the dissipative term depending on R taking into account. This is not the case since the equation is not related to the classical definition of the mechanical energy and has no direct connection to Newton's third law. Some authors such as Fuchs [26, p.20] defines rightfully this equation as a law of balance. This definition is well chosen since the equation of energy transfer depends on a function which quantifies this unbalance as the voltage difference between two capacitors, the difference of level of the fluids in two tanks or the temperature difference between two heat baths. The interesting fact is the possibility to minimize the heat losses during the transfer of energy between the source and receiver by processing the energy transfer by small steps to the point to almost recover the totality of the energy of the source as shown in the references [25,27,28]. Indeed, the integration of the equation 41 gives:

$$\frac{1}{2}MV^2(t) + E_P[\mathbf{R}(t)] - \left(\frac{1}{2}MV^2(0) + E_P[\mathbf{R}(0)]\right) = -\int_0^t aV^2(t')dt' \tag{43}$$

The term in the left hand side of the equation is the mechanical energy of the system which decreases due to the dissipative term. The dissipative term can be written in a different form if we consider a mass variation [25]:

$$\frac{d}{dt}\left(\frac{1}{2}m(t)V^2\right) = \frac{1}{2}\frac{dm}{dt}V^2 + m(t)V\frac{dV}{dt} \tag{44}$$

We obtain an equation that can be integrated:

$$\frac{1}{2}m(t)V^2(t) - \frac{1}{2}m(0)V^2(0) = \frac{1}{2}\int_0^t \frac{dm}{dt'}V^2(t')dt' + \int_0^t m(t')V(t')\frac{dV}{dt'}dt' \tag{45}$$

Using the equation of motion:

$$m(t)\frac{dV}{dt} = F_i - aV \tag{46}$$

we get:

$$\frac{1}{2}m(t)V^2(t) - \frac{1}{2}m(0)V^2(0) = \frac{1}{2}\int_0^t \frac{dm}{dt'}V^2(t')dt' + \int_0^t F_i V dt' - \int_0^t aV^2 dt' \tag{47}$$

If we substitute the preceding equation in equation 46, we obtain a final equation:

$$\left[\frac{1}{2}m\mathbf{V}^2 + E_P\right]_0^t = -\frac{1}{2}\int_0^t \frac{dm}{dt'}\mathbf{V}^2 dt' \tag{48}$$

The above equation is an exact mathematical formulation where the term in the left hand side of the equation is the classical mechanical energy of the system while in the right hand side, we have a dissipative term if the condition $dm/dt > 0$ is verified. The speed limit c of a charged particle in a particle accelerator results from the braking force included in the dissipative term. If $dm/dt = 0$, we recover the conservation law of energy in the laboratory frame. The above equation is important since we can cancel the dissipative term as explained in the references [25,28] opening the way to a break of the light speed barrier. However, to achieve that goal, the stepping process implies a very high switching process which is not technically easy to do, we were inspired to propose this approach by the experimental work of Gupta [29].

The experiment in a particle accelerator demonstrates without ambiguity that an accelerated charged particle gain a large kinetic energy as it approaches the speed of light which results in an apparent increase of mass $m(t) = \gamma(t)m_0$. However, we must point out that equation 46 is defined in the laboratory frame. Therefore, the gamma factor $\gamma(t)$ cannot result from the special relativity theory since no change of reference frame or space-time units and Lorentz transformation are concerned in the experiment where all the measurements are done in the laboratory frame.

2.4 The partition of forces in Special Relativity Theory

In this section, we deal with a controversial subject concerning the validity of the special relativity theory. Too often, this theory is the object of claims which are in direct contradiction not only with classical physics but also with experimental facts as we shall see. Some physicists defend the validity of this theory by making assertions more pertaining to religious faith than to science. Quite often, they are unable to refute the critics and prefer to manipulate the scientific truth by accusing opponents of doing pseudo-science. A good example of non-ethical behavior concerns the three experiments refuting the special relativity theory namely the Sagnac experiment [34], the Michelson-Gale experiment [35] and the Hafele and Keating experiment [36] which are quoted as experiments in agreement with the special relativity theory. These three experiments used the so-called Sagnac effect which was discovered by Sagnac in 1913.

The Sagnac experiment proves the anisotropy of the speed of light with respect to an absolute space since we have $c \pm v$, not c, where $v = \omega r$ is the rotational velocity of the disk in the Sagnac experiment or the Earth's surface in the Gale experiment. We can calculate the effect from a classical point of view by taking into account the travel time t_1 of light which moves with the rotation $ct_1 = 2\pi r + vt_1$ while the travel time t_2 for light moving against the rotation is $ct_2 = 2\pi r - vt_1$, the difference in travel time is:

$$t_1 - t_2 = \frac{\gamma^2}{c^2} 4\pi r^2 \omega \approx \frac{4\pi r}{c}(\beta + \beta^3) \tag{49}$$

With the definitions $\beta = v/c$ and $\gamma^2 = 1/(1-\beta^2)$. We recover the gamma factor γ^2 which has nothing to do with the special relativity theory since this factor results from a standing wave effect that we will recover again in the calculation of the Lienard potential. It is obvious from the preceding equation that the Sagnac effect is a first order effect in β. Therefore, the time dilation effect cannot explain the effect since the gamma factor contributes at a lower order in β^3. A recent paper published in 2015 [37] restates this evidence. The proponents of special relativity invented a discontinuity in simultaneity to explain the effect in spite of the fact that there is no clock and synchronisation procedure in this experiment. In spite of the experimental evidence, many authors claim that the Sagnac effect is a purely relativistic effect. These experiments imply that an observer can deduce his motion with respect to space by an internal experiment contrary to the teaching of the special relativity theory.

We can find in our book [18] a critical review of the problems raised by the special relativity theory. We complete this work in this section by giving simple proofs: mathematical, physical arguments and above all experimental results which prove that this theory cannot be correct.

Let us consider a lucky physicist who ignores the existence of the special relativity theory. He wants to solve the following equation to calculate the Lienard-Wiechert potential $\Phi(\mathbf{r}, t)$ induced by a charged particle moving with a constant velocity $\mathbf{U}$ with respect to the laboratory frame where our physicist is located

$$\Delta\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -4\pi q\delta(\mathbf{r} - \mathbf{V}t) \tag{50}$$

where $\mathbf{r}_0 = \mathbf{r} - \mathbf{V}t$ is the distance between the charge and the observation point $\mathbf{r}$. For a rectilinear motion in the direction z, the solution of the above equation [18, p.509] is:

$$\Phi(\mathbf{r}, t) = \frac{q}{[(z - Vt)^2 + r^2(1 - \beta^2)]^{1/2}} \tag{51}$$

With the definitions $\beta = V/c$ and $\gamma^2 = 1/(1 - \beta^2)$. The gamma factor in this equation has nothing to do with the special relativity theory as for the Sagnac effect because no change of reference frame, no change in space-time units, no Lorentz transformation are involved in this calculation. This gamma factor results from the presence of standing waves in vacuum [18, p.11] which is a logical explanation since the potential is a solution of a second order differential equation.

Bertozzi [18, p.306] performed an experiment in 1964 with a good accuracy in which the speed of electrons with kinetic energies in the range 0.5 to 15 MeV was determined by measuring the time required for the electrons to traverse a given distance while the kinetic energy $E_K = m_0c^2(\gamma - 1)$ was determined by calorimetry measurements. His result shows that the dependence of the kinetic energy on the speed of the electrons is in good agreement with the formula:

$$\beta^2 = 1 - \left(\frac{1}{1 + E_K/m_0c^2}\right)^2 \tag{52}$$

This experiment demonstrates without ambiguity that accelerated charged particles gain large kinetic energies as they approach the speed of light which results in an apparent increase of mass $m = \gamma(t)m_0$. One more time, we must point out that the above equation and the measurements are done in the laboratory frame. Therefore, the gamma factor $\gamma(t)$ cannot result from the special relativity theory: no change of reference frame or space-time units or Lorentz transformation are concerned in the measurements. Indeed, almost all the experiments dealing with the special relativity theory are done in the laboratory frame, that is to say, in the unique Earth reference frame where the quantities such as the position, the velocity and acceleration are defined and measured especially when these quantities are related to the Newton's third law.

The partition of forces is totally ignored in all textbooks dealing with the special relativity theory with the notable exception of the books by Landau [38, p.226] and Cornille [18, p.480]. As explained in several papers [17,18] this fact results from the covariance and the relativity principles which negate the Newton's third law. Indeed, it is often stated in the literature that the equality of action and reaction has no place in relativistic mechanics. For example, French [39, p.224] in his book states: The equality of action and reaction has almost no place in relativistic mechanics. It must essentially be a statement about the forces acting on two bodies, as a result of their mutual interaction at a given instant. And, because of the relativity of simultaneity, this phrase has no meaning.

The conservation laws of momentum and energy imply that Newton's third law must be satisfied. There is certainly a contradiction when relativistic physicists affirm that Newton's third law does not apply in relativistic dynamics and use it when dealing with the collision of relativistic particles. It is important to note that both particles are observed in the same reference frame, namely the laboratory frame. Therefore, no change of reference frame can be invoked to explain the contradiction. As pointed out by P. Beckmann [40, p.77]: The fact remains that the Einstein's theory has some explaining to do. For a theory that does not recognize the equality of action and reaction cannot, without apology, invoke the conservation of momentum

The covariance principle is also criticized as a principle which has no physical purpose as stated by Moussa [41, p.59]: We recall that classical forces are known in the frame linked to the Earth. Therefore if we keep that frame, the above relations have no practical use. Relativistic physicists invented an infinite set of inertial frames in relative rectilinear motion to one another where any experiment can be analyzed from these different frames where the observer can jump freely from one frame to another to do his own measurements which must satisfy the Lorentz transformation. This is a thought experiment which has never be done in the real world since all the measurements are done in the Earth reference frame where the velocity of the particle is defined. Physicists make a confusion between a frame and a reference frame since there are an infinity of frame where the coordinate system of the particle velocity can be projected.

Moreover, the relativity principle for inertial reference frames in relative motion is defeated by the existence of external forces. Brillouin [42, p.45] reached the same conclusion when he says that the usual statement of the relativity principle requires that frames of reference be extremely heavy to avoid the recoil effect. Einstein's relativity principle refers to laws of physics but initial conditions have to be taken into account. These initial conditions are "fact-like" rather than "law-like", they are not invariant since they depend on the external forces applied to the center of mass of the system. I suggest to any physicist who disagrees with the preceding analysis to jump from a train moving with a uniform velocity along a railroad and after a stay at the hospital to think again about the meaning of the relativity principle.

We can prove the absurdity of the covariance principal with a simple example by considering two twin walking at the same rate one on Earth and the other one in a train which has a rectilinear uniform velocity $\mathbf{U}$. The covariance principle in special relativity implies that the momentum $\mathbf{P}_0$ of the twin in the train is now $\mathbf{P}_1$ on Earth, it follows that there is a change of power which can be calculated with the Lorentz transformation:

$$\mathbf{U}_1 \cdot \mathbf{F}_1 = \frac{1}{D}(\epsilon \mathbf{U} \cdot \mathbf{F}_0 + \mathbf{U}_0 \cdot \mathbf{F}_0) \tag{53}$$

$$\mathbf{F}_1 = \frac{\gamma^{-1}}{D}\mathbf{F}_0 + \epsilon \left[\frac{\gamma}{c^2}\mathbf{U}_0 \cdot \mathbf{F}_0 + \frac{\epsilon}{\mathbf{U}^2}(\gamma - 1)\mathbf{U} \cdot \mathbf{F}_0\right] \mathbf{U} \tag{54}$$

with the following definitions:

$$D = 1 + \epsilon \mathbf{U} \cdot \mathbf{U}_0/c^2 \qquad \gamma = (1 - \mathbf{U}^2/c^2)^{-1/2} \tag{55}$$

where $\epsilon = \pm 1$ is a coefficient. In classical Newtonian mechanics, the kinetic energies of the twin is the same in the change reference frame while it is obvious from the above formulation that the kinetic energy of the twin in the train has not the same numerical value that his kinetic energy in the Earth frame. There is a violation of the conservation law of energy in the change of reference frame. The solution of the paradox is given in the reference [18, p.81].

