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Cosmological Exact Solutions of Petrov Type D. A Mixture of Fluids of Dark Energy and a Nonlinear Element that Emerges as a Zeldovich Fluid and that Transforms into a Dust Type Fluid

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Abstract

In this article, cosmological exact solutions to Einstein's equations are obtained for an anisotropic symmetry of Petrov type D from a non-linear fluid that responds to the equation of state of type $\mu_1 - P_1 = \frac{B}{a}\sqrt{\frac{P_1}{\mu_1}}$ where $\mu_1 = \mu - \Lambda$, $P_1 = P + \Lambda$, μ, P and Λ are the volumetric energy density, the pressure, and a constant linked to the concept of dark energy; B and a are constants. That equation of state and what it represents in terms of time limits (when $t \to 0$ and when $t \to \infty$) are analyzed. Two general solutions different from each other because of the initial expansion degree that a coordinate can have in relation to a perpendicular plane to this one are obtained. The solution, as time increases, transforms asymptotically into an isotropic space-time of FRWL type with a dark energy fluid. Temperature, Hubble parameters, and deceleration are also obtained and studied in relation to time transformations.

Keywords: cosmology, exact solution, Einstein, temperature, non-lineal, Kretschmann, singularity, Hubble, deceleration

1 Introduction

Discoveries since the late 20th century until the start of 21st century related to cosmological observations mainly due to the observation of Ia supernovae and gathered data by COBE, WMAP, and still active Planck satellites, as well as its analysis, have allowed significant advancement and interest in cosmology. In this regard, other aspects and its respective literature have been discussed in other articles; for example [1].

One aspect of great theoretical interest is the type of transition that the universe could have had since its beginning until now and that could have in the future. Research regarding this topic is being developed around hypotheses related to the kind of matter that the universe has had. One possible hypothesis is to consider that the universe matter is a mixture of fluids, some of which could have been more relevant than others in different historical periods; accordingly, a fluid of dark energy type prevails nowadays, and thus, the universe accelerates. Another possible hypothesis is the existence of a nonlinear fluid that has transitioned continuously through time from a fluid with an equation type of linear state (between pressure and energy density) to another one or other linear fluids. What was above-mentioned is equivalent to studying a universe model that has gone through several possible mixtures of different linear fluids in different times within a continuously process, so all different scenarios from the universe history unify into a unique fluid. Previous hypotheses are used altogether in this work to consider a mixture of dark energy with a nonlinear fluid; that represents a universe where the pressure of the production of particles that are created P_c is due to the nonlinear fluid since the dark energy fluid does not produce changes in this one [2]. On the other hand, from general discussions [3] about particle formation specially in close proximity with a cosmological singularity, it has been established that particles with null inertial mass do not come from nondisrupted solutions of FRWL type (conformally flat models) either with nonnull inertial mass with the exception of disrupted and nonhomogeneous cases [4], but in anisotropic and homogeneous models both can emerge.

The nonlinear fluid resulting from the solution in this work behaves closely to a Zeldovich fluid (stiff matter) in proximity with the beginnings ($t \to 0$) and continuously as time increases, it becomes into a dust type fluid $P_1 = 0$. It undergoes this transformation through time as a similar fluid to a hard Universe $1/3 < P_1/\mu_1 < 1$, radiation $P_1/\mu_1 = 1/3$ and ordinary fluids $0 < P_1/\mu_1 < 1/3$.

2 Symmetry, Einstein's Equations and Solutions

The symmetry used in this work is anisotropic and homogeneous of Petrov type D and has the form [1]

$$ds^2 = Fdt^2 - t^{2/3}K(dx^2 + dy^2) - \frac{t^{2/3}}{K^2}dz^2, \qquad (1)$$

where F and K are functions of t. Einstein's tensor components ($G_\alpha^\beta = R_\alpha^\beta - \frac{1}{2}\delta_\alpha^\beta R$) different from zero, of (1), are

$$G_0^0 = \frac{4\,K^2 - 9\,t^2\dot{K}^2}{12t^2K^2F}, \tag{2}$$

$$G_1^1 = -\frac{3Kt\dot{K}\left(2F - \dot{F}t\right) + 3Ft^2\left(2K\ddot{K} - 5\dot{K}^2\right) + 4K^2\left(\dot{F}t + F\right)}{12t^2K^2F^2}, \tag{3}$$

$$G_2^2 = G_1^1 = -\frac{G_3^3}{2} + \frac{9\,Ft^2\dot{K}^2 - 4\,K^2\dot{F}\,t - 4\,K^2F}{8t^2K^2F^2}, \tag{4}$$

where the points over the functions represent derivatives by time.

