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Emanator Theory is Shown to be an Optimal Martingale Process at the Fractal Edge of Chaos, Where the Gravitational Constant G is Hypothesized to be a Multiscale Fractal Coupling Parameter

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Abstract

Strong experimental evidence is established for the achiral emanation process to be Martingale. The significance of this is that statistical mechanics processes that are Martingale can have equilibria and other well-defined limit phenomena, precisely what is needed for emanator theory to describe a physical theory. Well-defined limit processes, together with the maximum noise propagation dimension of 29* derived in [1,2], indicates that the oddities of the Dirac Large Number Hypothesis [3] should be re-examined. In doing so we arrive at the fractal G hypothesis, which explains the origin of the gravitational constant.

Keywords: Trigintaduonion, Martingale, Gravitational Constant, fractal, emanator theory, emanation, the standard model, U(1), SU(2), SU(3), alpha, fine-structure constant, Feigenbaum constant, Planck's constant, quantum matter, path integral, propagation, Cayley algebra.

Introduction

Computational results on achiral emanator processes with over 10^9 computational emanation steps was initially reported in [4]. Further evaluation of those results show that the Trigintaduonion evolution, component-wise, is exactly like a random walk. A random walk process is Martingale [5], thus strong experimental evidence is established for the achiral emanation process to be Martingale. The significance of this is that statistical mechanics processes that are Martingale can have equilibria and other well-defined limit phenomena, precisely what is needed for emanator theory to describe a physical theory.

The Dirac Large Number Hypothesis [5] has been a mystery that has been noted by many other physicists [6-16]. At issue is the odd occurrence of a lot of ratios of different 'fundamental' lengths in the theory, that tend to group in families where their ratio is 10^40, or 10^60, or 10^80, etc. (depending on how you want to group the terms). This 'fractal' behavior at large scale (a classic fractal trait being repeating structures at different scales) has not been couched as a fractal phenomenon previously because there wasn't a context to warrant such a supposition. But here we know the maximal information emanation hypothesis is likely to force the emanation process to operate at the edge of chaos, where the process becomes fractal or has a well-defined fractal limit (given the Martingale property), such that we arrive at the fractal G hypothesis, which will explain the origin of the gravitational constant.

Background

Existence of generalized unit-norm propagation structure that is 10D [17]

Unit-norm right product propagation is trivial for the division algebras since norm(XY) = norm(X) $\times$ norm(Y). From this it is apparent that we have an automorphism group given by the norm itself (A(XY)=A(X)A(Y)), and in the case of the octonions this automorphism group is G2 [18]. It can be shown that SU(3) is in G2 [18]. Let's now consider the situation with a higher-order Cayley algebra, the Sedenions, 'S'. We obviously don't have norm(S_1S_2) = norm(S_1) $\times$ norm(S_2) in general, as this would then allow S to join the ranks of the division algebras, and it is proven that such don't exist above the Octonions [19]. Can we still have a propagation structure? Is it possible to have a 'base' sedenion for which norm(S_{base})=1, and to have a right propagator (product) sedenion also norm(S_{right})=1, such that norm(S_{base} x S_{right}) =1? The answer is yes (see appendix of [1] and [17]), when the sedenion has the (chiral) form of an octonion crossed with a real octonion: S_{chiral} = (O,O_{real}) or S_{chiral} = (O_{real},O). Can we continue this to arrive at a propagation structure on the Trigintaduonions? Again the answer is yes, with the chiral form generalizing off the chiral Sedenion as might be expected: T_{chiral} = (S_{chiral}, S_{real}) or

(S_{real}, S_{chiral}) [17]. It is proven that this extension process will go no further [17]. What happens is that due to the chiral form we are still able to re-express all T products (or S) as collections of terms involving tri-octonionic products (which have nice properties as described in [17]), and this can no longer occur above the (chiral) trigintaduonion level.

Note that most other analyses of propagator constructs stay within the division algebras, thus never go past the octonions, or past 8D. In the excellent documentary by Spinal Tap they describe their amps going to '11', not just 10 like everyone else. Mindful of that, my propagator goes to 10D, not just 8D. So some interesting things are bound to happen:

Chiral Trigintaduonion emanation involves 137 independent octonionic terms
This is too lengthy to conveniently repeat in the background here, see [1].