The confusion get worst when dealing with the definitions of relative and absolute velocities. Did the relative and absolute velocities exclude one another or they exist at the same time since an absolute velocity is usually defined with respect to material bodies while a relative velocity is defined with respect to the ether. We will give the definitions based on mathematics, namely:

- An absolute velocity follows Newton's third law, its numerical value is invariant in a change of reference frame and does not depend on the existence of vacuum which is defined as an absolute space.

- A relative velocity does not follow Newton's third law, its numerical value is not invariant in a change of reference frame and can depend in a non reciprocal manner from other particles or vacuum if we define a reference frame supposed to be at rest with the vacuum.

Of course, both velocities coexist at the same time because of the partition of forces which implies the rejection of the Mach's principle and his critics about the Newton's concept of absolute space.

Indeed, the special relativity theory affirms that it is not possible to reveal our motion with respect to space by an internal experiment. However, there are at least 14 experiments from 1728 to 2007 [34-37,43-58] that contradict this assertion, proving that one can measure the motion of Earth by internal experiments. Depending on the kind of experiments and the accuracy achieved, one could measure the spin rotation, the orbital rotation, and the galactic motion of Earth through the ether. We must also stress that space cannot be relative when rectilinear motion is concerned and becomes absolute when rotational motion is observed. There is also a contradiction in the general theory of relativity when Einstein conceptualizes space as a medium and at the same time denies the existence of such medium in special relativity in his earlier papers.

2.5 The partition of forces in electromagnetism

The partition of forces is well-known in classical mechanics however most physicists do not acknowledge the fact that there is no conservation law of energy for external forces violating Newton's third law. As an example, consider the case of two charged particles q_1 and q_2 moving with velocities $\mathbf{U}_1$ and $\mathbf{U}_2$ relative to a given reference frame. The charge q_1 exerts on q_2 a Lorentz's force where the fields $\mathbf{E}_1$ and $\mathbf{B}_1$ are the electric and magnetic fields produced by q_1 at the position occupied by q_2. Conversely, the charge q_2 produces on q_1 a Lorentz's force. We get the partition:

$$\begin{aligned} \mathbf{F}_{12} &= -q_1 \nabla \Phi_2 \qquad \mathbf{F}_{11} = -\tfrac{q_1}{c}\left(\tfrac{\partial \mathbf{A}_2}{\partial t} - \mathbf{U}_1 \wedge \mathbf{B}_2\right) \\ \mathbf{F}_{21} &= -q_2 \nabla \Phi_1 \qquad \mathbf{F}_{22} = -\tfrac{q_2}{c}\left(\tfrac{\partial \mathbf{A}_1}{\partial t} - \mathbf{U}_2 \wedge \mathbf{B}_1\right) \end{aligned} \tag{56}$$

However, these two Lorentz's forces have different directions and magnitudes since we have:

$$\frac{d\mathbf{P}_1}{dt} + \frac{d\mathbf{P}_2}{dt} = \mathbf{F}_{11} + \mathbf{F}_{22} = \mathbf{F}_e \neq 0 \tag{58}$$

The above equation can be written in a relativistic form often encountered in the literature:

$$\frac{d}{dt}\left(m_1\gamma_1\mathbf{U}_1 + \frac{q_1}{c}\mathbf{A}_2\right) + \frac{d}{dt}\left(m_2\gamma_2\mathbf{U}_2 + \frac{q_2}{c}\mathbf{A}_1\right) = 0 \tag{59}$$

From this equation, one can deduce that field theory attributes momentum to the electromagnetic field to allow a particle to interact only with fields at the position of the particle. It precludes the possibility of instantaneous particle interactions except as an approximation. Therefore, the interaction between the particles proceeds by a transfer of momentum from one particle to the field, then the field transports the momentum at light speed to the position of the second particle where it can be transferred from field to the other particle. However, this transfer is not symmetric since the above equation can be rewritten as follows:

$$\frac{d\mathbf{P}_1}{dt} + \frac{d\mathbf{P}_2}{dt} = \mathbf{F}_e = -\frac{d}{dt}\left(\frac{q_1}{c}\mathbf{A}_2\right) - \frac{d}{dt}\left(\frac{q_2}{c}\mathbf{A}_1\right) \neq 0 \tag{60}$$

We can also define a relativistic equation of power for matter and radiation:

$$\frac{d}{dt}[(m_1\gamma_1 c^2 + q_1\Phi_2) + (m_2\gamma_2 c^2 + q_2\Phi_1)] = 2\frac{\partial E_P}{\partial t} + \frac{q_1}{c}\mathbf{A}_2 \cdot \frac{\partial \mathbf{U}_1}{\partial t} + \frac{q_2}{c}\mathbf{A}_1 \cdot \frac{\partial \mathbf{U}_2}{\partial t} \tag{61}$$

With a Lorenz gauge, the definition of the generalized potential E_P implies the equalities:

$$E_P = q_1(\Phi_2 - \frac{1}{c}\mathbf{U}_1 \cdot \mathbf{A}_2) = q_2(\Phi_1 - \frac{1}{c}\mathbf{U}_2 \cdot \mathbf{A}_1) \tag{62}$$

There is a violation of the conservation law of energy even if the potential and the velocities do not depend explicitly on time.

2.6 The partition of forces defined in a rotating reference frame

All the previous equations have been defined in a reference frame where the coordinate axes are fixed relative to distant stars. Now, we consider the well-known case where these equations are defined in a frame rotating around an axis with the angular velocity ω_e:

$$\mathbf{\Gamma}_m = \frac{d\mathbf{U}_r}{dt} + 2\boldsymbol{\omega}_e \wedge \mathbf{U}_r + \boldsymbol{\omega}_e \wedge (\boldsymbol{\omega}_e \wedge \mathbf{r}_m) + \frac{d\boldsymbol{\omega}_e}{dt} \wedge \mathbf{r}_m \tag{63}$$

The above equation represents the sum of four accelerations which are respectively known in the literature as: relative acceleration, Coriolis acceleration, centrifugal acceleration and angular acceleration. These forces are often defined in the literature as pseudo-forces or fictitious forces. We must emphasize that there is nothing fictitious about any of these forces. They give real physical effects since the pseudo forces are felt by the rotating observer but he can't find any interactive source concerning this effect and hence cannot identify it as an interaction force. This is the reason why some authors classify these forces as nonphysical forces since they arise from kinematics and are not due to physical interactions. For example, centrifugal force increases with distance r, whereas real forces always decrease with distance.

We can also quote the Foucault's pendulum experiment where an observer on Earth note that the plane of the pendulum would rotate and complete one full circuit in one sideral day. The plane is thus fixed with respect to the fixed stars and not to the Earth. The behavior of Foucault's pendulum is historically important since it was the first experiment in 1851, confirmed later by the Sagnac's experiment in 1913, as the most direct evidence that the Earth rotates.

Let us now consider the following identities

$$\boldsymbol{\omega}_e \wedge (\boldsymbol{\omega}_e \wedge \mathbf{r}_m) = (\boldsymbol{\omega}_e \cdot \mathbf{r}_m)\boldsymbol{\omega}_e - \omega_e^2 \mathbf{r}_m = \frac{1}{2}\nabla[(\boldsymbol{\omega}_e \cdot \mathbf{r}_m)^2 - \omega_e^2 \mathbf{r}_m^2] \tag{64}$$

$$(\boldsymbol{\omega}_e \wedge \mathbf{r}_m)^2 = \omega_e^2 \mathbf{r}_m^2 - (\boldsymbol{\omega}_e \cdot \mathbf{r}_m)^2 \tag{65}$$

It follows that the centrifugal force derives from a potential:

$$\boldsymbol{\omega}_e \wedge (\boldsymbol{\omega}_e \wedge \mathbf{r}_m) = -\frac{1}{2}\nabla(\boldsymbol{\omega}_e \wedge \mathbf{r}_m)^2 \tag{66}$$

By definition, we have the relations:

$$\mathbf{W}_m = \boldsymbol{\omega}_e \wedge \mathbf{r}_m \Rightarrow \nabla \wedge \mathbf{W}_m = 2\boldsymbol{\omega}_e \tag{67}$$

$$\frac{d\boldsymbol{\omega}_e}{dt} = \frac{\partial \boldsymbol{\omega}_e}{\partial t} \qquad \frac{\partial \mathbf{r}_m}{\partial t} = 0 \Rightarrow \frac{d\boldsymbol{\omega}_e}{dt} \wedge \mathbf{r}_m = \frac{\partial \mathbf{W}_m}{\partial t} \tag{68}$$

$$\frac{d\mathbf{U}_r}{dt} = \frac{\partial \mathbf{U}_r}{\partial t} + \frac{1}{2}\nabla \mathbf{U}_r^2 - \mathbf{U}_r \wedge \nabla \wedge \mathbf{U}_r \tag{69}$$

If we report all the relations previously defined in equation 63, we obtain the total acceleration defined in the rotating reference frame written in a form which reminds us of the expression of the Lorentz force [30-33]:

$$\frac{D\mathbf{U}_m}{Dt} = \frac{\partial \mathbf{U}_m}{\partial t} + \frac{1}{2}\nabla(\mathbf{U}_r^2 - \mathbf{W}_m^2) - \mathbf{U}_r \wedge \nabla \wedge \mathbf{U}_m \tag{70}$$

knowing that $\mathbf{U}_m = \mathbf{U}_r + \mathbf{W}_m$. Indeed, if we define $\mathbf{P} = m_0\mathbf{U}_m, q\Phi = -m_0(\mathbf{U}_r^2 - \mathbf{W}_m^2)/2$ and $q\mathbf{A}/c = -m_0\mathbf{U}_m$, the preceding equation turns into an equation of motion whose right-hand member is the Lorentz force:

$$\frac{d\mathbf{P}}{dt} = -\frac{q}{c}\frac{\partial \mathbf{A}}{\partial t} - q\nabla\Phi + \frac{q}{c}\mathbf{U}_r \wedge \nabla \wedge \mathbf{A} \tag{71}$$

The similitude between the equations 41 and 42 suggests that equation 41 can be partitioned in the same manner as donne in electromagnetism for the Lorentz force. We must point out a common mistake made by all authors in mechanics textbooks who add without precaution the components of classical forces defined in a fixed reference frame with the components of pseudo-forces defined in the rotating reference frame as explained in our book [18, p.42].

2.7 The partition of forces in Quantum Mechanics

Most textbooks dealing with quantum mechanics except in our book [18, p.193] do not speak on the partition of forces in this theory while many systems studied in quantum mechanics consist of two particles. Here are a few examples: the hydrogen atom's electron and proton, two atoms making a diatomic molecule, and the probe and target particles described in a scattering event or a collision. Let us consider the case of a scalar function $\Psi(\mathbf{r}_1, \mathbf{r}_2, t)$ depending on two space variables and one time variable for a system of two particles whose masses are m_1 and m_2.

$$m\mathbf{r} = m_1\mathbf{r}_1 + m_2\mathbf{r}_2 \qquad \mathbf{R} = \mathbf{r}_1 - \mathbf{r}_2 \tag{72}$$

where $m = m_1 + m_2$ is the total mass of the system. By definition, we have the equality $\Psi(\mathbf{r}_1, \mathbf{r}_2, t) = \Psi_0(\mathbf{R}, \mathbf{r}, t)$. After a few calculations, [18, p.194], we obtain the identity:

$$\frac{1}{m_1}\Delta_1\Psi + \frac{1}{m_2}\Delta_2\Psi = \frac{1}{m}\Delta_r\Psi_0 + \frac{1}{M}\Delta_R\Psi_0 \tag{73}$$

where $M = m_1 m_2/(m_1 + m_2)$ is the reduced mass.

