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# Envariance Symmetry in Quantum Mechanics and Statistical Mechanics

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## Abstract

A quantum symmetry known as entanglement assisted invariance or envariance is introduced. It has recently been of interest in understanding the process of performing quantum measurements. An apparatus which interacts with other physical systems, the environment, exchanges a single state with physical states equal in number to that of possible outcomes of the experiment. Correlations between the apparatus and environment give rise to a type of selection rule which prohibits the apparatus from appearing in a superposition corresponding to different eigenvalues of the pointer basis of the apparatus. Eigenspaces of this observable make a natural basis for the apparatus and determine the observable of the measured quantum system. It is also discussed how statistical mechanics can be expressed in terms of this symmetry.

**Keywords:** probability, correlations, operator, measurements, basis, eigenstates, envariance

## 1 Introduction

Understanding of the measurement process applied to quantum mechanical systems [1-4] can be greatly increased by treating an apparatus, also thought

of as an environment, quantum mechanically. The interaction of the system and apparatus can be studied together quantum mechanically, including also the presence of an environment, or other physical systems which can interact with the apparatus [5-6]. In this process, states of the apparatus become correlated with the system and influence what is observed. Both can become correlated with an immediate environment. This may perhaps better thought of as a second apparatus. Thus the apparatus should itself be described by quantum mechanical laws [7]. Von Neumann's approach to the problem has a particular foundation. A correlation is established between states of the apparatus and states of the system. Let us call the apparatus  $A$ , the system  $S$  and the environment  $E$ . If the apparatus states  $|A_s\rangle$  are regarded quantum mechanically, there is nothing to prevent the state of  $A$  being presented in terms of an alternate orthonormal basis  $|A_r\rangle$  composed of superpositions of states  $|A_s\rangle$ ,

$$|A_r\rangle = \sum_s \langle A_s | A_r \rangle A_s. \quad (1.1)$$

The state of the combined system is described by a superposition of states  $|A_s\rangle \otimes |s\rangle$ , where  $|s\rangle$  describe states of the system. Then the combined system can be given in terms of new states  $|A_r\rangle$  as

$$\begin{aligned} \sum_s \tau_s |A_s\rangle \otimes |s\rangle &= \sum_r \langle A_r | A_s \rangle \sum_s \tau_s |A_s\rangle \otimes |s\rangle = \sum_r |A_r\rangle \otimes \sum_s \tau_s \langle A_r | A_s \rangle |s\rangle \\ &= \sum_r \kappa_r |A_r\rangle \otimes |r\rangle. \end{aligned} \quad (1.2)$$

In (1.2), definition  $\kappa_r = \sum_s \tau_s \langle A_r | A_s \rangle$  provides a set of relative states  $\{|r\rangle\}$ . These constitute normalized but not necessarily mutually orthogonal states of system  $S$ , relative to the arbitrarily chosen basis set  $|A_r\rangle$  of the apparatus. Is it possible for the quantum system to end up in one of the states  $|r\rangle$  rather than  $|s\rangle$ . If all the  $\tau_s$  are of the same magnitude, whenever  $|A_r\rangle$  is orthonormal, the collection of relative states  $|r\rangle$  is as well. So the apparatus, which has to be correlated with the state of the system, contains not only information about observable  $\hat{S} = \sum_s q_s |s\rangle \langle s|$ , but many other observables  $\hat{R} = \sum_r r_r |r\rangle \langle r|$  as well. These are defined on the Hilbert space of the system [8-10].

However generally  $\hat{R}$  and  $\hat{S}$  will not commute. Quantum mechanics does not permit the simultaneous measurement of two noncommuting observables with arbitrary accuracy. What then in a real world apparatus does determine the seemingly unique pointer basis  $|A_r\rangle$  which records the corresponding relative states  $|p\rangle$  of the system? The new element is that interaction of the

quantum apparatus with the environment also produces correlations. Correlations with the environment impose certain kinds of selection rules which prevent the apparatus from existing in a superposition of states corresponding to different eigenvalues of this special apparatus basis.

The idea is to introduce a new quantum symmetry called entanglement assisted invariance or envariance. It is studied as it applies to the measurement process in quantum mechanics. It also provides a new, consistent way in which to understand quantum statistical mechanics. Statistical mechanics has a deep roots in thermodynamics as well. Interaction with the environment is an important component of the concept. It distinguishes the model apparatus from the quantum system. Eigenspaces of the pointer observable provide a natural basis for the pointer of the quantum apparatus and determine the observables of the measured quantum system. The observation or monitoring of the apparatus by the environment terminates in the apparent reduction of the wave packet. Correlations among states of the pointer basis and those of relative states of the system are preserved in the end mixed-state density matrix [10]. Decay of those elements of the apparatus-system density matrix, which are off-diagonal in the apparatus observable, is a result of the natural evolution of the system-apparatus-environment combination. Selection rules need not be imposed from outside [11-13].

In statistical physics the description of canonical thermal equilibria is usually derived from Boltzmann's  $H$ -theorem, the ergodic hypothesis, or maximization of the statistical entropy in equilibrium [15]. However, none of these concepts are particularly well stated for quantum systems. Statistical physics developed when the fundamental physical theory was classical mechanics. Concepts such as ensembles consisting of infinitely many versions of the same system then come up. Progress on this problem has occurred by demonstrating that representations of microcanonical and canonical equilibria can be obtained from a fully quantum mechanical analysis. This means taking account of symmetry considerations such as entanglement and consequently envariance. After studying this symmetry in detail, it is shown how envariance can give rise to microcanonical and canonical states.