Achiral Trigintaduonion unit-norm propagation has maximum perturbation α [25,26]
This is too lengthy to conveniently repeat in the background here, see [25,26].

Achiral emanation using a 72-card deck
There are 4 chiralities, so to get an achiral emanator candidate, minimally need a "4-card deck" to emanate in the four chiralities, with emanator equal to normalized sum. The actual deck appears to require a normalized sum over sub-chiralities, as will be explicitly enumerated in what follows. Regardless of the form of the achiral sum over chiral variants, since each chiral emanation has 29 free components, their norm sum will again have 29 free components. Thus, the form for α^{-1}shown above is the complete, non-approximating, result since it only need rely on 29 free component number. Further analysis of the emanation deck, beyond 'dealing a singe card' is in [1].

Single-step achiral 72-deck emanation has noise propagation dimension ≅29* [1]
Obtaining an achiral emanation from a collection of chiral emanations requires that all chiralities be summed over (there are four) as well as sub-chiralities (there are 72). Noise analysis requires collecting of first-order terms. Analysis of noise transmission (when dealt a 'single-card hand') indicates 29* dimensions, where:

$$29^{*} \cong 29 + \left(\frac{4\pi}{72}\right)\left[1 + \left(\frac{\pi}{137\cdot 29}\right)\left(\left(\frac{\pi}{72}\right) + \left(\frac{3}{72}\right)\right)\right]$$

The above result was obtained in [4] to describe the 72-card chiral 'deck' of chiral emanation products for a single-step emanation. In [1] this is reviewed and elaborated further.

'Edge of chaos' maximal perturbation hypothesis [4]
Consider the 'edge of chaos' maximal perturbation in each of the 29* dimensions to be at position C_∞ (see Appendix in [X] for relevant background on Mandelbrot Set), which is on the negative real axis, i.e., at π rotation to have –1 factor, ***thus at maximal antiphase***. This results in the relation for maximal perturbation at maximal antiphase (maximum reference angle with sign chosen positive by convention) has a lower bound on α given by:

$$\alpha_0^{-1} = \left(\sqrt{C_\infty}\right)^{29^*}.$$

where

$$C_\infty = 1.4011551890920506004 \ldots$$

This ties $1/\alpha$ to the second Feigenbaum constant C_∞in the context of the Mandelbrot set. It is well known that the Feigenbaum constants are universal, and part of a description of a universal transition to chaos regime. The Mandelbrot set is also universal [21], and maximal in that its fractal boundary has maximal fractal dimension of 2 [21], a detail that will be important in the meromorphic matter description given later.

For C_∞, most references only provide $C_\infty = 1.401155189 \ldots$, and a higher precision tabulation is not readily found, so use is made of the relation

$$C_n = a_n(a_n - 2)/4,$$

together with the tabulation on a_∞ [22]:

$$a_\infty = 3.5699456718709449018 \ldots$$

The resulting C_∞ is:

$$C_\infty = 1.4011551890920506004 \ldots$$

The resulting α_0^{-1} is:

$$\alpha_0^{-1} = 137.03599933370198263 \ldots$$

For the multi-card analysis we have:

$$\alpha^{-1} = \alpha_0^{-1} + \alpha_1^{-1} + \cdots$$

where α_0^{-1} involves the sum over emanation by one-card. For sum on two-chiral products we have further 'noise' contribution α_1^{-1}. With the multi-card modifications, albeit small, there is the complication of shift from 72-deck to 78-deck, and whether there is a chiral step ('card') type that can be exactly repeated (i.e., are cards from the deck played with replacement when considering a multi-card flop sequence). There may be a reason why the sums must be done without card-replacement. This might be because card replacement would allow degenerate tri-octonionic product terms, again throwing off the 137 braid term total, perhaps, leading to non-optimality. This is being explored in further work and will not be discussed further at this time.