The quantum mechanical treatment of the two-body problem begins with the Schrödinger equation written as follows:

$$\frac{1}{m_1}\Delta_1\Psi + \frac{1}{m_2}\Delta_2\Psi + \epsilon_j\frac{2}{\hbar}\frac{\partial\Psi}{\partial t} - \frac{2}{\hbar^2}E_P\Psi = 0 \tag{74}$$

where $E_P(\mathbf{r}_1, \mathbf{r}_2, t)$ is the potential energy of interaction between two particles whose rest masses are m_1 and m_2. Substituting the particular solution $\Psi(\mathbf{r}_1, \mathbf{r}_2, t) = \Psi(\mathbf{r}_1, \mathbf{r}_2)e^{j\omega t}$ in the preceding equation produces an inhomogeneous Helmholtz equation:

$$\frac{1}{m_1}\Delta_1\Psi + \frac{1}{m_2}\Delta_2\Psi + \frac{2}{\hbar^2}\left(E_T - E_P\right)\Psi = 0 \tag{75}$$

where we have used the Einstein relation $E_T = -\epsilon\hbar\omega$ with $\epsilon = \pm 1$ depending on the sign of E_T. The Schrödinger equation can now be split into parts relating to the center of mass $\mathbf{r}$ and to the relative position $\mathbf{R}$:

$$\frac{1}{m}\Delta_r\Psi_0 + \frac{1}{M}\Delta_R\Psi_0 + \frac{2}{\hbar^2}\left(E_T - E_P\right)\Psi_0 = 0 \tag{76}$$

By definition, the potential function $E_P(\mathbf{R})$ does not explicitly depend on time. Therefore, the total energy can be split into the sum of two constants $E_T = E_{tr} + E_{TR} = -\hbar(\epsilon_r\omega_r + \epsilon_R\omega_R)$. The wave function $\Psi_0(\mathbf{R}, \mathbf{r}) = \Psi_R(\mathbf{R})\Psi_r(\mathbf{r})$ can be factorized and substituted in the preceding equation.

When divided by $\Psi_R\Psi_r$, we get a sum of two terms which only depend respectively on $\mathbf{r}$ and $\mathbf{R}$. Therefore, each term must be equal to a constant which implies the system of equations:

$$\Delta_r\Psi_r + \frac{2m}{\hbar^2}E_{tr}\Psi_r = 0 \qquad \Delta_R\Psi_R + \frac{2M}{\hbar^2}\left(E_{TR} - E_P\right)\Psi_R = 0 \qquad (77)$$

The first equation describes the motion of a free particle in the absence of any external force. The second equation is used to calculate the stationary states of the electron in a hydrogen atom. Therefore, what we observe is how fundamental the Newton's third law is in quantum mechanics. Concerning the measurement theory in quantum mechanics, we must insist that there are two kinds of observations: those who perturb the behavior of system under scrutiny, and those who do not perturb the behavior of system under scrutiny. Of course, the perturbing measurement occurs when the observer is in mutual interaction with the system under scrutiny, which implies both reciprocity and Newton's third law, even if external forces are present. Claiming that an observer always interferes in the measurement process in quantum mechanics is simply untrue, because external forces are non reciprocal.

2.8 The partition of forces in gravitation

The law of gravitation was formulated by Newton and published in 1687. In accordance with this law, two point particles or two bodies attract each other with a force that is directly proportional to the masses of these bodies and inversely proportional to the square of the distance between them

$$\frac{d}{dt}MV = \mathbf{F}_i = -\nabla E_G \qquad E_G = -G\frac{m_1 m_2}{R} \qquad (78)$$

with the definitions:

$$M = \frac{m_1 m_2}{m_1 + m_2} \qquad \mathbf{R} = \mathbf{r}_1 - \mathbf{r}_2 \qquad \mathbf{V} = \frac{d\mathbf{R}}{dt} \qquad (79)$$

where the gravitational constant G has for value $G = 6.67\times10^{-11}Nm^2/kg^2$. We note that equation 78 with the concept of reduced mass M is the adequate formulation for the gravitational force since this force satisfies the Newton's third law. Therefore, this force follows the conservation law of energy. We can generalize Newton's gravitation theory to a system of N particles or bodies by using Jacobi's coordinates as defined in the references [2, p.164,18, p.102].

The masses m_1 and m_2 may depend on vacuum if we define inertia as a property of vacuum as explained in the references [59-64]. We can conceive that the gravitational attractive force as for the Casimir effect results from the pushing force due to the motion of zero point energy flowing into two material particles. Newton's law of gravitation was not adequate to account for certain gravitational experiments, namely: the motion of Mercury perihelion, the bending of a light ray, the gravitational red shift of spectroscopic lines. It was suggested in the references [65] to add a velocity term in the definition of the gravitational potential:

$$E_G = -G\frac{m_1 m_2}{R}\left(1 + \frac{\mathbf{V}^2}{c^2}\right) \tag{80}$$

to solve the problem. Assis in his book [19, p.321] by analogy with the Weber electrodynamics proposed another potential function:

$$E_G = -G\frac{m_1 m_2}{R}\left(1 - 3\frac{\mathbf{V}^2}{c^2}\right) \tag{81}$$

Newton's theory of gravitation does not address the motion of galaxies relative to the vacuum. In contrast, Einstein's general theory of relativity does not permit a splitting or partition of forces. This is understandable, given that in general relativity, the vacuum is treated not empty space, but as a dynamic medium capable of being deformed by the presence of mass and energy. Nevertheless, this omission is problematic, especially considering that the partitioning of forces is applicable to all other fundamental interactions, many of which are significantly stronger than gravity.

More critically, general relativity is a nonlinear and, under certain conditions, it violates the conservation of energy [66]. This is not a minor issue if energy conservation is considered the most fundamental law in physics. The question arises: why should gravitational forces be exempt from a principle that applies so broadly elsewhere? The partition of gravitational forces can also be examined from an experimental perspective. Observations show that light from distant galaxies is increasingly redshifted with distance, a phenomenon interpreted as a Doppler shift resulting from cosmic expansion. Cosmologists maintain that galaxies beyond the Milky Way appear to be receding from us at velocities nearly proportional to their distance, a relationship first predicted by Georges Lemaitre in 1927 [67] and confirmed observationally by Edwin Hubble in 1929 [68], now known as Hubble's Law.

More recently, it was discovered that the universe is not only expanding but doing so at an accelerating rate. This observation poses a challenge to both Newtonian and Einsteinian gravity, as both describe gravity as an inherently attractive force. The source of this acceleration remains a mystery.

When cosmologists refer to "recessional velocities," they typically do not specify whether this motion adheres to Newton's third law, a principle that underpins Mach's principle. If the motion is truly reciprocal, then it implies the presence of an internal repulsive force derived from an unknown potential. In this case, "dark energy", placeholder term referring to vacuum energy, would be an insufficient explanation. The only potential that satisfies all necessary conditions is the one derived from centrifugal effects as shown in equation 66, which increases with distance. This would imply the existence of a center of rotation in the universe.

If, on the other hand, the internal force between galaxies is zero, then the recessional velocity remains constant while the center of mass of the two galaxies accelerates relative to the vacuum, assuming an external force is acting. In such a case, energy would be supplied by the vacuum, as described by equation 22, leading to a violation of energy conservation. Due to the immense distances between galaxies, it is extremely difficult for cosmologists to distinguish between mutual acceleration and the acceleration of a system's center of mass, especially if the galaxies differ significantly in mass.

General relativity, as a field theory, is not well-suited to account for phenomena such as the cosmic microwave background radiation or the structure of the vast vacuum between galaxies. Alternative theories have been proposed, most notably plasma cosmology, which is based on electromagnetic forces and plasma currents, both of which are directly observed and measured at multiple scales in the universe. Proponents of the plasma cosmology claim that their model can solve many of the cosmos problems that astro-physicists are unable to solve.

One of the leading proponents of plasma cosmology was nuclear physicist Anthony Peratt [69], whose book Physics of the Plasma Universe (1992) helped popularize the model. Another major contributor was Hannes Alfven [70-72] who profoundly expanded our understanding of space electromagnetism. Alfven showed that plasma dynamics observed in laboratory conditions can be scaled up to galactic dimensions. These findings make it increasingly untenable to ignore the role of electric forces in space. Alfven explained the auroras, correctly described the Van Allen radiation belts, identified previously unrecognized electromagnetic attributes of Earth's magnetosphere, explained the structure of comet tails. Alfven meet great difficulties to gain acceptance of his ideas. Nevertheless, he was awarded the Nobel Prize in 1970 for his contribution in MHD. Alfven's pioneering work showed that plasmas (ionized gases with freely moving charged particles) are central to the behavior of the cosmos.

A third and fourth group of proponents are Wallace Thornhill and David Talbott, coauthors of The Electric Universe (2007) [73], which presents a comprehensive case for plasma-based cosmology. Plasma cosmology proposes that vast electric currents play a central role in structuring the universe from galaxies and galaxy clusters down to stars and planetary systems. Many of its principles are grounded in laboratory plasma experiments and electrical discharge behavior.

Laboratory-based research projects such as the Safire Project have explored the role of electricity in stellar and planetary systems, lending some credibility to plasma cosmology. While many mainstream astrophysicists dismiss these ideas as pseudoscience, it is an undeniable fact that plasma makes up over 99% of the observable universe and is capable of explaining electromagnetic emissions across the entire spectrum.

Plasma cosmology also accommodates the partition of forces in electromagnetism. Residual electrostatic or magnetic fields in space can generate repulsive forces strong enough to account for the observed accelerated expansion of the universe. For context, the electric force between charged particles is 39 orders of magnitude stronger than the gravitational force.

A concrete example of force partitioning in plasma physics involves the Barium release experiments [74-80] conducted in Earth's ionosphere. These experiments aimed to study Earth's electric field by releasing dense plasma clouds at altitudes with high conductivity. The behavior of these clouds, including edge striation and instabilities, offered insights into plasma dynamics. For a plasma cloud composed of ions, electrons, and neutral particles, three separate equations of motion are required to model their behavior accurately.

$$n_i m_i \frac{d\mathbf{U}_i}{dt} + \nabla(n_i k_b T_i) = Z n_i q_i(\mathbf{E} + \frac{1}{c}\mathbf{U}_i \wedge \mathbf{B}) + \mathbf{F}_{ie} + \mathbf{F}_{in} \tag{82}$$

$$n_e m_e \frac{d\mathbf{U}_e}{dt} + \nabla(n_e k_b T_e) = n_e q_e(\mathbf{E} + \frac{1}{c}\mathbf{U}_e \wedge \mathbf{B}) + \mathbf{F}_{ei} + \mathbf{F}_{en} \tag{83}$$

$$n_n m_n \frac{d\mathbf{U}_n}{dt} + \nabla(n_n k_b T_n) = \mathbf{F}_{ne} + \mathbf{F}_{ni} \tag{84}$$

with the definition $\mathbf{F}_{kl} = -n_k m_k \omega_{kl}(\mathbf{U}_k - \mathbf{U}_l)$

We assume now that the neutral particles have a rectilinear uniform motion where the constant velocity $\mathbf{U}_n$ is defined with respect to the center of mass of the Sun. Then the preceding equations become:

$$n_i m_i \frac{d\mathbf{V}_{in}}{dt} + \nabla(n_i k_b T_i) = Z n_i q_i(\mathbf{E}_n + \frac{1}{c}\mathbf{V}_{in} \wedge \mathbf{B}) - n_i m_i \omega_{in} \mathbf{V}_{in} \tag{85}$$

$$n_e m_e \frac{d\mathbf{V}_{en}}{dt} + \nabla(n_e k_b T_e) = n_e q_e(\mathbf{E}_n + \frac{1}{c}\mathbf{V}_{en} \wedge \mathbf{B}) - n_e m_e \omega_{en} \mathbf{V}_{en} \tag{86}$$

with the definitions $\mathbf{U}_i = \mathbf{U}_n + \mathbf{V}_{in}$, $\mathbf{U}_e = \mathbf{U}_n + \mathbf{V}_{en}$ and $\mathbf{E}_n = \mathbf{E} + \mathbf{U}_n \wedge \mathbf{B}/c$. This set of equations is often used in the literature to analyze the striation effect.