## 2 Measurement Performed On Combined Two-State Systems

### 2.1 Construction Of An Accurate Model

A pair of two state systems are defined in order to create a model which emphasizes some of the main physical aspects of the model. The first two-state system goes by the name spin and the second is referred to as atom. The spin system has a basis denoted by the states  $\{|\uparrow\rangle, |\downarrow\rangle\}$ , also written as  $\{(1, 0)^t, (0, 1)^t\}$ . It describes a state parallel or antiparallel to the  $z$ -axis. However, there are other bases which can be formed out of linear combinations of these states which result in orthonormal states such as

$$\begin{pmatrix} |\alpha\rangle \\ |\beta\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix}, \quad \begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} |\alpha\rangle \\ |\beta\rangle \end{pmatrix}. \quad (2.1)$$

As well there is the basis  $\{|\rightarrow\rangle, |\leftarrow\rangle\}$

$$\begin{pmatrix} |\rightarrow\rangle \\ |\leftarrow\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix} \begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix}, \quad \begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \begin{pmatrix} |\rightarrow\rangle \\ |\leftarrow\rangle \end{pmatrix}. \quad (2.2)$$

The second two-state system is called *atom* in order to make a connection with an object that may be present in an experiment. It is an object which consists of two states  $\{|g\rangle, |e\rangle\}$ . These states are called ground and excited. Let us assume the atom has the same energy no matter which of these states it is in. Neither of these systems have self-Hamiltonians. As in the case of spin, there are alternative bases as may be expected. They are formed out of linear combinations of these two

$$\begin{pmatrix} |+\rangle \\ |-\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} |e\rangle \\ |g\rangle \end{pmatrix}, \quad \begin{pmatrix} |e\rangle \\ |g\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} |+\rangle \\ |-\rangle \end{pmatrix}. \quad (2.3)$$

There is the related set as well,

$$\begin{pmatrix} |\perp\rangle \\ |\top\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix} \begin{pmatrix} |e\rangle \\ |g\rangle \end{pmatrix}, \quad \begin{pmatrix} |e\rangle \\ |g\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \begin{pmatrix} |\perp\rangle \\ |\top\rangle \end{pmatrix}. \quad (2.4)$$

The spin is regarded as the quantum system under observation. The role of the apparatus is played by the atom. An interaction Hamiltonian is responsible for coupling the apparatus-atom to the spin with coupling strength  $\alpha$ . It has the following form

$$\hat{H}^{AS} = \alpha (|\perp\rangle\langle\perp| - |\top\rangle\langle\top|) \otimes (|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|). \quad (2.5)$$

In terms of Pauli matrix  $\sigma_3$ , (2.5) it can be written as

$$\hat{H}^{AS} = \alpha(|\perp\rangle|\top\rangle)\sigma_3 \begin{pmatrix} |\perp\rangle \\ |\top\rangle \end{pmatrix} \otimes (|\uparrow\rangle|\downarrow\rangle)\sigma_3 \begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix}. \quad (2.6)$$

With respect to the bases which define (2.6), it can be written as

$$\hat{H}^{AS} = \alpha \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix} \quad (2.7)$$

The upper block works on the  $|\perp\rangle$  and the lower on  $|\top\rangle$  such that  $\sigma_3$  operates on the spin part.

The evolution of a state vector expressed in terms of the basis states  $\{|\uparrow\rangle, |\downarrow\rangle, |\perp\rangle, |\top\rangle\}$  under the influence of  $\hat{H}^{AS}$  is determined by the evolution operator  $\hat{U}$  defined by

$$\hat{U} = e^{i\hat{H}^{AS}t/\hbar} = \begin{pmatrix} e^{i\alpha\sigma_3 t/\hbar} & 0 \\ 0 & e^{-i\alpha\sigma_3 t/\hbar} \end{pmatrix}. \quad (2.8)$$

Let us investigate the effect of  $\hat{U}$  on a state such as an initial state defined as

$$|\varphi_i\rangle = (a|\uparrow\rangle + b|\downarrow\rangle) \otimes |+\rangle. \quad (2.9)$$

It is to be evolved over a time interval  $[0, T]$  such that  $T > 0$ . Using (2.1)-(2.4) we write (2.9) in terms of the basis vectors that define  $\hat{H}^{AS}$ . It will be helpful to introduce a dimensionless time  $\tau = \alpha t/\hbar$  in the following

$$\hat{U}|\varphi_i\rangle = \frac{1}{2}(1-i) \begin{pmatrix} ae^{i\tau} \\ be^{-i\tau} \end{pmatrix} \otimes |\perp\rangle + \frac{1}{2}(1+i) \begin{pmatrix} ae^{-i\tau} \\ be^{i\tau} \end{pmatrix} \otimes |\top\rangle. \quad (2.10)$$

Going to the basis  $\{|e\rangle, |g\rangle\}$ , it is clearly seen that  $|\varphi_i\rangle$  can be transformed into a correlated state

$$\begin{aligned} \hat{U}|\varphi_i\rangle &= \frac{1}{2}(1-i) \begin{pmatrix} ae^{i\tau} \\ be^{-i\tau} \end{pmatrix} \otimes \frac{1}{\sqrt{2}}(|e\rangle + i|g\rangle) + \frac{1}{2}(1+i) \begin{pmatrix} ae^{-i\tau} \\ be^{i\tau} \end{pmatrix} \otimes \frac{1}{\sqrt{2}}(|e\rangle - i|g\rangle) \\ &= \frac{1}{2\sqrt{2}} \left[ \left( (1-i) \begin{pmatrix} ae^{-i\tau} \\ be^{i\tau} \end{pmatrix} + (1+i) \begin{pmatrix} ae^{-i\tau} \\ be^{i\tau} \end{pmatrix} \right) |e\rangle + \left( (1+i) \begin{pmatrix} ae^{i\tau} \\ be^{-i\tau} \end{pmatrix} + (1-i) \begin{pmatrix} ae^{i\tau} \\ be^{-i\tau} \end{pmatrix} \right) |g\rangle \right] \\ &\quad (2.11) \\ &= \frac{1}{\sqrt{2}} \left[ \left( \begin{pmatrix} a \\ b \end{pmatrix} \cos \tau + \begin{pmatrix} a \\ -b \end{pmatrix} \sin \tau \right) |e\rangle + \left( \begin{pmatrix} a \\ b \end{pmatrix} \cos \tau - \begin{pmatrix} a \\ b \end{pmatrix} \sin \tau \right) |g\rangle \right]. \end{aligned}$$

It can be stated that the interaction Hamiltonian  $\hat{H}^{AS}$  evolving the state over  $(0, \tau_1)$  where  $\tau_1 = \pi/4$  transforms the initial product state (2.11) into a correlated state vector

$$|\varphi_f\rangle = \hat{U}(\tau_1)|\varphi_i\rangle = a|\uparrow\rangle \otimes |e\rangle + b|\downarrow\rangle \otimes |g\rangle \quad (2.12)$$

This is still a pure state and so correlations between system and apparatus have already been established. However so far the measurement could not have produced a definite outcome. First the correlated apparatus-system state vector  $|\varphi_f\rangle$  in (2.11) returns to the initial  $|\varphi_i\rangle$  if the same interaction continues for a further time  $t_2 = 3\pi/4$ ,

$$\hat{U}(\frac{3}{4}\pi)|\varphi_i\rangle = -\frac{1}{\sqrt{2}}\begin{pmatrix} a \\ b \end{pmatrix} \otimes (|e\rangle + |g\rangle) = -\begin{pmatrix} a \\ b \end{pmatrix} \otimes |+\rangle. \quad (2.13)$$

The apparatus could not have decided at the instant characterized by (2.12) which outcome of the measurement was the state  $|\uparrow\rangle$  and which  $|\downarrow\rangle$ . If the initial direct product is to reemerge after  $\tau_1 + \tau_2 = \pi$ , all outcomes of the measurement should have been present at  $\tau_1$ .