Chiral trigintaduonion emanation leads to the standard model of particle physics and to quantum matter

The chiral trigintaduonions T, with right product operation ((T x T) x T)…, used previously for maximum information transmission, are here shown to be $H \times O$ when arranged for achiral emanation. When considering a sum over chiral emanations to obtain an achiral emanator, with T as phase factor, we have the exponentiation operation exp(iT), which leads to a theory that is $C \times H \times O$. As such, we have the foundation for the associative operator algebra of the Standard Model: $U(1) \times SU(2)_L \times SU(3)$. A complication with T products is you can have zero divisors. A framework is adopted to remove the zero divisors by requirement of maximum domain of analyticity on the log trigintaduonion multiplication, resulting in a description for the meromorphic precipitation of matter. In this process a fundamental quantum is indicated from the zero-divisor residue terms. Analyticity in the form of a Wick rotation also provides a mechanism whereby we can transition to a dimensionful action and quantum and arrive at an explanation for the critical 'smallness' of Planck's constant. The emanator is $T_{em} \cong H \times O$, and provides a description of a possible meromorphic origin for point-like matter.

Getting an associative algebra from the repeated operation of a non-associative algebra is first described in [18] in the context of repeated octonion products: ((OxO)xO)…, where the algebra SU(3) can result. This is found to be equivalent to fixing one of the octonion imaginary components in such a repeated-product operation [18,23]. Dixon shows in [23] that the $C \times H$ x O product algebra lays the foundation for the associative operator algebra of the Standard Model: $U(1) \times SU(2)_L \times SU(3)$. In later work this is explored in the form of ideals [24]. Emanator theory, described in a brief background to follow, is based on unit-norm propagation at maximal dimension. It turns out this maximal dimension is not octonion-based but trigintaduonion-based [4], although it does have an octonion sub-algebra: $T_{em} \cong H \times O$, as will be shown here.

In prior work [1,4,17,25-28] we hypothesized maximal algebraic information flow, where the "emanation" of information is represented as multiplication by an element of an algebra in two steps: (i) take the maximal current-state element that is a unit-norm trigintaduonion; and (ii) perform the emanator step that consists of an achiral sum of multiplications with chiral trigintaduonion emanators. In prior efforts this was considered without the complication of zero divisors. We will see that zero divisors act as "sources", so the prior work was effectively analysis of sourceless information flow.

In [1] we show $T_{em} \cong H \times O$ (which will lead to the standard model) and we consider zero divisors and their impact on the maximal information flow and in doing so see a

mechanism for meromorphic precipitation of quantum matter with dimensionful action. When we consider
$\exp(iT_{em})$, we get a mathematical object that is C×H×O, thus whose product algebra becomes U(1)xSU(2)xSU(3) according to Dixon [23]. If this route to the standard model is unsatisfactory, consider that the T emanation sum pre-normalization can be grouped as a 2x2 real matrix that is finite, thus SU(2) with normalization: $T_{em} \cong$ [2x2 real]×O;
where repeated steps of
$\exp(iT_{em}) \rightarrow$ unit norm Cx[2x2 real]xO
re-group as repeated steps of CxCxCx…, and [2x2 real]x[2x2 real]x…, and OxOxO…., with unit normalization, thus the product algebra is
U(1)xSU(2)xSU(3).

Methods

The Dirac Large Numbers Hypothesis

Recall the hypothesized relation [1]:

$$\alpha^{-1} = \left(\sqrt{C_\infty}\right)^{29^*}, \quad where\ dim. = 29^*.$$

Here we are imagining maximum information flowing via 29* effective dimensions, where the phenomenology is concerned with the *number* of dimensions and arrives at a dimensionless parameter α. We might as readily ask for the maximum information that would flow through the 137* independent tri-octonionic terms at chiral-trigintaduonion component level. Here the phenomenology is concerned with the amount of ‘noise’ that each terms could carry to arrive at a dimensionful parameter ‘X’:

$$X^{-1} = \left(\sqrt{C_\infty}\right)^{137^*}$$

Evaluation leads to

$$X = 8.05077 \times 10^{-11} [dimensionful\ unit].$$

In SI units the gravitational constant G is:

$$G = 6.674 \times 10^{-11} [N\, m^2\, Kg^{-2}],$$

so, is the nearness of these numbers a coincidence? Taken alone, yes certainly, but taken together with a collection of other odd ‘coincidences’, noted over the past 100 years by Weyl [6], Eddington [7], Dirac [4], Narlikar [8], Cetto et al. [9], Raychaudhuri [10], Jordan [11], Shemi-Zadeh [12], Peacock [13], Andreev & Komberg [14], and the review by Ray et asl [15] and by Valev [16], maybe there’s something to this.