The partition of forces in physics with the existence of external forces implies to question the role plays by the vacuum from the particle level to the cosmic scale. If we consider vacuum or ether or dark energy, whatever the name, as an empty space not only without matter and radiation but also without some mysterious substance then we cannot define vacuum as a medium.

This is not a trivial question as we shall demonstrate hereafter. The abolition of the ether concept is often credited to Einstein. This was not Einstein's position, on the contrary, in a conference held in Leyden's University in 1920 [81], he stated the absolute necessity of the ether and he was particularly clear on this point when he said:

- (p.68) The negation of ether is not necessary required by the principle of special relativity. We can admit the existence of ether but we have to give up to attribute it a particular motion.

- (p.69) The hypothesis of the ether as such does not contradict the theory of special relativity.

- (p.74) From the theory of general relativity space is endowed with physical properties, in that case therefore an ether exists. According to the theory of general relativity, space without ether is inconceivable because the propagation of light would be impossible.

The question about the compatibility of the existence of an ether with relativity has been also considered by Dirac [82,83] who presented the concept of a covariant ether. The position of Einstein is quite logic because if vacuum is not a medium how can we imagine the space-time curvature of vacuum since coordinate lines are not physically present in vacuum. Cosmologists avoid carefully to answer the question.

There is also another problem that most physicists avoid to speak concerning the radiation of transverse electromagnetic waves in the cosmos. A transverse wave can only travel through solid, then why transverse light wave can travel through air and vacuum. This implies that the ether must behave as if it were an elastic solid with a rigidity which had to be incredibly high in order to transmit waves at the fantastic speed of light. On logical grounds, such a medium was compelled to slow down planetary motions around the Sun.

A search in the scientific literature concerning the answer to that question was given by Bekefi [84, p.150] in plasma physics where the author notes that a longitudinal wave can create a transverse wave: In an infinite homogeneous plasma, the energy exchange between longitudinal and transverse waves occurs at the microscopic level, and is essentially the result of the medium's grainy nature. Thus, it is not necessary to consider the vacuum as a kind of elastic solid to sustain transverse waves. We give a more complete answer to this question in several papers published from 1990 to 1994 [7,8,18,85] where we assume the presence of scalar inhomogeneous waves in vacuum. This scalar longitudinal field is characterized by the definition of the phase

$$\phi(\mathbf{r},t)=\int_{t_0}^{t}\alpha(\mathbf{r},t')\omega(\mathbf{r},t')dt'-\int_{\mathbf{r}_0}^{\mathbf{r}}\alpha(\mathbf{r}',t_0)\mathbf{k}(\mathbf{r}',t_0)\cdot d\mathbf{r}' \tag{87}$$

where the quantity $\alpha(\mathbf{r},t)$ is an integrating factor. In this approach, there is no space-time curvature of vacuum but a space-time deformation of the standing longitudinal waves making the vacuum. This approach has two merits: first if a Fourier mode is solution of a scalar wave equation, then one can deduce the Maxwell's equations [18, p.356] by using successively more complicated potential definitions, hence sound generates light so to speak. Secondly, we prove that the deformation of the scalar wave is quantized. Thus the picture of light as a transverse vibration in the ether, analogous to transverse waves on a string, can be reconciled with the existence of the ether. We can conclude this section by pointing out the fact that numerous experiments have proved that vacuum is a vibrational medium able to explain the Casimir effect, the Lamb shift, and the Van de Waals forces.

3 Definition of the concept of work

3.1 Work of the force along an arbitrary trajectory

The action of a force $\mathbf{F}[\mathbf{r}(t),t]$ on a particle over a period of time dt produces a motion $d\mathbf{r}$ of the particle along a trajectory. The work $dW=\mathbf{F}\cdot d\mathbf{r}$ of this force is defined by taking the scalar product of the force and the associated motion $d\mathbf{r}$. The integral of work on a trajectory give the variation of kinetic energy:

$$W=E_K[\mathbf{r}(t),t]-E_K[\mathbf{r}(t_0),t_0]=\int_{\mathbf{r}(t_0)}^{\mathbf{r}(t)}\mathbf{F}[\mathbf{r}'(t),t]\cdot d\mathbf{r}' \tag{88}$$

It is important to point out that the definition of work is independent of the nature of the force. If we substitute $d\mathbf{r}'=\mathbf{U}[\mathbf{r}'(t),t]dt'$ in the preceding integral, we obtain:

$$E_K[\mathbf{r}(t),t]-E_K[\mathbf{r}(t_0),t_0]=\int_{t_0}^{t}\mathbf{F}\cdot\mathbf{U}dt' \tag{89}$$

This gives the relation:

$$\frac{dE_K}{dt}=\mathbf{U}\cdot\mathbf{F} \tag{90}$$

The preceding relation can de deduced from Newton's second law. Its expression will depend on the definition of kinetic energy. In classical mechanics, $E_K=m_0\mathbf{U}^2/2$, thus resulting in Newton's second law:

$$\frac{d\mathbf{P}}{dt} = \mathbf{F} \tag{91}$$

with the definition $\mathbf{P} = m_0\mathbf{U}$

We know that a macroscopic magnetic force does work since we can define a magnetic potential function $E_B = -\mathbf{M} \cdot \mathbf{B}$ or the potential energy for two magnetic dipoles with moments $\mathbf{M}_1$ and $\mathbf{M}_2$ which has for expression:

$$E_B[\mathbf{R}(t)] = \frac{1}{R^3}\left[\mathbf{M}_1 \cdot \mathbf{M}_2 - \frac{3}{R^2}\left(\mathbf{M}_1 \cdot \mathbf{R}\right)\left(\mathbf{M}_2 \cdot \mathbf{R}\right)\right] \tag{92}$$

However, at the microscopic scale, we know that the work of a magnetic force applied to a free charge q moving with the velocity $\mathbf{V}$ in a given reference frame does not work since we have:

$$\int_{\mathbf{r}(t_0)}^{\mathbf{r}(t)} \mathbf{F}[\mathbf{r}'(t), t] \cdot \delta\mathbf{r}' = \int_{\mathbf{r}(t_0)}^{\mathbf{r}(t)} q\left(\mathbf{E} + \mathbf{V} \wedge \mathbf{B}/c\right) \cdot \delta\mathbf{r}' = \int_{\mathbf{r}(t_0)}^{\mathbf{r}(t)} q\mathbf{E} \cdot \delta\mathbf{r}' \tag{93}$$

The contribution of the magnetic force is zero since we have the definition $\mathbf{V} = \delta\mathbf{r}'/dt$. This has been obtained because we take in consideration only one charge in the calculation.

Let us now consider the case of two charges q_e and q_i with the condition $q_e = -q_i$ moving with the same velocity $\mathbf{U}_e = \mathbf{U}_i$ (rigid motion) with respect to a given reference frame. By using classical Maxwell's equations, one can demonstrate that the magnetic forces of interaction between the two charges bounded together by a dielectric of thickness R are given by the relations:

$$\mathbf{B}_e = \frac{q_e}{cR^3}\mathbf{U}_e \wedge \mathbf{R}_{ei} \qquad \mathbf{B}_i = \frac{q_i}{cR^3}\mathbf{U}_i \wedge \mathbf{R}_{ie} \tag{94}$$

$$\mathbf{F}_{ie} = \frac{q_i}{c}\mathbf{U}_i \wedge \mathbf{B}_e \qquad \mathbf{F}_{ei} = \frac{q_e}{c}\mathbf{U}_e \wedge \mathbf{B}_i \tag{95}$$

knowing that $\mathbf{R}_{ei} = -\mathbf{R}_{ie}$ and $\mathbf{U}_e = \mathbf{U}_i$, we have $\mathbf{B}_e = -\mathbf{B}_i$ and $\mathbf{F}_{ei} = -\mathbf{F}_{ie}$. Therefore, the external force $\mathbf{F}_e = \mathbf{F}_{ei} + \mathbf{F}_{ie}$ and the work $dW = \mathbf{F}_e \cdot d\mathbf{r}$ are zero.

This is no more the case if the velocities of the particles are different since now we have $\mathbf{U}_e = \mathbf{U}_i + \mathbf{V}$ where $\mathbf{V}$ is the velocity of electrons defined with respect to the rest frame of the ions. If $md\mathbf{r}/dt = m\mathbf{U} = m_e\mathbf{U}_e + m_i\mathbf{U}_i$ defines the motion of the center of mass of the two particles, then the work of the external force $\mathbf{F}_e = \mathbf{F}_{ei} + \mathbf{F}_{ie}$ becomes

$$dW = \mathbf{F}_e \cdot d\mathbf{r} = \frac{q_i^2}{c^2R^3}\left[(\mathbf{U}_i \cdot \mathbf{R}_{ie})\mathbf{V} - (\mathbf{V} \cdot \mathbf{R}_{ie})\mathbf{U}_i\right] \cdot d\mathbf{r} \neq 0 \tag{96}$$

This work is not null, therefore the statement that a magnetic force does not work only applies to the definition of work along a trajectory for free individual particles. This subject has been discussed in several papers [86-89]. This point of view is important since the partition of forces involves at least two particles and can be applied to the concept of work. Indeed, the partition of forces is clearly stated in the reference [90] where the author says: In brief, the key point is that there are two major categories of work, center-of-mass work and particle work. Therefore, the partition of forces gives two relations:

$$E_K[\mathbf{r}(t), t] - E_K[\mathbf{r}(t_0), t_0] = \int_{\mathbf{r}(t_0)}^{\mathbf{r}(t)} \mathbf{F}_e[\mathbf{r}'(t), t] \cdot d\mathbf{r}' \tag{97}$$

$$E_K[\mathbf{R}(t)] - E_K[\mathbf{R}(t_0)] = \int_{\mathbf{R}(t_0)}^{\mathbf{R}(t)} \mathbf{F}_i[\mathbf{R}'(t)] \cdot d\mathbf{R}' \tag{98}$$

If we make the substitutions $d\mathbf{R}' = \mathbf{V}[\mathbf{R}'(t)]dt'$ and $\mathbf{F}_i = -\nabla E_P[\mathbf{R}(t)]$ in the preceding integral, we recover at once the conservation law of mechanical energy:

$$E_K[\mathbf{R}(t)] - E_K[\mathbf{R}(t_0)] = -\int_{\mathbf{R}(t_0)}^{\mathbf{R}(t)} \nabla E_P[\mathbf{R}'(t)] \cdot d\mathbf{R}' = -(E_P[\mathbf{R}(t)] - E_P[\mathbf{R}(t_0)]) \tag{99}$$

The definition of work does not bring anything new with respect to the differential approach. However, the preceding equation is useful to prove easily that any force deriving from a potential function gives zero work on a closed curve. The equations 97 and 98 have the same mathematical form but not the same meaning since the numerical values of the quantities in equation 97 depend on the choice of a reference frame while for equation 98, the quantities are invariant. Depending on how we define an isolated system, we give an example hereafter for the smot experiment, some physicists use the word external force applied to a system for a force deriving from a potential. In that case, we must use the Jacobi coordinates [18, p.102] to avoid confusion with the present partition where the external force always violates Newton's third law.

The work equation 97 is sometimes called "pseudo-work energy equation" by Sherwood and others authors [91-93]. This is a misnomer and we must keep the term "pseudo-work" for the definition hereafter. I think the authors use unconsciously this term to avoid to point out the violation of the conservation law of energy for external forces. Indeed, the author of the reference [92, p.1001] is particularly clear when he says these three equations provide an additional proof that the CM equation is not really an energy equation. The similarity between the work equations 97 and 98 seems to imply that we have defined a conservation law of energy for equation 97. However, for a closed curve, the right hand side of equation 97 is not zero, therefore, the function in the left hand side becomes a multivalue function.

The ambiguity comes from the word "conservation". Let us clarify the subject by considering the case of a physicist who fills the tank of his car with gas and who drives his car until the tank is empty. The driver will say that he has transformed the energy of the gas into kinetic energy and heat due to mechanical frictions in the motor. He will state the conservation of energy, that is to say, the constancy of energy with respect to time in the transformation of one form of energy in another one. The driver did not care about the motion through space of the center of mass of his car and Earth. The energy used by this motion whether the force applied to the center of mass violates or not the Newton's third law, cannot be a conserved quantity for the driver since this energy is either not constant or not brought by the driver. We will review again this problem when we will analyze the electrostatic pendulum.