At the stage described by state  $|\varphi_f\rangle$ , it is as yet undetermined which possible states are distinguished by the measurement of the system. Transform the state  $|\varphi_f\rangle$  to the basis  $\{|+\rangle, |-\rangle\}$  to observe that

$$|\varphi_f\rangle = a|\uparrow\rangle \otimes \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) + b|\downarrow\rangle \otimes \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle) = \frac{1}{\sqrt{2}}[(a|\uparrow\rangle + b|\downarrow\rangle) \otimes |+\rangle + (a|\uparrow\rangle - b|\downarrow\rangle) \otimes |-\rangle]. \quad (2.14)$$

The states  $|+\rangle$  and  $|-\rangle$  called atom are correlated with definite states of spin

$$|S_1\rangle = a|\uparrow\rangle + b|\downarrow\rangle, \quad |S_2\rangle = a|\uparrow\rangle - b|\downarrow\rangle. \quad (2.15)$$

The two states  $|S_1\rangle$  and  $|S_2\rangle$  are distinct from  $|\uparrow\rangle$  and  $|\downarrow\rangle$ , which is the basis of (2.12) registered by the apparatus. When the spin state before the measurement was least certain, corresponding to  $a = b = 2^{-1/2}$ , the fixed correlated state vector using (2.1) can be expressed as

$$|\varphi_f\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle \otimes |+\rangle + |\beta\rangle \otimes |-\rangle). \quad (2.16)$$

This can be done as well in many other equivalent ways.

The atom at the stage of  $|\varphi_f\rangle$  in (2.12) and (2.16) does not contain the information about the spin observable that was supposed to be recorded. It is not possible to claim that the measurement in the normally used sense has already happened. Although the argument following these equations applies

directly to the two-state measurement interactions, it is possible to modify them. It is concluded that in a closed apparatus-system object which evolves unitarily, a reduction of the wave packet cannot be accomplished. Measurement is supposed to be a process which produces information. It is the transfer of information between the spin and atom that has taken place, and this information can be quantified. The pointer basis of the apparatus which eliminates ambiguity in the choice of the recorded variable has to be developed now [14].

## 2.2 Effect Of An Environment

Consider the influence of the environment  $E$  consisting of  $N$  two-level atoms. Atom  $k$  has the Hilbert space spanned by the basis set  $\{|e_E\rangle_k, |g_E\rangle_k\}$ . Suppose the self-Hamiltonians of the system taken individually, and the interaction Hamiltonian between the atoms is zero. The only part of the Hamiltonian which remains is the apparatus-environment interaction  $H^{AE}$  which separates as

$$\hat{H}^{AE} = \sum_k H_k^{AE}. \quad (2.17)$$

Suppose the components  $\hat{H}_k^{AE}$  are assumed to have the form

$$\hat{H}_k^{AE} = g_k (|e\rangle\langle e| - |g\rangle\langle g|) \otimes (|e_E\rangle\langle e_E| - |g_E\rangle\langle g_E|)_k \prod_{j \neq k} \mathbf{1}_j. \quad (2.18)$$

The eigenstates have the special property that they are direct products. The components of the direct product belong respectively to Hilbert spaces of the apparatus and the environment atoms. When the environment constructed this way interacts with the apparatus, superselection rules arise in a natural way. They make it impossible for the apparatus to be detected in a superposition of ground and excited states. Thus, let the interaction of the apparatus and environment start at  $t = 0$ . Before  $t = 0$ , no correlations with the environment exist. The combined system-apparatus-environment state vector would have the form,

$$|\Psi(0)\rangle = |\varphi_f\rangle \prod_{k=1}^N \otimes [\alpha_k |e_E\rangle_k + \beta_k |g_E\rangle_k]. \quad (2.19)$$

The set of states  $|e\rangle, |g\rangle, |e_E\rangle_k, |g_E\rangle_k$  are the eigenstates of the interaction Hamiltonian. This is the one that acts on the combined system for  $t > 0$ . This allows state  $|\Psi\rangle$  to be expressed at arbitrary time  $t$  in the form

$$|\Psi(t)\rangle = a |\uparrow\rangle \otimes |e\rangle \prod_{k=1}^N \otimes [\alpha_k e^{ig_k t/\hbar} |e_E\rangle_k + \beta_k e^{ig_k t/\hbar} |g_E\rangle_k]$$

$$+b|\downarrow\rangle\otimes|g\rangle\prod_{k=1}^N\otimes[\alpha_k e^{ig_k t/\hbar}|e_E\rangle_k + \beta e^{ig_k t/\hbar}|g_E\rangle_k]. \quad (2.20)$$

This follows along the same lines as the set (2.10)-(2.11) using an evolution operator  $\hat{U}_k$  as matrix exponential of the Hamiltonian, which breaks up into blocks of similar exponential form. The transition between  $|\Psi(0)\rangle$  and  $|\Psi(t)\rangle$  establishes the correlation between the state of the apparatus and the state of the environment. The apparatus observable  $\hat{\Lambda}$  which is most reliably recorded by the environment is usually called the *pointer* observable. For this interaction,  $\hat{\Lambda}$  would have the form, with  $\lambda_1 \neq \lambda_2$  real

$$\hat{\Lambda} = \lambda_1|e\rangle\langle e| + \lambda_2|g\rangle\langle g|, \quad (2.21)$$

It can be said the pair of states  $\{|e\rangle, |g\rangle\}$  defines the pointer basis.