Consider relation that motivated Dirac to suggest the large number hypothesis. Working with ‘constants’ that are seen in the micro and macro scale, such as the Hubble constant H, with age of the universe $\sim H^{-1}$, the mass of the nucleon (proton) m_p, we have the following:

$$H^{-1} \cong 4.3 \times 10^{17}: the\ age\ of\ the\ universe \text{ (macro-scale constant)}$$

$$\tau = \frac{e^2}{(m_e c^3)} \cong 10^{-23}: the\ strong\ time\ scale \text{ (micro-scale constant)}$$

where m_e is the electron mass, c is the speed of light, e^2 is the electrostatic coupling between a single electron and proton, for example, e.g., exhibiting electrostatic force e^2/r^2. The gravitational force between an electron an proton, on the other hand, is $Gm_e m_p/r^2$:

$$\frac{Gm_e m_p}{r^2}: gravitational\ force \text{ (macro-scale)}$$

$$\frac{e^2}{r^2}: electrostatic\ force \text{ (micro scale)}$$

Consider another ratio of ‘fundamental’ constants operational on some system that spans the micro and macro realms (e.g., the Universe vs internal, or the macro gravitational system versus its micro system of hydrogen). Consider the mass of the observable Universe M in a ratio with the nucleon mass, which gives $\sim 10^{80}$, thus the square root is used to group it with the $\sim 10^{40}$ ratios. There are going to be obviously going to be large number ratios if we take “universe” / ”elemental particle” in some operation, so this isn’t what is surprising. What is odd is that these large numbers ratios appear to group as $\sim 10^{40}$, $\sim 10^{60}$, and $\sim 10^{80}$. For the above cases we have:

$$\sim 10^{40} \approx \frac{H^{-1}}{\tau} \approx \frac{e^2}{Gm_e m_p} \approx \sqrt{\frac{M}{m_p}}\ .$$

Results

Emanation is Martingale

In analysis of zero-crossing events in Tbase (a necessary condition for a zero-divisor event with a Tchiral product) [4] it was noted that the behavior precisely matched, component-wise, that of a random walk towards the zero-crossing event. Experimentally, this shows that the Emanation process is a Random Walk process. But Random Walk processes are known to be Martingale [5], thus Emanation is Martingale. Systems that are Martingale have limits, such as the familiar equilibrium limit. The hypothesis of universal thermality (see Disc.) is consistent with this existence of equilibria result.

Emanator theory projects to a quantum theory with a complex propagator and associated Action functional theory. This is proposed to be a reified (dimensionful units) emergence that is guided by the dictates of the quantum deFinetti constraint (to have a complex propagator [29]); analyticity (for maximal continuation with maximal information flow); and the product-algebra gauge group of the standard model with representation and particle families according to the extended standard model that includes massive sterile neutrinos (to arrive at 22 parameters). The emanator projection is seen as a fundamental aspect of the process. In this context, the suggestion of thermal universality emergence with analytic time may not seem so extreme.

Fractal G Hypothesis

If we take the Dirac relations as part of the fractal nature of reality (with optimization pushing to “the edge of chaos” and inducing fractal effects), with repeating structure at different sales and similarity relations between ‘things’ at different scales a possibility. Given this perspective, interpreting the constant ‘G’ as a multiscale fit parameter across all of these domains, such that the single observed G suffices, is an interesting prospect. Note that not all the terms in the ratios involve G so we can solve for G, approximately, accordingly.