3.2 Work of a force along a moving curve

There exists another definition of work that we call "pseudo-work" which is different from the preceding definition. Curiously, the fact that those two definitions of work differ is never explicitly underlined in the literature, even though this second definition is currently used in electromagnetism and magnetic motors. This definition concerns the case of work performed by forces applied to particles which are obliged to move along a curve $C(t)$ while this curve moves in the laboratory frame with the velocity $\mathbf{U} = d\mathbf{r}/dt$. This new work definition is used to calculate the induction force $\mathbf{F}_i = q\mathbf{E}_i$ applied to each free charge q present in the circuit $C(t)$ when the circuit is submitted to an external magnetic field. In order to demonstrate the induction force, it suffices to consider the Faraday law written as a function of the vector potential:

$$q\int_{C(t)} \mathbf{E}_i \cdot \delta\mathbf{r} = \int_{C(t)} \mathbf{F}_i \cdot \delta\mathbf{r} = -\frac{q}{c}\frac{d}{dt}\int_{C(t)} \mathbf{A} \cdot \delta\mathbf{r} \tag{100}$$

$$\frac{d}{dt}\int_{C(t)} \mathbf{A} \cdot \delta\mathbf{r} = \int_{C(t)} \left[\frac{\partial \mathbf{A}}{\partial t} + \nabla(\mathbf{U}\cdot\mathbf{A}) + (\nabla \wedge \mathbf{A}) \wedge \mathbf{U}\right] \cdot \delta\mathbf{r} \tag{101}$$

We can rewrite equation 100 by using the preceding definition, we get:

$$q\int_{C(t)} \mathbf{E}_i \cdot \delta\mathbf{r} = q\int_{C(t)} \left[-\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \frac{1}{c}\nabla(\mathbf{U}\cdot\mathbf{A}) + \frac{1}{c}\mathbf{U} \wedge \nabla \wedge \mathbf{A}\right] \cdot \delta\mathbf{r} \tag{102}$$

We observe that the work delivered by the electromotive force $\mathbf{F}_i$ along a closed circuit is not zero. Moreover, the work of the magnetic force is not zero $(\mathbf{U} \wedge \nabla \wedge \mathbf{A}) \cdot \delta\mathbf{r} \neq 0$

The vector $\mathbf{F}_i$ looks like a Lorentz force

$$\mathbf{F}_i = q\left[-\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \nabla(\Phi_0 + \frac{1}{c}\mathbf{U}\cdot\mathbf{A}) + \frac{1}{c}\mathbf{U}\wedge\nabla\wedge\mathbf{A}\right] \quad (103)$$

with the following definitions:

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \nabla(\Phi_0 + \frac{1}{c}\mathbf{U}\cdot\mathbf{A}) \qquad \mathbf{B} = \nabla\wedge\mathbf{A} \quad (104)$$

where the presence of the scalar potential Φ_0 in the definition of $\mathbf{E}$ originates from the fact that the induced electric field $\mathbf{E}_i$ is defined a gradient apart. However, the induction force, as written above and calculated on the basis of the integral formulation of Maxwell's equations, does not match with the classical expression of the Lorentz force

$$\mathbf{F}_l = q\left(-\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \nabla\Phi + \frac{1}{c}\mathbf{U}\wedge\nabla\wedge\mathbf{A}\right) \quad (105)$$

for two reasons:

The first reason is that the equality $\Phi = \Phi_0 + (\mathbf{U}\cdot\mathbf{A})/c$ is not necessarily verified. However, the difference from these two terms disappears as the work associated to gradients cancels for closed circuits.

The second reason deals with the meaning of the charge velocity in the expression of the magnetic force since for the Lorentz force, the velocity of the charge q is $\mathbf{U}$ while in the classical induction force expression, the velocity of the charge q is $\mathbf{V} = \delta\mathbf{r}/\delta t$ which defines the electronic current along the circuit $C(t)$. The velocities $\mathbf{U}$ and $\mathbf{V}$ are different, which makes it possible for the induced magnetic force to perform work $\mathbf{F}_i\cdot\delta\mathbf{r} \neq 0$, contrary to the case of the magnetic Lorentz force $\mathbf{F}_l\cdot d\mathbf{r} = 0$ whose work is always null. We recall that all the quantities defined in the above equations must be defined in the reference frame where the velocity $\mathbf{U}$ is defined.

3.3 Work of a force in the case of a rotating frame

We will now examine the work for forces defined in a rotating frame. We recall the expression given in equation 70:

$$m\frac{D\mathbf{U}_m}{Dt} = \mathbf{F}_p = m\left(\frac{\partial \mathbf{U}_m}{\partial t} + \frac{1}{2}\nabla[\mathbf{U}_r^2 - (\boldsymbol{\omega}_e\wedge\mathbf{r})^2] - \mathbf{U}_r\wedge\nabla\wedge\mathbf{U}_m\right) \quad (106)$$

knowing that $\mathbf{U}_m = \mathbf{U}_r + \boldsymbol{\omega}_e\wedge\mathbf{r}$ with the definition $\mathbf{U}_r = \delta\mathbf{r}/dt$. We can now calculate the work of the force $\mathbf{F}_p$ in the rotating reference frame.

$$W_p = \int_{\mathbf{r}(t_0)}^{\mathbf{r}(t)} \mathbf{F}_p(\mathbf{r},t)\cdot\delta\mathbf{r} \quad (107)$$

where we have the condition $(\mathbf{U}_r \wedge \nabla \wedge \mathbf{U}_m) \cdot \delta\mathbf{r} = 0$. Let us now assume that there is a force $\mathbf{F}_a$ defined in the fixed frame applied to the point particle of mass m, then the most general relation to define work in the fixed reference frame is

$$W = \int_{\mathbf{r}(t_0)}^{\mathbf{r}(t)} (\mathbf{F}_a + \mathbf{F}_p) \cdot d\mathbf{r} \tag{108}$$

With the definition $d\mathbf{r} = \delta\mathbf{r}_r + \boldsymbol{\omega}_e \wedge \delta\mathbf{r}dt$. As for the magnetic force, the Coriolis force will work in the fixed frame. We recall that we have to take into account a rotational matrix for the force $\mathbf{F}_p$ to add the components of both vectors. Some authors explained the macroscopic work of the magnetic or the Coriolis force by the hidden work of the electrostatic or the centrifugal force which both derive from potential functions. However, the result is the same, the work is not zero because if this work is zero then no magnetic motor in the world would work.

There is a formal analogy between equations 103,105 and 106, however, these equations differ on a fundamental point, namely the equations 103 and 105 are defined in a fixed frame while equation 106 is defined in the rotating frame. Therefore, we can ask the question whether the rotating frame is a mathematical frame or a physical frame which has mass which is a legitimate question if these forces are considered by some authors to be fictitious. After a search in Internet, we find out a series of interesting papers written by several authors [94,95] and Tombe [96-98] who deals with the subject.

Tombe considers a point particle in motion in a fixed polar frame. Then, he writes the position vector $\mathbf{r} = r\mathbf{e}_r$ relative to a chosen polar origin where the unit vector $\mathbf{e}_r$ is in the radial direction and where r is the radial distance. Taking the time derivative and using the product rule, we obtain the velocity term $d\mathbf{r}/dt = \dot{r}\mathbf{e}_r + r\omega\mathbf{e}_\theta$ where $\mathbf{e}_\theta$ is the unit vector in the transverse direction. Taking the time derivative for a second time, we obtain the expression for acceleration in the fixed polar frame:

$$\frac{d^2\mathbf{r}}{dt^2} = (\frac{d^2r}{dt^2} - r\omega^2)\mathbf{e}_r + (2\omega\frac{dr}{dt} + r\frac{d\omega}{dt})\mathbf{e}_\theta \tag{109}$$

The author notes that centrifugal and Coriolis terms both appear in the above equation. Mainstream physicists will however deny that these centrifugal and Coriolis terms correspond in any way to the fictitious centrifugal and Coriolis forces which they promote in the literature for rotating frames. However, the argumentation given by Tombe is partially correct since a coordinate system need not have any relationship to a given reference frame. But although a reference frame and a coordinates system are not related, a choice of a reference frame with its origin is needed to define the position vector where the components of the vector can be projected in any polar frame where its origin can be the point particle of the center of rotation.

4 Superposition of fields and conservation law of energy

A medium is said to be linear if it obeys the linear superposition principle, namely the field due to several sources is the sum of the fields produced by each source. This principle is a consequence of the linearity of the wave equation in the medium. As pointed out by Jackson [99, p.10] this principle is exploited so often in electromagnetism and in quantum mechanics that it is taken for granted. There are, of course, circumstances where non-linear effects occur but here we are only concerned with fields in the vacuum at the microscopic level inside atoms and nuclei. However, this principle seems to not apply to field energy. Indeed, the total electric energy associated to the total electric field $\mathbf{E}(\mathbf{r},t) = \mathbf{E}_1(\mathbf{r},t) + \mathbf{E}_2(\mathbf{r},t)$ has for expression:

$$8\pi E_T = \int_V \mathbf{E}_1^2 \delta V + \int_V \mathbf{E}_2^2 \delta V + 2\int_V \mathbf{E}_1 \cdot \mathbf{E}_2 \delta V \tag{110}$$

In most textbooks in physics, the non-linearity of field energy is not discussed. Even in the professional literature, we have been able to find only a few relevant papers [100-108] dealing with the subject. Before examining the compatibility of the principles of superposition and energy conservation, let us discussed two simple examples.

4.1 Case of quasi electrostatic fields

The electric fields at the point $\mathbf{r}$ due to two charges q_1 and q_2 respectively located at $\mathbf{r}_i(t)$ in the laboratory frame are given for $\mathbf{U}_i(t)/c = (d\mathbf{r}_i/dt)/c \ll 1$ by the expressions:

$$\mathbf{E}_1(\mathbf{r}) = q_1 \frac{\mathbf{R}_1}{R_1^3} \quad \text{with} \quad \mathbf{R}_1 = \mathbf{r} - \mathbf{r}_1 \tag{111}$$

$$\mathbf{E}_2(\mathbf{r}) = q_2 \frac{\mathbf{R}_2}{R_2^3} \quad \text{with} \quad \mathbf{R}_2 = \mathbf{r} - \mathbf{r}_2 \tag{112}$$

The linear superposition of fields means that the total electric field is the vector sum of the above two fields. The total energy can be calculated by squaring the total electric field. As it is well-known, the proper energies become infinite if the particles are reduced to points. However, the potential energy calculated from the relation

$$E_P = \frac{1}{4\pi}\int_V \mathbf{E}_1 \cdot \mathbf{E}_2 \delta V = \frac{q_1 q_2}{4\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{\mathbf{R}_1 \cdot \mathbf{R}_2}{R_1^3 R_2^3}\delta \mathbf{r}^3 = \frac{q_1 q_2}{R_{12}} \tag{113}$$

is finite since the definition $\mathbf{R}_{12} = \mathbf{r}_1 - \mathbf{r}_2$ implies $\mathbf{R}_{12} \neq 0$ because two particle points do not superpose. By reading the literature on this subject, one get the impression that the mutual energy term is the cause of the violation of the conservation law of energy. This is just the opposite case. The internal force $\mathbf{F}_i$ between two particles is derivable from the potential function $E_P(\mathbf{R})$. Therefore, with the definition $\mathbf{V}(t) = d\mathbf{R}_{12}/dt$, the equation of motion for the reduced mass M has for expression:

$$M\frac{d\mathbf{V}}{dt} = -\nabla E_P \tag{114}$$

One can multiply both sides of the above equation by $\mathbf{V}$ to get a conservation law of power (and energy)

$$\frac{d}{dt}\left(\frac{1}{2}M\mathbf{V}^2 + E_P\right) = 0 \tag{115}$$

By definition, we have $P = dE/dt$, therefore we have the identity:

$$\frac{dE_T}{dt} = \frac{dE_{11}}{dt} + \frac{dE_{22}}{dt} + 2\frac{dE_{12}}{dt} = P_{11} + P_{22} + 2P_{12} \tag{116}$$

where the terms E_{ii} and E_{ij} are respectively the self energy and the mutual energies of the fields. We recover the equations 11 and 12 of our preceding discussion where the mutual energy term $2E_{12}$ satisfies the conservation law of energy because of the existence of the potential function. We can follow two different view points by telling that the superposition of fields violates the conservation law of power (and energy) due to the presence of the proper terms or we can also say that the superposition of the fields defines another conservation law of power (and energy) by taking into account the proper terms. We will discuss hereafter several experiments that show that the two points of view are not totally equivalent.