If simultaneously the apparent reduction of the state vector is accomplished, the state of the apparatus-system has to be described by the density matrix upon tracing over  $E$  setting  $\tau = t/\hbar$ ,

$$\begin{aligned} \rho^{SA} &= \text{Tr}_E |\Psi(t)\rangle\langle\Psi(t)| = \text{Tr}_E\{|a|^2|\uparrow\rangle\langle\uparrow| \otimes |e\rangle\langle e| \cdot \prod_{k=1}^N[|\alpha_k|^2 + |\beta_k|^2] \\ &+ ab^*|\uparrow\rangle\langle\downarrow| \otimes |e\rangle\langle g| \prod_{k=1}^N\otimes[\alpha_k e^{ig_k\tau}|e_E\rangle_k + \beta_k e^{-ig_k\tau}|g_E\rangle_k][\alpha_k^* e^{ig_k\tau}\langle e_E| + \beta_k^* e^{-ig_k\tau}\langle g_E|] \\ &+ a^*b|\downarrow\rangle\langle\uparrow| \otimes |g\rangle\langle e| \prod_{k=1}^N\otimes[\alpha_k e^{-ig_k\tau}|e_E\rangle_k + \beta_k e^{ig_k\tau}|g_E\rangle_k][\alpha_k^* e^{-ig_k\tau}\langle e_E| + \beta_k^* e^{ig_k\tau}\langle g_E|] \\ &+ |b|^2|\downarrow\rangle\langle\downarrow| \otimes |g\rangle\langle g| \prod_{k=1}^N[|\alpha_k|^2 + |\beta_k|^2]. \end{aligned} \quad (2.22)$$

The trace of the second line for example is

$$\begin{aligned} &\text{Tr}_E \prod_{k=1}^N [\alpha_k e^{ig_k\tau}|e_E\rangle_k + \beta_k e^{-ig_k\tau}|g_E\rangle_k][\alpha_k^* e^{ig_k\tau}\langle e_E| + \beta_k^* e^{-ig_k\tau}\langle g_E|] \\ &= \prod_{k=1}^N [|\alpha_k|^2 e^{2ig_k\tau} + |\beta_k|^2 e^{-2ig_k\tau}] = \prod_{k=1}^N [\cos(2g_k\tau) + i(|\alpha_k|^2 - |\beta_k|^2)\sin(2g_k\tau)]. \end{aligned} \quad (2.23)$$

To summarize, it has been shown that

$$\rho^{SA} = \text{Tr}_E |\Psi(t)\rangle\langle\Psi(t)| = |a|^2|\uparrow\rangle\langle\uparrow| \otimes |e\rangle\langle e| + \mu(t)ab^*|\uparrow\rangle\langle\downarrow| \otimes |e\rangle\langle g|$$

$$+\mu^*(t)a^*b|\downarrow\rangle\uparrow|\otimes|g\rangle\langle e|+|b|^2|\downarrow\rangle\langle\downarrow|\otimes|g\rangle\langle g|. \quad (2.24)$$

In (2.24), function  $\mu(t)$  is the correlation amplitude,

$$\mu(t) = \prod_{k=1}^N [\cos(2g_k\tau) + (|\alpha_k|^2 + |\beta_k|^2) \sin(2g_k\tau)]. \quad (2.25)$$

The quantity (2.25) depends on the initial conditions of the environment via the various probabilities of finding the system in one of the eigenstates of the interaction Hamiltonian

$$p(|e_E\rangle_k) = |\alpha_k|^2, \quad p(|g_E\rangle_k) = |\beta_k|^2. \quad (2.26)$$

Property (2.26) is important. It gives an indication that the ability of  $\mu(t)$  to dampen correlations is the same for a mixture where only (2.26) may be given.

In fact, the correlation amplitude  $\mu(t)$  can also be found from the scalar product

$$\mu(t) = \langle E_e(t) | E_g(t) \rangle. \quad (2.27)$$

The two states in (2.27) are defined to be

$$|E_e(t)\rangle = \prod_{k=1}^N \otimes [\alpha_k e^{ig_k\tau} |e_E\rangle_k + \beta_k e^{-ig_k\tau} |g_E\rangle_k], \quad |E_g(t)\rangle = \prod_{k=1}^N \otimes [\alpha_k e^{-ig_k\tau} |e_E\rangle_k + \beta_k e^{ig_k\tau} |g_E\rangle_k]. \quad (2.28)$$

The two distinct records made by the environment of two alternative outcomes of the measurement are represented by (2.28). The time dependence of  $\mu(t)$  is very important to successful damping of the off-diagonal correlation terms. It is clear that  $|\mu(t)|^2 \leq 1$  as well  $\mu(0) = 1$  and

$$\langle \mu(t) \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \mu(t) dt = 0, \quad \langle |\mu(t)|^2 \rangle = 2^{-N} \prod_{k=1}^N (1 + (|\alpha_k|^2 - |\beta_k|^2)^2). \quad (2.29)$$

The last result in (2.29) implies that unless the initial state of the environment coincides with one of the eigenstates of the Hamiltonian, the expected absolute value of the correlation amplitude  $|\mu(t)|^2$  is much less than the initial value. Relatively small environments are quite effective in giving rise to an exact definition of the pointer variable.

As long as  $N$  is finite, a theorem from the theory of periodic functions implies the absolute value of  $\mu(t)$  will return arbitrarily close to one. An almost periodic function attains any value in its range infinitely many times. There is a close analogy between the problem of recurring correlations exemplified by

the existence of translation numbers  $T_\epsilon$  requiring  $1 - |\mu(t)|^2 < \epsilon$  for given  $\epsilon$  at both  $t = 0$  and  $T_\epsilon$ , but not in  $(0, T_\epsilon)$ .

This example shows how the interaction of the apparatus and the environment can cause an effective reduction of the state vector. Correlations established between the apparatus and the environment have taken place at the expense of the previously attained correlations between the apparatus and the system. Putting the final density matrix in a form such that the apparatus contains information about some arbitrary two states of the spin is not possible when all off-diagonal terms in  $\rho^{AS}$  vanish even when  $a = b = 2^{-1/2}$ . It is important to stress that through the interaction of the apparatus with the environment, simultaneously both the apparent reduction of the pure state density matrix into a mixture and the determination of the observable recorded by the apparatus is achieved. The dual role of the environment is therefore equivalent to imposition of superselection rules. In this way they make a very natural appearance [7].