Thus, the parameter we know as G is hypothesized to actually be a fractal fit parameter, albeit still one of the 22 parameters determined in the emanation process. G is not like the other 22 constants in that it doesn’t describe elementary particle mass, say, but gives a parameterization of the fractal structures that occur at different scales. In the derivation of $|\boldsymbol{h}^*|$ we see that maximal information flow occurs ‘at the edge of chaos’, i.e., the description is fractal, so this is consistent. As with the $|\boldsymbol{h}^*|$ derivation capturing the extreme smallness of Planck’s constant $\boldsymbol{h}$ up to the actual constant chosen to give precisely $\boldsymbol{h}$ $(\boldsymbol{h} = \boldsymbol{c}|\boldsymbol{h}^*|)$, here we have the significant smallness of the gravitational constant captured in the term X up to the actual constant chosen to give precisely G (e.g., $G = kX$).

Discussion

Universal Thermality

There is a fundamental complex structure that still remains from the Emanator formalism upon projection from the higher-dimensional Cayley algebras into the maximum propagation described, for the alpha maximum-perturbation trigintaduonion. That complex structure is ‘contact’ analytic and is realized under conditions where limits involving that structure are taken to zero (or to some fixed value at component-level). Two main examples where this has been most significant are in (i) shifting a QFT to a thermal QFT by shifting to imaginary time related to the inverse temperature [30]; and (ii) in the use of dimensional regularization to renormalize QED and QCD [31]. Thus, the process of emanator theory settling on the maximum information propagation dimension is one where higher order complex structure (from non-propagate-able dimensions) is still accessible for regularization processes.

The significance of the remnant complex structure is ***universal thermality***. Thermality via complex structure and Lorentz Invariance on that chiral Trigintaduonion are more fundamental constructs than QFT (based on the Standard Model) or GR (with manifold dynamics according to the Standard Cosmological Model). This is because the limits defining such thermality would exist for a single chiral Trigintaduonion propagation step, and we don't 'see' QFT until enough emanation steps have occurred such that the product group gauge that begins to resolve is the gauge group of the standard model, thus giving rise to the QFT of the Standard Model.

Universal thermality explains the odd behavior seen upon 'complexification', such as BH geometry complexification giving the Hawking temperature of a Black Hole [32] consistent with the full quantum field theory analysis at the BH horizon demonstrating thermal emission with that same temperature done by Hawking [33]. Given the Hawking radiation, a BH is clearly unstable and will evaporate over time. If the BH is 'placed in a box' or has not-flat asymptotic geometry, however, then the 'closed' system may reach an equilibrium and be stable. This is precisely what is considered by Hawking and Page (1983) [34] upon examining the thermodynamics of AdS BH's. Further work along these lines was done in [35-37]. In these efforts a Hamiltonian formulation for Gravity is made and then complexification yields a partition function describing an ensemble of BH solutions. Stability, in the form of positive heat capacity, can then be shown.

A Dark fitter: virile neutrinos

Recall for 'light' matter we have $U(1) \times SU(2)_L \times SU(3)$, and for 'dark' matter, with the sterile neutrino we have just $SU(2)_R$ singletons that only interacts gravitationally. Consider now virile neutrinos where we do this differently. We still can't work with a U(1) part since this would indicate bare-charge properties not seen. If we add SU(3), however, to get $SU(2)_R\times SU(3)$, then we have a possible candidate for *clumpy* cold dark matter (CDM), not just CDM. The Hagedorn temperature for virile neutrinos would be the same as the mass energy of the lightest RH neutrino, so very low in the scheme of things. When analyzing strings the Hagedorn temperature marks a phase transition at which long strings are copiously produced. This provides a mechanism for virile neutrinos at or near the Hagedorn temperature to produce strings, e.g., clumpy CDM. If very near the Hagedorn temperature, the string formation could lead to galactic scale structures that would lock galactic rotation curves in the odd manner observed.

Conclusion

Emanator Theory results from a hypothesized maximal information propagation and this means maximal analyticity, maximal domain, etc. As a process, Emanator theory is also hypothesized to operate up to "the edge of chaos" to permit maximal perturbation (noise) domain. When taken with the results showing that Emanator theory is Martingale, thus has well-defined limits, we then must wonder if there are well-defined multi-scale (fractal) limits. In other words, is there a relation that would tie the micro scale constant (truly) fundamental constants (as they are counted in the 22) with the cosmological scale 'constants' that have settled out, at macro scale, in the current evolution of the Universe? In this context, the Gravitational constant G is hypothesized to be a multiscale fractal coupling parameter.

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