4.2 Case of light wave interferences

One of the first paper dealing with the paradox was written in 1950 [100], after that, only a few papers [101-108] can be found in the scientific literature dealing with the conservation law of energy for light wave interferences. The subject was fully analyzed by Vajda in 1998 [107] and by Cornille in 1999 [2,18]. The problem with light or sound sources is more difficult to understand than the preceding case since the two sources are fixed. Therefore, neither a relative motion between them nor a recoil of the sources upon radiation is permitted. The phenomenon of light interference in space and time results from the superposition of two vibrations defined by its monochromatic electric field. After calculation [2,18] and averaging on time, we get the relation:

$$< 2E_t(\mathbf{r}, t) >= \mathbf{E}_{01}^2(\mathbf{r}) + \mathbf{E}_{02}^2(\mathbf{r}) + 2\mathbf{E}_{01}(\mathbf{r}) \cdot \mathbf{E}_{02}(\mathbf{r}) \cos\phi(\mathbf{r}) \tag{117}$$

with the definition $\phi(\mathbf{r}) = (\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r} + \theta$ where the quantity $\theta = \alpha_1 - \alpha_2$ represents the phase difference between the two sources. The term $\cos(\phi)$ is at the origin of the fringe effect. Therefore, the non-linearity of energy results from the mutual interference term since we have:

$$2E_{12} = \frac{1}{8\pi} \int_V \mathbf{E}_{01}(\mathbf{r}) \cdot \mathbf{E}_{02}(\mathbf{r}) \cos\phi(\mathbf{r}) \delta\mathbf{r}^3 \tag{118}$$

Some authors [101,102] doing a one-dimensional study of the problem state incorrectly that the above integral is zero due to the oscillatory behavior of the functions inside the integral. In that case, the conservation of energy is satisfied. To show that this is not the case consider two punctual sources S_1 and S_2 located at a distance $2d$ where the origin is hall-way between the sources as shown in figure 2.

According to Einstein [109] outgoing radiation in the form of spherical waves does not exist. Because if an atom radiates a spherical wave it could no recoil. Although no isotropic light source of transverse wave does exist, this does not change the demonstration if we use such a source. Moreover, the following demonstration does rigorously apply to the case of sound waves as we shall see. Therefore, a spherical wave has for expression:

$$\mathbf{E}_{0i}(\mathbf{r}) = \frac{\mathbf{E}_i}{r_i} e^{-(a^2 r_i + b^2/r_i)} \tag{119}$$

The presence of the parameters a and b is necessary to obtain a finite energy solution in the following calculation:

$$E_{12} \approx \frac{\mathbf{E}_1 \cdot \mathbf{E}_2}{32\pi} \int_0^\infty \int_0^{4\pi} \frac{1}{r^2} e^{-2(a^2 r_i + b^2/r_i)} (e^{j\phi} + e^{-j\phi}) r^2 dr d\Omega \tag{120}$$

From the figure 2, we get the relations $\mathbf{r}_1 = -\mathbf{d} + \mathbf{r}$ and $\mathbf{r}_2 = \mathbf{d} + \mathbf{r}$, it follows the phase relation $\phi(\mathbf{r}) = \mathbf{k}_1 \cdot \mathbf{r}_1 - \mathbf{k}_2 \cdot \mathbf{r}_2 \approx -4kd \cdot \mathbf{r}/r$ knowing that $\mathbf{k}_i = k\mathbf{r}_i/r_i$. After integration over the solid angle, we get:

$$E_{12} \approx \frac{1}{4} \mathbf{E}_1 \cdot \mathbf{E}_2 \frac{\sin(kd)}{kd} \int_0^\infty e^{-2(a^2 r + b^2/r)} dr \tag{121}$$

If we now integrate over r, we obtain:

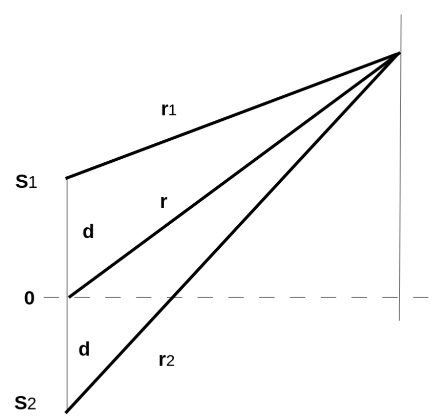

Figure 2: Interference of light

$$E_{12} \approx \mathbf{E}_1 \cdot \mathbf{E}_2 \frac{\sin(kd)}{kd} \frac{b}{2a} K_1(4ab) \tag{122}$$

where K_1 is the modified Bessel function of second specie. We note that the above equation is dimensionally a potential equation since it depends on the distance $2d$ between the two sources as for equation 113. Finally, the total energy has for value:

$$E_T \approx \left[\mathbf{E}_1^2 + \mathbf{E}_2^2 + 2\mathbf{E}_1 \cdot \mathbf{E}_2 \frac{\sin(kd)}{kd}\right] \frac{b}{2a} K_1(4ab) \tag{123}$$

The mutual energy is not zero contrary to the statements of the literature. This energy will be zero only in the case where the distance $2d$ between the two sources tends to infinity. The magnetic field must be taken into account in the preceding calculation, however, it turns out that its contribution is exactly the same as the contribution of the electric field. To explain the paradox, we need the full machinery of Maxwell's equations analyzed in details by Zao [110] and in the references [2, p.179,18, p.376].

4.3 Case of the sound wave interferences

A study of interferences of two loudspeakers is presented in the reference [111] where the authors derived a formula giving the total power as a function of the distance $2d$ between two sound sources:

$$P(t) = 2P_0(t) \left[1 + \frac{\sin(2kd)}{2kd} \cos\theta\right] \tag{124}$$

where P_0 is the power radiated by a single source and θ is the phase shift between the two sound waves. Equation 113 indicates that the total power of radiation of two in-phase $\theta = 0$ loudspeakers at the same location $d = 0$ is four times $P = 4P_0$ greater than the total power of one loudspeaker. This fact has been confirmed experimentally by Gander [112]. Only for $2kd \gg 1$, do we recover the value $P = 2P_0$ regardless of the phase difference θ.

The problem has been examined again in a recent paper [113], where the authors state that the experimental results and the mathematical analysis do not contradict the law of conservation of energy by explaining the phenomena as the influence of the medium on the radiated power. They sustain their view point by considering the oscillations of the tines of a simple tuning fork. The authors affirm that the increase in amplitude is provided by the wave sources which each draw more power from their power supplies. However, the mutual extra power cannot be given by the loudspeakers since the power delivered to each loudspeaker is a fixed quantity defined by the amplifier. Certainly aware of the problem, the authors say:

> If the supplies are not capable of producing this additional power, then the combined wave amplitude will never achieve the value of 2A in the first place.

This explanation cannot be correct since we know that in any sound or light radiation process, the radiation is detached from the source. That is the reason why the speed of radiation is determined by the medium and does not depend on the motion of the source, once radiated. Indeed, how each radiation will be informed of the presence of the other source since the interference is a local process and how this information will be conveyed back to the sources and what is the physical process involved for each source to react from this information.

If an excess of energy appears as the result of the interference of waves, then we can ask the question where does the excess come from. In the case of longitudinal waves propagating in a medium one might imagine that the excess of energy is derived from the heat of the medium although this is not correct. But for electromagnetic waves, this explanation does not hold since the electromagnetic waves propagate in vacuum where there is no medium. But if we assume that there is a medium that fills the vacuum, then we get some basis for the explanation by introducing the so called zero-point energy. We can get an understanding if we compare the characteristics of the energy propagation of waves, and other forms of energy propagation. It is important to understand that in sound waves the energy is not transmitted by macroscopic movements of the particles that compose the medium. The particles do not move macroscopically, but oscillate around their original neutral positions, only the energy moves through macroscopic distances. This is the reason why a sound wave can move thousands of meters in a few seconds even though the air does not move macroscopically.

5 Physics of closed and open systems

A review of the scientific literature reveals a surprising scarcity of publications addressing the physics of closed and open systems aside from a few notable exceptions in biology [114]. This relative silence may stem from the profound implications that a serious examination of such systems would have on both, and our foundational understanding of physics. Acknowledging the role and reality of closed and open systems could challenge some of the most deeply held scientific assumptions, suggesting the potential for technologies such as overunity devices, novel space propulsion methods, and even perpetual motion machines. More radically, it could imply that our prevailing theories, special relativity, quantum mechanics, and general relativity, are either fundamentally flawed or incomplete.

Thermodynamics, as conventionally taught, forms a cornerstone of modern physics and technology. However, a critical issue lies in the scope of the first law of thermodynamics, which is a local law, both in space and time, and strictly valid only for isolated systems that comply with Newton's third law. By contrast, closed and open systems defined as systems that exchange energy and/or matter with their surroundings do not fulfill these conditions. Therefore, the first law cannot be directly or universally applied to them without careful reinterpretation.

Before proceeding, let us recall the two different definitions of efficiency:

Classical Thermodynamic Efficiency

In physics, efficiency typically refers to the ratio of useful output energy to input energy in a system. It is expressed as: Efficiency $= 100 \times E_{out}/E_{in}$. This definition applies strictly to isolated systems, where the conservation of energy dictates that efficiency cannot exceed 100%.

Coefficient of Performance (COP)

A different metric "COP" is commonly used for systems like heat pumps and refrigerators. It is defined as the ratio of the heating or cooling provided to the surroundings over the electrical energy consumed at the input of the heat pump. We have: Cop of heat pump = 1 + Cop of the refrigerator. The Cop of the refrigerator can be less than one or greater than one. The above formula also shows that the Cop of the heat pump can never be less than one.

Importantly, these systems appear to challenge the classical thermodynamic view because they move heat from a lower to a higher temperature reservoir seemingly against the natural thermodynamic gradient that dictates heat flow from hot to cold. Indeed, the function of the refrigerator is to move the heat from the low temperature reservoir to the high temperature reservoir.

Traditional definitions of efficiency apply to systems like engines and boilers, where energy is converted from one form (e.g., chemical, electrical) into another (typically mechanical or thermal). In contrast, refrigerators and heat pumps redistribute energy rather than convert it. As such, the Cop framework is more appropriate for analyzing non-isolated systems and may also provide insight into potential over-unity systems. To explore this further, we describe two simple experiments that can be reproduced at the university level to convince students that technologies with Cop greater than 1 exist:

- The SMOT (Simple Magnetic Overnuity Toy)
- The Pendulum Experiment

Both aim to demonstrate that systems can exhibit a coefficient of performance greater than one. Indeed, this is already a reality for heat pumps, which commonly achieve Cop values greater than 3. Furthermore, we will show that the pendulum experiment challenges the conservation of energy, as it exhibits behavior inconsistent with this foundational law under conventional assumptions. Following this, we will outline the physical conditions required to achieve over-unity performance and perpetual motion, and conclude by analyzing experimental approaches to space propulsion based on external force interactions.

5.1 The SMOT experiment

Watson's SMOT is a simple device which demonstrates the possibility to get over-unity efficiency. Easily built with readily available square magnets. This device is a permanent magnet propulsion system wherein a steel ball is propelled up an inclined plane between two rows of permanent bar magnets. These magnets being in spaced relationship with all north poles of the first row facing in the same direction and with all south poles in the second row facing in the same but opposite direction to the first row.