### 3 The Pointer Basis

Observers who look at the pointer of the ideal apparatus are made aware that the system is in one of the eigenstates of the observable, and not in some relative state chosen arbitrarily. Quantum mechanics alone when applied to a composite made up of an apparatus and a system cannot in principle determine which observable has been measured, as we have seen. It will become clear that the choice of what has been measured comes about when one realizes two things. First the apparatus interacts with its environment by means of a specific interaction Hamiltonian  $\hat{H}_{AE}$ . Secondly the observer consults only the pointer of the apparatus and the state of the environment.

The apparatus-environment interaction may be regarded as an additional measurement in its own right which can establish measurable correlations between the apparatus and the environment. Information about the environment destroys the information about the premeasured quantum system  $S$ . Commutation of the Hamiltonian  $\hat{H}_{AE}$  with the observable of the apparatus  $\hat{\Pi}$  ensures this variable will not be perturbed. Only the basis made up of the eigenstates of operator  $\hat{\Pi}$  called the pointer basis, contains nothing but the information about the quantum system itself. The combined apparatus-system is now represented by a mixture which is diagonal in a product basis consisting of the eigenvectors of the pointer basis of the apparatus and corresponding relative states of the system. In fact the pointer basis of the apparatus is chosen by the

form of the apparatus-environment interaction. This is the basis which contains a reliable record of the states of the system. This has to determine the set of relative states of the system in a unique way correlated with the apparatus. As well the apparatus-environment correlations prevent the observation of the  $AS$  system combination in a superposition. It becomes a mixture diagonal in the basis assembled from the pointer basis eigenstates and the corresponding relative states of the system. The exact details of the environment state itself are not to obtain the pointer basis; the form of the apparatus-environment interaction suffices for that.

If the three systems are described by a combined density matrix  $\hat{\rho}_{SAE}$  of these systems the density matrix is the solution to the evolution

$$-i\hbar\dot{\hat{\rho}}_{SAE} = [\hat{\rho}_{SAE}, \hat{H}_S + \hat{H}_A + \hat{H}_E + \hat{H}_{SA} + \hat{H}_{AE} + \hat{H}_{SE}]. \quad (3.1)$$

To get (3.1), it has been assumed that all the interactions are pairwise so  $\hat{H}_{SAE} = 0$  and the environment can be considered a quantum system. The last point should be clarified as to what is meant by the term environment, that is, which degrees of freedom must be considered in determining the mixture to which the wavefunction collapses.

The environment is defined as being made up of all those degrees of freedom which contribute greatly to the evolution of the state of the apparatus. If it is agreed that the environment may in principle be regarded as isolated, a basis  $|E\rangle$  spanning its Hilbert space can be introduced. It should be possible to formulate a criterion that excludes those degrees of freedom whose total contribution to the total apparatus-environment interaction may be ignored.

It can also be assumed the quantum system itself remains isolated from the environment. If this is violated after the premeasurement has occurred, the apparatus will contain the information about which state the quantum system was, but not necessarily is any more. Suppose  $\hat{H}_{SA}$  acts only for a very short time during which  $\hat{H}_{SA}$  dominates  $\hat{H}_{AE}$  and a correlation of the form  $|A_0\rangle \otimes |\psi\rangle$  is established, where  $|\psi\rangle$  pertains to  $S$ . After this the interaction between system and apparatus is nonexistent. All the vectors of the pointer basis correspond to a common, degenerate energy eigenstate  $\hat{H}_A|A_p\rangle = E|A_p\rangle$ , where the eigenvalue does not depend on  $p$ . Physically, this is the case where no energy is exchanged between the system and the apparatus.

Right after the correlation between the system and apparatus has been established, the density matrix for the  $SAE$  combination evolves as

$$-i\hbar\dot{\hat{\rho}}_{SAE} = [\hat{\rho}_{SAE}, \hat{H}_S + \hat{H}_A + \hat{H}_E + \hat{H}_{AE}] = [\hat{\rho}_{SAE}, \hat{H}_S + \hat{H}_A + \hat{H}_E] + [\hat{\rho}_{SAE}, \hat{H}_{AE}]. \quad (3.2)$$

The first commutator bracket can be ignored, which follows as the time evolution of the time evolution of the states  $|A_p(t)\rangle$  leaves invariant the diagonal entries of the density matrix. Therefore, evolution of the apparatus due to  $\hat{H}_A$  does not destroy information about the system.

The second commutator in (3.2) introduces correlations between the apparatus and the environment. Diagonal terms of the density matrix remain left-invariant only if it commutes with the projection operators that appear on the diagonal. This means that if states  $|A_p\rangle$  are to remain correlated with the relative state of the quantum system, operator  $\hat{H}_{AS}$  must satisfy the commutation relation,

$$[\hat{H}_{AE}, \sum_p \gamma_p |A_p\rangle\langle A_p|] = 0, \quad (3.3)$$

for any choice of coefficients  $\gamma_p$ . Define the pointer observable for a real  $\gamma_p$  as

$$\hat{\Pi} = \sum_p \gamma_p |A_p\rangle\langle A_p|. \quad (3.4)$$

So (3.4) can be rewritten by stating that the pointer basis  $\{|A_p\rangle\}$  is a complete set of eigenfunctions of the operator  $\hat{\Pi}$  that commute with the pointer Hamiltonian  $\hat{H}_{AE}$ ,

$$[\hat{\Pi}, \hat{H}_{AE}] = 0. \quad (3.5)$$

The interaction Hamiltonian then depends only on one apparatus observable,  $\hat{\Pi}$ , so any interaction Hamiltonian which has the form

$$\hat{H}_{AE} = \sum_p \sum_{\sigma > \eta} |A_p\rangle\langle A_p| \otimes (\zeta_{\sigma\eta}^{(p)} |\eta\rangle\langle\sigma| + \zeta_{\eta\sigma}^{(p)} |\sigma\rangle\langle\eta|) \quad (3.6)$$

does have a form that satisfies (3.5). As well the states and  $\zeta_{\eta\sigma}^{(p)}$  in (3.6) may depend explicitly on time due to the interaction with the remote environment. If the interaction remains diagonal in the pointer basis, it will not disturb correlations of the apparatus with the states of the system relative to the pointer basis.