The perfect-fluid model used in cosmology represents a fluid without viscosities, isentropic ($P = P\,(\mu)$) and shear stress which could be written as

$$T_{\alpha\beta} = (\mu + P)\,u_\alpha u_\beta - g_{\alpha\beta}P, \tag{5}$$

where $T_{\alpha\beta}$ is the energy momentum tensor of the perfect fluid, u_α the tetradimensional speed, $g_{\alpha\beta}$ the metric tensor, μ and P the energy density and the fluid pressure respectively.

The equation of state for the analyzed fluid is taken from the form

$$\mu_1 - P_1 = \frac{B}{a}\sqrt{\frac{P_1}{\mu_1}}, \tag{6}$$

where $\mu_1 = \mu - \Lambda$, $P_1 = P + \Lambda$ and Λ is the constant linked to the concept of dark energy.
Considering a fluid with a tetradimensional speed $u_\alpha = (u_0, 0, 0, 0)$, the components of the energy momentum tensor (22) different from zero are

$$T_0^0 = \mu, \tag{7}$$
$$T_1^1 = T_2^2 = T_3^3 = -P, \tag{8}$$

so it implies that Einstein's equations $G_\alpha^\beta = \kappa T_\alpha^\beta$, must meet that $G_1^1 = G_3^3$, so from (3) and (4); it is obtained

$$\dot{K}K\left(2F - \dot{F}t\right) - 2Ft\left(-K\ddot{K} + \dot{K}^2\right) = 0. \tag{9}$$

Consequently,

$$K = K_0 e^{C_1\int\frac{F^{1/2}}{t}dt}, \tag{10}$$

without loss of generalities, the constant K_0 in (10) will be considered equivalent to 1 and $C_1 = \pm 2/3$; for each possible value of C_1, a different model is obtained.

From Einstein's equations $G_\alpha^\beta = \kappa T_\alpha^\beta$, the equality $T^{\mu\nu}{}_{;\mu} = 0$ (see [5]). From (10) and (6), it is obtained, for any C_1, that the solution of F is

$$F = \left(3\,\Lambda\,t^2 + 3\,B\sqrt{a^2+t^2} + 1\right)^{-1}, \tag{11}$$

Considering Einstein's equations, the solution (11), (10), G_0^0 and G_1^1 (see [5]), it is obtained that pressure P is

$$P = P_1 - \Lambda, \quad \textit{where} \quad P_1 = \frac{Ba^2}{t^2\sqrt{a^2+t^2}} \tag{12}$$

and density μ is

$$\mu = \mu_1 + \Lambda, \quad \textit{where} \quad \mu_1 = \frac{B\sqrt{a^2+t^2}}{t^2}. \tag{13}$$

in (11), (12) and (13), $a > 0$ a constant with a temporal characteristic (like time) that substantiates the nonlinearity of the fluid μ_1 suppressing the possibility of this one to be isobaric for all times. $B > 0$ represents the constant of the energy density of a dust type fluid where μ_1 transforms as time increases. Moreover, the nonlinear fluid μ_1 initially ($t \to 0$) is of Zeldovich type and its constant is the product of Ba.

The function K in (10) can be written as

$$K = e^{\pm 2\sigma/3}, \tag{14}$$

where σ is determined differently, as shall be shown below, in dependence on the existent relation between the fluid constants (Λ, a, B). For values of $t \to 0$, σ can be considered as $\sigma = \frac{\ln(t)}{\sqrt{1+3\,Ba}}$ where σ_0 is a constant of integration that does not play an important role in the solution and that can be equivalent to zero redefining coordinates x, y and z. If $3Ba \neq 1$, so

$$\begin{aligned}\sigma &= -\frac{1}{2s_-}\arctan\left(\frac{Q_1}{2\sqrt{A}s_-}\right) - \frac{1}{2s_+}\ln\left(\frac{Q_2 + 2\,s_+\sqrt{A}}{\sqrt{a^2+t^2}-a}\right) - \sigma_0, \quad \textit{where} \\ \sigma_0 &= -\frac{1}{2s_-}\arctan\left(\frac{(2\,a\Lambda - B)\sqrt{3}}{2\sqrt{\Lambda}s_-}\right) - \frac{\ln\left(3\,B + 2\,s_+\sqrt{3}\sqrt{\Lambda} + 6\,a\Lambda\right)}{2s_+},\end{aligned} \tag{15}$$