Two such arrays are made and fastened to a wooden board so that they form an angle. The two permanent bar magnets generate an inhomogeneous magnetic field $\mathbf{B}$ between the two bar magnets since they are not parallel. Bisecting that angle is an aluminum track on which a steel ball can roll freely. The length of the aluminum track supporting the ball is configured to allow the ball to escape the track where the potential magnetic energy is minimum in order that the ball drop out of the magnetic field.

The steel ball has a magnetic moment $\mathbf{M}$ and a mass m, therefore the potential energy of the interaction between the metallic ball and the magnetic field is $E_B = -\mathbf{M} \cdot \mathbf{B}$. The steel ball is submitted to two conservative forces: the gravitational and the magnetic forces which derive from a potential $\mathbf{F}_i = -\nabla E_B$. If we considered the ball, the magnets and Earth as an isolated system, then we have the following definition of the mechanical energy:

$$E_M = E_K + E_G + E_B = \text{Ct} \Rightarrow \Delta E_M = \Delta E_K + \Delta E_G + \Delta E_B = 0 \qquad (125)$$

Knowing that $\Delta E_G = -mgh$ and $\Delta E_B < 0$, the efficiency will be $\Delta E_K / |\Delta E_G + \Delta E_B| = 1$ However, it is more physical to consider the ball and Earth as a closed system $\Delta E_K + \Delta E_G = -\Delta E_B$ with a cop $= \Delta E_K / |\Delta E_G| = 1 + \Delta E_B / \Delta E_G > 1$

In the presence of the magnets, the steel ball will climb on the ramp because the magnets induce in the ball a magnetic moment and an interaction force appeared between the ball and the inhomogeneous magnetic field that does work against the gravitational field. The cop is over-unity for a closed system. Indeed, a $Cop = 1.14\%$ was measured as demonstrated in [115]. The work done by the magnets is free-energy because the magnetic energy is not consumed since the magnetic force derives from a reversible potential energy and therefore, the motion can be repeated indefinitely without consuming any energy provided by the magnets that is to say the magnetic energy is not decreased in the process. The smot experiment is a fundamental experiment because it proves that an efficiency higher than 100% is possible since at the input we have a small potential energy given by the observer and at the output the observer recover a kinetic energy which is higher than the input potential energy.

We may think to build an over-unity magnetic motor with a string of smots in series along a closed loop. We will examine hereafter if such a thing is possible. Imagine a wheel of four spokes with balls on the end of each spoke whose ends pass through four smots at the minimum of their potential energy. Now as each spoke enters the smot, it is sucked through and speeding up. The wheel is gaining energy from the smots. But, the picture is not complete since we have to take into account the magnetic braking force between the four smots which slows down on exit the motion of the wheel.

It is important to stress that we have four magnetic independent potential functions, therefore the work of each magnetic force must be calculated for a part of the closed loop where the ball is present. Indeed, the variation of the kinetic energy of the rotor along a closed curve is given by the work relation:

$$\sum_{n=1}^{4}\Big(\int_0^{L_m}\mathbf{F}_m\cdot\mathbf{dr}+\int_0^{L_p}\mathbf{F}_p\cdot\mathbf{dr}\Big)+\int_0^{2\pi R}\mathbf{F}_g\cdot\mathbf{dr}=\sum_{n=1}^{4}\Big(\int_0^{L_m}\mathbf{F}_m\cdot\mathbf{dr}+\int_0^{L_p}\mathbf{F}_p\cdot\mathbf{dr}\Big) \tag{126}$$

We have neglected in the preceding equation the loss of energy due to friction of the wheel which is a small quantity. The work of the gravitational force along a closed curve is zero. For the magnetic case, all the smots are placed in order that the scalar product $\mathbf{F}_m \cdot \mathbf{dr}$ is always positive inside the smots while the term $\mathbf{F}_p \cdot \mathbf{dr}$ is always negative outside the smots.

However, there is a difference between the work of the two magnetic forces $\mathbf{F}_m$ and $\mathbf{F}_p$ along the closed curve since the field strength of the magnetic force $\mathbf{F}_p$ falls off as $1/R^4$ outside the smots and each ball leaves the smot at the minimum of the potential energy. Therefore, we can expect that the negative contribution of $\mathbf{F}_p$ along the closed loop will not cancel totally the positive work of the magnetic force $\mathbf{F}_m$.

$$\sum_{n=1}^{4}\big(\int_0^{L_m}\mathbf{F}_m\cdot\mathbf{dr}+\int_0^{L_p}\mathbf{F}_p\cdot\mathbf{dr}\big)>0 \tag{127}$$

We can find in internet [116] a similar smot motor which works as an over-unity motor.

5.2 Electrostatic pendulum experiment

We replicated the pendulum experiment first done by Brown in 1929 [117] and reported our results for the first time in 1996 [12-18]. We will review again this experiment in order to bring a new insight concerning the problem of energy conservation. The pendulum of mass m_1 as shown in figure 3 consists of two metallic balls $2R = 5cm$ weighting $m = 500g$ each suspended by two nude copper wires $l = 2m$ and diameter $d = 0.5mm$ to the ceiling of the laboratory. An insulating rod is used between the balls in order to keep the balls at a fixed distance. The rod cancels the electrostatic attraction between the two metallic balls.

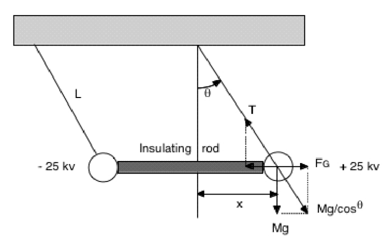

Figure 3 : Various forces acting on a double solid pendulum

Figure 3: Various forces acting on a double solid pendulum

The thin nude wires create a strong electric field which ionizes air around the wires. This local field is stronger than the average field between the wires.

As soon as we turn on the bipolar Glassman HT8 HV power supplies connected to the two wires, we can observe a motion towards the positive wire. A reversal of the direction of motion is observed when the polarity of the voltage source is inverted. A stationary state is observed with a measured current of $I = 1.5mA$ and a displacement of the pendulum $x = 8mm$. For a pendulum of mass $m = 1kg$, we get a calculated force 4×10^3 dynes or $4g$ at $50kV$.

In classical circuit theory, the DC power supply with voltage V_S, the wires with resistance R_s and the capacitor $C = 10pF$ with a parallel resistance $R_p = 33M\Omega \gg R_s$ to account for the leakage current between the wires form a closed system where the conservation law of energy has for expression:

$$\int_0^T R_e I^2(t)dt + \int_0^T V_c(t)I(t)dt = \int_0^T V_e(t)I(t)dt \tag{128}$$

with the definitions:

$$R_e = \frac{R_s}{\alpha} \qquad V_e = \frac{V_S}{\alpha} \qquad \alpha = 1 + \frac{R_s}{R_p} \approx 1 \tag{129}$$

and the currents:

$$I_p = \frac{V_S}{R_s + R_p}(1 - e^{-\alpha t/R_s C}) \qquad I_c = \frac{V_S}{R_s} e^{-\alpha t/R_s C} \tag{130}$$

The energy equation above can be written in the symbolic form:

$$E_R(T) + E_C(T) = E_S(T) \tag{131}$$

Therefore, the energy E_S given by the power supply during the transport of energy is converted into energy stored E_C in the capacitor and into heat E_R dissipated in the resistances. This is an energy balance law.

The gravitational potential energy between the pendulum and Earth is given by the equation:

$$E_G[\mathbf{R}(t)] = -G\frac{m_1 m_2}{R} \approx -G\frac{m_1 m_2}{R_E}(1 - \frac{h}{R_E}) = -m_1 g R_E - m_1 g h(t) \tag{132}$$

where we have posed $R(t) = R_E + h(t)$. For the Earth's mass m_2, we verify the condition $m_1 \ll m_2$. For a classical pendulum, the potential and kinetic energies have for expression:

$$E_K[\theta(t)] = \frac{1}{2} m_1 l^2 \left(\frac{d\theta}{dt}\right)^2 \qquad E_G[\theta(t)] = m_1 g l(1 - \cos\theta) = m_1 g h(t) \tag{133}$$

The equation of motion and the force for the displacement x of the pendulum are:

$$m_1 \frac{d^2\theta}{dt^2} = \frac{q}{l} \sin\theta \qquad F = m_1 g \tan\theta = m_1 g x/l \tag{134}$$

We know that the mechanical energy is conserved during the motion of the pendulum if there is no frictional force and no external force applied to the pendulum.

$$E_M = E_K[\theta(t)] + E_G[\theta(t)] = \mathrm{Ct} \tag{135}$$

Once we have given an initial impulse to the pendulum, it will oscillate back and forth for ever with a constant magnitude if there is no other force applied to the system. Now If an external force is applied to the center of mass of the pendulum and Earth, there is a motion of this point which is not observable due the important value of the Earth mass. However, the effect on the reduced mass M is observable since we have now to take into account the stimulated force which has for expression:

$$\mathbf{F}_S = \frac{1}{m}(m_2 \mathbf{F}_{11} - m_1 \mathbf{F}_{22}) \approx \mathbf{F}_{11} \tag{136}$$

We can now write a law of conservation of power of the pendulum by taking into account the work of the stimulated force $\mathbf{F}_S$:

$$\frac{d}{dt}(\frac{1}{2}MV^2 + E_P) = \mathbf{V} \cdot (\mathbf{F}_S + \mathbf{F}_T) \tag{137}$$

where $\mathbf{F}_T$ is the tension in the string which does not work $\mathbf{V} \cdot \mathbf{F}_T = 0$ and $\mathbf{V}$ is the velocity of the pendulum in the Earth's reference frame.

After integration over time of this equation seems to define an equation of conservation of energy. In fact this equation states the violation of the conservation law of energy since the work of the external force is not given by an external observer as for example when a parent push the swing of his child sited on a swing. The external power resulting from the work of the stimulated force pumps energy from the plasma for free. Indeed, when the power supply is switched off, a kinetic energy $E_K = MV^2/2 = Mlg(1-\cos\theta) \approx m_1 g x^2/2l$ is recovered. This energy is not given by the power supply but is taken from the plasma. For two balls charged at $50kV$, the kinetic energy of the pendulum is $E_K = 1.57 \times 10^{-4} J$ while the electrostatic potential energy is $E_C = CV^2/2 = 1.25 \times 10^{-2} J$. The kinetic energy due to the stimulated force is not taken from the power supply since in classical circuit theory no motion of the capacitor is taken into account during the charging process. We also applied the high voltage in a pulsed manner in synchronism with the oscillatory motion of the pendulum. It results in an amplification of the displacement of the pendulum which reaches a magnitude of $\pm 5cm$. The observation of this amplification implies the existence of the stimulated force which increases the preceding kinetic energy to $E_K = 0.61 \times 10^{-2} J$ for free while the average power delivered by the power supply decreases by a factor 2.

The pendulum experiment does not show over-unity efficiency due to the dissipative power $P_D = V_s I_p = 75W$ necessary to sustain the creation of the plasma in air. The pendulum experiment proves the existence of forces which do not satisfy Newton's third law. It results from the above calculation that these forces induced inside plasma or matter can be used to do space propulsion. This external force is generated as explained in several papers [12-18] from the magnetic Lorentz force which does not follow Newton's third law.

5.3 The free energy devices and the perpetual motion machines

Historically, what are now called "free-energy machines" were once referred to as perpetual motion machines. It is important to clarify that the term "free-energy" used in this context should not be confused with thermodynamic free energy, a well-defined concept in physics. Rather, "free-energy" typically refers to technologies that have the potential to dramatically reduce individual energy costs, often with relatively low investment.

Over the years, numerous claims have been made regarding free-energy devices, claims that are generally considered impossible according to the established laws of physics. Nevertheless, the present author has conducted experimental research in this field for over 22 years in collaboration with other scientists. The decision not to publish these results stems from a desire to avoid unproductive debate with skeptics, individuals who often dismiss such work without ever conducting a single experiment themselves.