It is then the environment-apparatus interaction that allows for the existence of the pointer basis. However it is not sufficient for the actual, successful functioning of the apparatus. The action that actually correlates the quantum state of the apparatus with that of the system state plays an important role. Immediately after the premeasurement the  $AS$  wave function should appear as  $\sum_p b_p |A_p\rangle \otimes |p\rangle$ , where  $p$  is an orthonormal basis composed of the eigenstates of the particular variable  $\hat{P}$ . Upon measurement, the measured system should transform into one of the mutually orthonormal eigenstates of the operator  $\hat{P}$ .

It was intended to show that when the environment, thought of as taking the additional role of an apparatus, is taken into account, a definite answer can be provided to the question: what mixture does the wave packet transform into. To describe the given world, there ought to be two distinguishable types of evolution. There is the reversible, deterministic kind, which has been confirmed in the 2, as well as the irreversible, random one which must be the source of what is experienced by consciousness.

## 4 Environment Induced Selection

The interaction Hamiltonian which couples system  $S$  to the environment  $E$  may commute with subspaces of the Hilbert space of the system. As a result of such interaction, the state vector of the system can remain pure only if it is entirely limited to one of these subspaces  $\mathcal{H}_n$ . Arbitrary superpositions with components spanning two or more subspaces decay into mixtures which are diagonal in the state vectors belonging to the individual, disjoint subspaces. The decay originates in the establishment of correlations between quantum system  $S$  and its environment  $E$ . Moreover, as long as the environment-system coupling remains stronger than the coupling introduced by the observer conducting a measurement, the set of the observables that can be measured on  $S$  is limited to those that leave the subspaces invariant.

Systems which forbid the existence of groups of pure states and restrict the possible observables in a way such as discussed here is said to admit superselection rules. Consequently, the idea is to see how interaction with the environment can impose such rules on  $S$ . The superselection rules once in place makes the system behave classically. Environment induced superselection rules can be used to justify the classical nature of the apparatus reading, but as well apply to an even greater class of classical observables of systems which are inherently quantum.

Suppose the combined Hilbert space of system  $S$  and environment  $E$  is of the form

$$\mathcal{H}_C = \mathcal{H}_s \otimes \mathcal{H}_E. \quad (4.1)$$

The evolution is given by a Hermitean operator defined on the Hilbert space. Let us suppose it breaks up into a self-Hamiltonian of the system  $\hat{H}_S$ , of the environment  $\hat{H}_E$  and an interaction Hamiltonian  $\hat{H}_{SE}$  written as,

$$\hat{H}_S + \hat{H}_E + \hat{H}_{SE} = \sum_i \chi_i |s_i\rangle \langle s_i| + \sum_j \varepsilon_j |e_j\rangle \langle e_j| + \sum_{i,j} \gamma_{ij} |s_i\rangle \langle s_i| \otimes |e_j\rangle \langle e_j|$$

$$+\lambda \sum_{i,j,i',j'} \sigma_{ii',jj'} |s_i\rangle\langle s_{i'}| \otimes |e_j\rangle\langle e_{j'}|. \quad (4.2)$$

When only the evolution of the diagonal part of the interaction Hamiltonian is considered, it is written as  $\hat{H}_{SE}^0$  and given by

$$\hat{H}_{SE}^0 = \sum_{i < j} \gamma_{ij} |s_i\rangle\langle s_i| \otimes |e_j\rangle\langle e_j|. \quad (4.3)$$

Setting  $\lambda = 0$  in (4.2) is equivalent to this case, and is somewhat of an idealization, where the diagonal  $\hat{H}_{SE}^0$  is much greater than the off-diagonal part of the interaction. Both  $\hat{H}_S$  and  $\hat{H}_{SE}^0$  are likely to be highly degenerate, which leaves additional freedom in the choice of the basis. Physically,  $\lambda \ll 1$  is equivalent to stating interactions 0 in real world physical systems destroy phase coherence between the system states on a time scale much shorter than the time scale of relaxation to thermal equilibrium.

Evolution of the combined system-environment state vector, which at  $t = 0$  was represented by a direct product state

$$|\Phi(0)\rangle = |\varphi_S\rangle \otimes |\psi_E\rangle = \sum_i \alpha_i |s_i\rangle \otimes \sum_j \beta_j |e_j\rangle. \quad (4.4)$$

is evolved by a unitary  $\hat{U}$  as done in section 2.1, it gives

$$|\Phi(t)\rangle = \sum_{i,j} \alpha_i \beta_j \exp[-i(\chi_i + \varepsilon_j + \gamma_{ij})t/\hbar] |s_i\rangle \otimes |e_j\rangle. \quad (4.5)$$

To provide an idea as to exactly how the superselection rules are manifested, trace the density matrix over  $E$

$$\rho^S(t) = \text{Tr}_E |\Phi(t)\rangle\langle\Phi(t)|. \quad (4.6)$$

The matrix elements of  $\rho^S(t)$  given in the representation  $\rho_{ij}$  are

$$\rho^S(t) = \sum_{i,j} \rho_{ij}(t) |s_i\rangle\langle s_j|. \quad (4.7)$$

In (4.7), the  $\rho_{ij}(t)$  are given as

$$\begin{aligned} \rho_{ii}(t) &= |\alpha_i|^2 \sum_k |\beta_k|^2 = |\alpha_i|^2, \\ \rho_{ij}(t) &= \alpha_i \alpha_j^* \exp(-i(\chi_i - \chi_j)t/\hbar) \cdot \sum_m |\beta_m|^2 \exp(-i(\gamma_{im} - \gamma_{jm})t/\hbar). \end{aligned} \quad (4.8)$$

The diagonal elements are time-independent when the off-diagonal part of the perturbing Hamiltonian is not present.

They can rotate on account of the factor  $\exp(-i(\chi_i - \chi_j)t/\hbar)$ , or more importantly they can decay as a result of a decrease in the correlation amplitude

$$\mu_{ij}(t) = \sum_k |\beta_k|^2 \exp(-i(\gamma_{ik} - \gamma_{jk})t/\hbar). \quad (4.9)$$

The net result of this second type of time dependence is to lower the absolute value of the correlation amplitude from one at time zero to a value much less than this for large  $t$ . The average of the correlation amplitude determined over a sufficiently long time interval approaches zero  $\langle \mu_{ij} \rangle_T \rightarrow 0$  as  $T \rightarrow \infty$  unless the frequencies  $\omega_m^{ij} = \gamma_{im} - \gamma_{jm}$  are equal to zero.

Demanding that all the  $\omega_m^{ij} = 0$  would be equivalent to the statement that interaction Hamiltonian  $H_{SE}^0$  has diagonal part zero. Let the correlation amplitude be expressed as

$$\mu_{ij}(t) = \sum_k p_k \exp(-i\omega_m^{ij}t/\hbar), \quad p_k = |\beta_k|^2. \quad (4.10)$$

When the environment is a mixture before to the interaction with the system,  $\{p_k\}$  gives probabilities for finding the environment in the states corresponding to distinct eigenvalues of  $\hat{H}_{SE}^0$ . Then (4.10) stays valid regardless of whether  $E$  is initially in a pure state or is in a mixed state. Since  $\mu_{ij}(t)$  is given by (4.9), the average absolute value is computed as

$$\langle |\mu_{ij}(t)|^2 \rangle = \frac{1}{T} \int_0^T |\mu_{ij}(t)|^2 dt \rightarrow \sum_{k,m} p_k p_m \delta(\omega_k^{ij} - \omega_m^{ij}). \quad (4.11)$$

Assuming all the  $\omega_k^{ij}$  are distinct, the standard derivation of correlation amplitude from average value is given as  $\sum_{k=1}^N p_k^2$ . Hence environments can cause correlations to damp out between those states of the system which diagonalize  $\hat{H}_{SE}^0$ .

It has been proved the environment will remove correlations between states which correspond to different eigenvalues of  $\hat{H}_{SE}^0$ . Many eigenvectors may correspond to the same eigenvalues  $\gamma_{ij}$  of  $\hat{H}_{SE}^0$  and they span a subspace  $\mathcal{H}_n$  of the system Hilbert space of the system. The entire Hilbert space of the system can be reconstructed from the individual subspaces. Pure states which belong to more than one subspace  $\mathcal{H}_n$  at a single instant are not admitted. This is the fundamental source of environment-induced superselection rules, and  $\mathcal{H}_S$  is a direct sum of these basic subspaces  $\oplus_n \mathcal{H}_n$ , with all the pure states in one and only one of the  $\mathcal{H}_n$ . As long as the coupling with some external apparatus is not too far in excess of the  $\gamma$ , the system may not be prepared

as measured in the state which does not remain invariant under the influence of the interaction with the environment. Only these observables which leave every  $\mathcal{H}_n$  invariant are admitted. Thus  $\hat{B}$  is an observable with respect to a system  $S$  in interaction with the environment  $E$  if and only if  $|\psi_n\rangle \in \mathcal{H}_n$  implies that

$$\hat{B}|\psi_n\rangle \in \mathcal{H}_n. \quad (4.12)$$

These two conditions can be thought of as equivalent to a more formal definition of the superselection rules.

The pointer observable  $\hat{\Lambda}$  can now be defined as any observable measurement which allows us to precisely determine the subspace  $\mathcal{H}_n$  which contains the state of the system. If  $\pi_n$  are a set of projection operators which project onto subspaces  $\mathcal{H}_n$  and  $\lambda_n$  are all real and distinct, by the spectral theorem, the pointer observable can be expressed as

$$\hat{\Lambda} = \sum_n \lambda_n \pi_n. \quad (4.13)$$

The projection operators can be constructed so that they are diagonal in the basis  $|s_k\rangle$ , which diagonalizes  $\hat{H}_{SE}^0$  and  $\hat{\Lambda}$  commutes with  $\hat{H}_{SE}^0$ ,

$$[\hat{\Lambda}, \hat{H}_{SE}^0] = 0. \quad (4.14)$$

## 5 Envariance And Statistical Mechanics

Traditionally thermodynamic equilibrium states are characterized by extremes of physical properties such as maximal thermodynamic entropy or randomness. The microcanonical equilibrium can be defined as the quantum state that is maximally envariant. This means it is envariant under all unitary operators on system  $S$ . By a theorem, a composite state  $|\Psi_{SE}\rangle$  can be written in the form of a Schmidt decomposition as

$$|\Psi_{SE}\rangle = \sum_k \alpha_k |s_k\rangle \otimes |\epsilon_k\rangle, \quad (5.1)$$

where  $\{|s_k\rangle\}$  and  $\{|\epsilon_k\rangle\}$  are orthocomplete sets in  $S$  and  $E$ , respectively. The procedure is to identify the special state that is maximally envariant.

In fact,  $|\Psi_{SE}\rangle$  is invariant under all unitary operators if and only if the Schmidt decomposition is even, so that the coefficients satisfy  $|\alpha_m| = |\alpha_n|$  for all  $m, n$  so  $|\Psi_{SE}\rangle$  can be expressed as

$$|\Psi_{SE}\rangle = C \sum_k e^{i\varphi_k} |s_k\rangle \otimes |\epsilon_k\rangle, \quad (5.2)$$

and the  $\varphi_k$  are phases. The classical concept of equilibrium ensembles translates into an equilibrium state that is envariant under the maximal number, that is, all unitary operations present.

The microcanonical state is usually identified as the state that is fully degenerate in terms of energy. Denote the Hamiltonian of the composite system by

$$H_{SE} = H \otimes I_E + I_S \otimes H_E. \quad (5.3)$$

The internal energy of  $S$  is given by the quantum mechanical average

$$E = \langle \Psi_{SE} | H \otimes H_E | \Psi_{SE} \rangle = \frac{1}{Z} \sum_k \langle s_k | H | s_k \rangle. \quad (5.4)$$

In equation (5.4),  $Z$  is the energy-dependent dimension of the Hilbert space of  $S$ . This is often called the microcanonical partition function. Since  $|\Psi_{SE}\rangle$  is envariant under all unitary maps, let us suppose, without loss of generality, that  $\{s_k\}$  represents the energy eigenbasis corresponding to  $H$ . Then the matrix elements are given by  $\langle s_k | H | s_k \rangle = e_k$  where  $E = e_k = e_m$  for all  $k, m \in \{1, \dots, Z\}$ . The fully quantum representation of the microcanonical state has been identified by using two conditions. The microcanonical equilibrium is not represented by a unique state, but by an equivalence class of maximally envariant states all with the same energy. Thus the microcanonical equilibrium of system  $S$  is a state that is envariant under all unitary operations on  $S$ . It is fully degenerate in energy with respect to  $H$ . In this approach, only this quantum symmetry is required and induced at a very basic level by entanglement or envariance.