Based on firsthand experience, I am convinced that these technologies are indeed possible. However, they are challenging to implement, requiring specific and sometimes unusual conditions. For those interested, I recommend consulting "The Free Energy Devices" by Patrick Kelly [132], an extensive 2,582-page resource worth reading by any open-minded physicist. If functional free-energy machines could be widely implemented, they would undoubtedly transform our world for the better.

Magnetic Motors and Over-Unity Devices

Among the most popular free-energy concepts found online are magnetic motors. We have previously discussed this type of motor in relation to the SMOT (Simple Magnetic Overunity Toy) experiment. It is theoretically possible to achieve over-unity operation using inhomogeneous magnetic fields. The principle behind this involves the magnetic field gradient force, a concept well-known in physics through experiments like the Stern-Gerlach experiment, which demonstrated spatial quantization by passing silver atoms through an inhomogeneous magnetic field.

Over-unity magnetic motors exploit the attractive and repulsive properties of magnetic poles to maintain continuous motion that can be harnessed to perform useful work. Free-energy is extracted by arranging permanent magnets in specific configurations. One such example is the Johnson Motor [133], which achieved a U.S. patent in 1979. This motor is based on the behavior of permanent magnets in inhomogeneous magnetic fields to create continuous rotational motion.

Another example is the Wankel magnetic motor, invented by Tekkosho in 1970. It is a clever variation of the SMOT experiment, utilizing an inhomogeneous magnetic field in a simple design. As detailed in a paper by Valone [134], the motor employs an electromagnet to assist the rotor in crossing a small gap where braking occurs. Theoretically, if the energy used in the coil is partially recovered, the device could exceed 100% efficiency. It's well known that the back EMF (electromotive force) recovery can return 60% to 90% of the energy used. This suggests that a Coefficient of Performance (COP) of 1.5 to 1.8 is potentially achievable. Currently, there are more than 140 known magnetic motor designs available online, though the efficiency measurements for most are not rigorously validated. Encouragingly, some universities have begun to study these motors using numerical simulations [135].

Perpetual Motion and the Laws of Physics

Behind the idea of perpetual motion lies a long-standing dream: to harness unlimited mechanical work for humanity. However, before seriously considering the feasibility of perpetual motion, it's essential to clarify what the term actually means. Literally, "perpetual motion" refers to continuous motion, something not at odds with Newton's First Law, which states that an object in motion will remain in motion unless acted upon by an external force. To question perpetual motion, in this literal sense, is to question Newton's foundational principle. However, for decades, "perpetual motion" has been used in a misleading way, implying the continuous creation of energy from nothing. Max Planck's definition [136] highlights this interpretation:

> It is in no way possible, either by mechanical, thermal, chemical, or other devices, to obtain perpetual motion, i.e., it is impossible to construct an engine which will work in a cycle and produce continuous work or kinetic energy from nothing

Planck's first clause, however, appears to contradict Newton's First Law. For example, if an object in space ejects another object, the ejected body continues in perpetual motion unless acted upon externally. This suggests that continuous motion per se is not forbidden. We explore this concept further in our prior work [17].

Planck's second point, however, is valid: no device has yet been shown to continuously produce useful work from nothing. Such a claim would indeed violate the principle of energy conservation. In this article, we begin by reviewing why perpetual motion is impossible in an inertial reference frame. We then show that, surprisingly, the situation may be different when considering a non-inertial reference frame.

Let us consider a gravitational force $\mathbf{F}[\mathbf{R}(t)]$ derived from a potential energy function $E_G(\mathbf{R})$, and examine the work done on a mass $M \approx m$ along a closed loop as defined in equation 99. Since the work integral over a closed loop in a conservative field is zero, the object's kinetic energy cannot increase over a full cycle. This leads us to conclude that perpetual motion within an inertial frame under the influence of conservative forces alone is impossible. Although Newton's First Law supports the existence of motion without friction, real-world machines are affected by energy losses such as friction and resistance, making it impossible to extract useful work indefinitely. The conservation of energy holds only if internal forces are central, and obey Newton's Third Law for translation. Thus, perpetual motion in inertial frames would imply a violation of energy conservation, confirming Planck's assertion, at least at first glance. However, there are exceptions.

Now there are two cases where the conservation law of energy is violated. The first case is when the potential energy depends explicitly on time. We have shown that the equation has for expression in that case:

$$\frac{d}{dt}(\frac{1}{2}m\mathbf{V}^2 + E_P) = \frac{\partial E_P}{\partial t} \tag{138}$$

Since the term in the right hand side of the above equation is not zero then a free-energy machine is possible. We recall that the gravitational potential energy does not depend explicitly on time. Therefore, classical mechanics does not forbid the existence of the so called free-energy devices or over-unity devices discussed in Internet provided they use forces that do not satisfy Newton's third law. The reader interested by this subject can consult the numerous web sites on free-energy devices. However, we can debunk the whole subject of over-unity devices by pointing out the existence of closed and open systems, a fact which is not well discussed in the literature. In the case of a closed system where Newton's third law is verified, the efficiency cannot be higher than 100% because the work of the external force is not taken into account. A good example is the heat pump with a Cop above 3. There is a second case as previously defined by equation 11:

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$$\frac{d}{dt}\left(\frac{1}{2}m\mathbf{U}^2\right) = \mathbf{U} \cdot \mathbf{F}_e \tag{139}$$

The central question is: how can we generate an external force? As demonstrated earlier, the magnetic Lorentz force does not comply with Newton's third law. This characteristic opens the possibility for its use in the construction of so-called "free-energy" devices. Some physicists may disagree with this perspective, arguing that any physical system can be considered closed by accounting for all other material particles in the Universe. However, this is not necessarily valid. One can always define the center of mass of all particles in the Universe; in this framework, the energy associated with the motion of this center cannot be attributed to the particles themselves, but must instead originate from the ether or, according to some, from the Big Bang, if one subscribes to that theory. Nevertheless, as shown through the use of Jacobi coordinates [18], the division between internal and external forces does not need to be applied universally to all particles. Rather, it can be treated as a local principle.

The situation changes notably when an object resides in a non-inertial reference frame, particularly one involving rotational motion and forces that violate Newton's third law. In such cases, the concept of a perpetual motion machine becomes more plausible.

In fact, in 1712, Johann Bessler publicly demonstrated a device he claimed to be a perpetual motion machine. His story is recounted in detail in a recent book by Collins [137]. Over the next eighteen years, Bessler presented his machine to the public, subjecting it to rigorous and repeated testing. Despite extensive scrutiny, no evidence of fraud was ever discovered.

In 1713, Bessler unveiled a larger wheel with a diameter of 1.5 meters and a thickness of 15 centimeters. This wheel could rotate up to 50 times per minute and was capable of lifting 40 pounds off the ground. It was displayed in Draschwitz near Leipzig. His subsequent model, presented in Merseburg, had a diameter of 2 meters and a thickness of 30 centimeters. This device received a certificate acknowledging it as a perpetual motion machine, affirming that sufficient precautions had been taken to rule out any deception. The certificate was signed by several scientists, some of whom were acquaintances of Isaac Newton. The case of an object located in a non-inertial reference frame where rotational motion is present is quite different especially if the force involved violates the Newton's third law. In that case a perpetual motion machine is possible.

Bessler ultimately failed to sell his invention, as he refused to reveal its inner workings until he received payment in full. His unwillingness to compromise prevented any successful negotiations, and he eventually died in poverty without ever disclosing his secret. From the discussion above, it is clear that the rotational motion of Bessler's wheel cannot be attributed solely to gravitational forces. With the wealth of information now available online, one can piece together clues about the mechanism behind Bessler's machine. We have developed a hydraulic version of the Bessler wheel. Preliminary experimental results show a coefficient of performance of 1.8. Our goal is to achieve a COP greater than 2 to create a self-sustaining system. A detailed paper will be published once all necessary improvements to the machine have been completed.

5.4 Space Propulsion

Two irish physicists, Trouton and Noble in 1903, two american physicists, Biefeld and Brown in 1923, two french physicists, Pages in 1921 and Cornille in 1998 demonstrated that a charged capacitor suspended to a thin wire to the ceiling of the laboratory spontaneously move with the observation of rectilinear and rotational motions. The basic aim of the 1903 Trouton-Noble experiment was to detect on Earth our absolue motion through the ether. The experiment has been replicated at the university of Lille [55,57,58] with present technology, free from the previous imperfections of the original Trouton-Noble experiment [45]. Contrary to previous results, a rotational motion has been observed since the sensitivity of the experimental set up has been improved by a factor 100 with respect to the original experiment. Namely, a continuous measurement of the rotation of the capacitor was observed when a high voltage $33.7kV$ is applied to the capacitor. This indicates that indeed the present experiment is the electrostatic equivalent of the Michelson-Morley experiment. Moreover, this experiment was done during long periods of time where the observed rotational motion was depending on the position of Earth in space [119-125]. This experiment alone disproves the validity of the special relativity theory.

T.T. Brown carefully conducted experiments for thirty years with charged bodies in air, oil and in a high vacuum. A retrospective discussion concerning the experiments done during many years by Brown at different laboratories throughout the world can be found in the book by A. Szames [126]. In our papers [127,128], we examine how this force can be used to do space propulsion. Most of the experimental work of Brown was known from his patents since none of his work was published in scientific magazines. The results have been discounted because they were attributed to ion wind and corona discharge. The critics formulated concerning the results of these experiments can be easily refuted because the ion wind effect is too small. Without a plausible theoretical explanation, the observation made by Brown received little attention from the scientific community until the year 2001. However, the Pages-Biefeld-Brown effect provides an experimental proof for the existence of an absolute space.

J.L. Naudin [129-131] worked on the concept of stimulated force applied to scientific projects among them electromagnetic propulsion which is referenced in the literature as the lifter project. Lifters build by J. L. Naudin are asymmetrical capacitors joined so as to form a triangle assembly capable of lifting their own weight. The lifter weighting $2.3g$ has a measured acceleration which increases from $0.8g$ to $1.3g$. Face to the evidence concerning these strange experimental results, the scientific community started to examine the subject with a growing interest as proved by the numerous publications [138-150] done through out the world. The publications confirm the reality of the experimental results observed previously by Pages, Brown and Cornille. However, most authors persist to explain the phenomenon by using equation 1 with $\mathbf{F}_{11} = \mathbf{F}_{22} = 0$ instead of equation 3 with $\mathbf{F}_{11} \neq \mathbf{F}_{22} \neq 0$ and $\mathbf{F}_{11}+\mathbf{F}_{22} = \mathbf{F}_e \neq 0$ where the stimulated motion results from the work of the external force.

Since a capacitor has two boundary layers of free charges $Q_i = -Q_e$ on the plates, it can be used to estimate the stimulated force of this capacitor with respect to a given reference frame located in a far from system. After some calculation [9,127,128], we get the relation:

$$\mathbf{F}_e \approx 2\frac{Q_e Q_i}{c^2 R^3}\mathbf{U}(\mathbf{R}\cdot\mathbf{V}) - 2\frac{Q_e Q_i}{c^2 R}\frac{d\mathbf{U}}{dt} \tag{140}$$

where $\mathbf{R} = \mathbf{r}_1 - \mathbf{r}_2$ is the distance between the space charges Q_e and Q_i. We note that the external force depends at the same time on both relative and absolute velocities of the charges.

6 Conclusion

In this paper, we demonstrate that the partition of forces is applicable to all major physical theories, with the exception of general relativity, where, we argue, it should also be considered. We highlight a common misinterpretation among physicists, who often associate the distinction between internal and external forces solely with the boundary between a system and its surroundings. Newton's third law provides a natural framework for classifying forces into two categories: internal (or mutual) forces, which comply with Newton's third law and uphold the conservation of energy, external (or proper) forces, which violate both principles. These two types of forces coexist simultaneously, and their separation has been validated through numerous experiments discussed in this work. Thus, the partition of forces is far from a trivial concept; acknowledging this fundamental principle has profound implications for our understanding of physical laws and the development of future technologies.

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