The canonical state can also be determined this way. Suppose the total system  $S$  can be separated into a smaller subsystem  $\mathcal{S}$  and its complement, a heat bath  $B$ . The Hamiltonian of  $\mathcal{S}$  is

$$H = H_{\mathcal{S}} \otimes I_B + I_{\mathcal{S}} \otimes H_B + H_{SB}, \quad (5.5)$$

where the contribution  $H_{SB}$  is an interaction term which makes possible energy exchange between  $\mathcal{S}$  and  $B$ . Suppose it is sufficiently small so that its contribution to the total energy can be neglected so  $E = E_{\mathcal{S}} + E_B$  and its effect on the composite equilibrium state neglected. This is usually called ultra-weak coupling in classical terms. In this case, every composite energy eigenstate  $|s_k\rangle$  can be written as a product

$$|s_k\rangle = |\hat{s}_k\rangle \otimes |b_k\rangle. \quad (5.6)$$

and  $|\hat{s}_k\rangle$  and  $b_k\rangle$  are energy eigenstates in  $\mathcal{S}$  and  $B$ . Now all orthonormal bases are equivalent under envariance so  $|s_k\rangle$  may be chosen as energy eigenstates of  $H$ .

For the canonical formalism, it is the number of states accessible to the total system  $\mathcal{S}$  under the condition total internal energy  $E$  is constant. When  $\mathcal{S}$  happens to be in a particular energy eigenstate  $|\hat{s}_k\rangle$ , the internal energy of the subsystem is given by the corresponding energy eigenvalue  $\hat{e}_k$ . For the total energy  $E$  to be constant, the energy of the heat bath  $E_B$  has to obey

$$E_B = E - \hat{e}_k. \quad (5.7)$$

To satisfy this, the energy spectrum of the heat reservoir must be at least as dense as that of the subsystem. The number of states  $\mathcal{N}$  accessible to system  $\mathcal{S}$  is then

$$\mathcal{N}(\hat{e}_k) = \frac{\mathcal{N}_B(E - \hat{e}_k)}{\mathcal{N}_\mathcal{S}(E)}. \quad (5.8)$$

In (5.8),  $\mathcal{N}(E)$  is the total number of states in  $\mathcal{S}$  consistent with (5.4) and the numerator is the number of states available to bath  $B$  determined by (5.7).

Suppose  $B$  consists of  $N$  non-interacting subsystems with identical eigenvalue spectra  $\{e_j^B\}_{j=1}^m$ . The initial energy (5.7) is

$$E = \hat{e}_k + n_1 e_1^B + n_2 e_2^B + \cdots + n_m e_m^B, \quad \sum_{j=1}^m e_j = N. \quad (5.9)$$

The degeneracy takes the form

$$\mathcal{N}(\hat{e}_k) = \frac{N!}{n_1! n_2! \cdots n_m!}. \quad (5.10)$$

This is a quantum envariant formulation of Boltzmann's counting formula for the number of energy states involved.

To obtain the Boltzmann-Gibbs result, consider the limit  $N \gg 1$ . Then by Starling's formula

$$\log(\mathcal{N}(\hat{e}_k)) \doteq N \log(N) - \sum_{j=1}^n n_j \log(n_j).$$

In the case of microcanonical equilibrium, this was satisfied by the state that is envariant under all unitary maps. Let us identify canonical equilibrium by the configuration of the reservoir  $B$  for which the maximal number of energy eigenvalues are occupied subject to the constraints

$$\sum_{j=1}^m n_j = N, \quad E - \hat{e}_k = \sum_{j=1}^m n_j e_j^B. \quad (5.11)$$

Introducing Lagrange multipliers  $\tau, \beta$ , it is required that

$$\delta\left(\sum_j n_j \log(n_j) + \mu \sum_j n_j + \beta \sum_j n_j e_j^B\right) = 0. \quad (5.12)$$

The solution to this variational constraint is the Boltzmann-Gibbs relation

$$n_j = c e^{\beta e_j^B}. \quad (5.13)$$

This represents the number of states in bath  $B$  with energy  $e_j^B$  for  $\mathcal{S}$  and  $B$  to be in thermodynamic canonical equilibrium. The temperature enters through the Lagrange multiplier  $\beta$ . This result is exact up to use of 1 approximation and depends only on the fact that the total  $\mathcal{S}$  be in microcanonical equilibrium as defined already by using the envariance concept.

## 6 Conclusions

The interaction of the environment with the system generates a correlation similar to that between the system and the apparatus. The environment can then be thought of as a higher-order apparatus which performs a 0 measurement on the state of the system destroying coherent superpositions. The eigenbasis of the pointer observable which is determined up to the coherent degeneracy of  $\hat{\Lambda}$  has been called the pointer basis.

When the eigenvalues of the self-Hamiltonian are highly degenerate. The eigenspaces  $\mathcal{H}_{\tau_i}$  which correspond to the distinct eigenvalues  $\tau_i$ , may contain, or be identical with or even be a subset of the eigenspaces of the observable  $\hat{\Lambda}$ ,  $\mathcal{H}_n$ . These possibilities are exhaustive as long as  $[\mathcal{H}_S, \hat{H}_{SE}^0] = 0$ . This follows from the commutation relation  $[\hat{\Lambda}, \hat{H}_{SE}^0] = 0$ . In the first instance, the interaction with the environment will remove part of the degeneracy in the spectrum of  $\hat{H}_S$ , similar to the splitting of levels observed in atomic physics. The energy levels of the system are shifted in the second case. In the last, the state may rotate under the influence of the self-Hamiltonian in  $\mathcal{H}_n$  without loss of coherence. What is most important in this is the redundancy of the record concerning the observable which is imprinted on the correlations. The interaction with the environment forces the system to be in one particular eigenstate of the pointer observable rather than in a superposition of such states. Thus the super selection rules need not be imposed from outside. This fact gives rise to rules which are induced by the environment [16-18]. Finally, envariance offers the possibility of establishing thermodynamics on a well defined set of ideas originating at a fundamental quantum level.

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