and functions A, Q_1, Q_2 and $s_\pm$ are

$A = 1 + 3\,B\sqrt{a^2+t^2} + 3\,\Lambda\,t^2$, $Q_1 = -2 + 3\,a\,(2\,a\Lambda + B) + 3\sqrt{a^2+t^2}\,(2\,a\Lambda - B)$,

$$Q_2 = 2 - 3\,a\,(2\,a\Lambda - B) + 3\,\sqrt{a^2+t^2}\,(2\,a\Lambda + B) \quad and \quad s_\pm = \sqrt{3Ba \pm 1},$$

if $s_- = 0$ y $6\Lambda a^2 \neq 1$, function σ has the form

$$\sigma = \frac{D_1 - a\sqrt{3\Lambda}}{6\,\Lambda\,a^2 - 1} - \frac{1}{\sqrt{2}}\left(arctanh\left(\frac{1}{\sqrt{2}D_1}\right) - arctanh\left(\frac{1}{a\sqrt{6\Lambda}}\right)\right), \qquad (16)$$

where $D_1 = \sqrt{\frac{\left(1-3\,\Lambda\,a^2+3\,\Lambda\,a\sqrt{a^2+t^2}\right)a}{\sqrt{a^2+t^2}+a}}$.

If $s_- = 0$ y $6\Lambda a^2 = 1$, function σ is

$$\sigma = \frac{1}{\sqrt{2}}\ln\left(\frac{|t|}{\sqrt{a^2+t^2}+a}\right) - \frac{a}{\sqrt{2}(\sqrt{a^2+t^2}+a)}. \qquad (17)$$

3 Analysis of Solutions

Two possible solutions for the metric (1) exist, in dependence on the sign used in (14), the solution with a positive sign represents the universe which near its beginning expands, so the plane xy expands with less intensity than the axis z. If the considered sign is negative, the contrary happens. The function K, for any possible solution, tends to $K \to 1$ when $t \to \infty$, so solutions tend to a transition to an isotropic space-time of FRWL type. The metric tends to

$$ds^2 \to \frac{1}{3\Lambda t^2}dt^2 - t^{2/3}(dx^2 + dy^2 + dz^2), \qquad (18)$$

and makes a change of coordinates, so $t = e^{\sqrt{3\Lambda}\eta}$ can be written as

$$ds^2 \to d\eta^2 - e^{2\eta\sqrt{\Lambda/3}}(dx^2 + dy^2 + dz^2) \qquad (19)$$

which is the usual form of the solution of a dark energy fluid in the isotropic and homogeneous FRWL symmetry.

When solutions $t \to 0$ are in proximity to the ones found for Zeldovich model are only for short times within the interval $t \in [5.4\cdot 10^{-44}, 0.25\cdot 10^{-23}]s$ (see [1]), and in the case of the studied fluid, this one does not present that restriction despite having a close behavior to the Zeldovich one in proximity to $t = 0$. The nonlinear nature of the fluid allows it to move states of linear fluids through time ($P = \lambda\mu$) from Zeldovich type ($\lambda = 1$) until the dust one ($\lambda = 0$), so $P_1 = \lambda_1(t)\mu_1$, where $\lambda_1(t) = \frac{a^2}{a^2+t^2}$. For example, the nonlinear fluid is similar to the one of radiation when $t \to a\sqrt{2}$. The total fluid is the sum of a dark energy fluid and a nonlinear fluid.

4 Singularities and Kretschmann's Invariant

In the study of possible singularities of a given space-time, the Kretschmann's invariant is used ($Krets = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$), and its importance has been discussed in [1]. For the found solutions (with $C_1 = \pm 2/3$), the invariant has the form

$$Krets_{\pm} = W_{\pm} + U, \tag{20}$$

where

$$U = \frac{8}{3\Lambda^2} + \frac{32}{27t^4} + \frac{B\left(\left(60\,a^4 + 15\,t^4 + 48\,t^2 a^2\right) B + 16\,\left(a^2+t^2\right)^{3/2}\right)}{9t^4\left(a^2+t^2\right)}$$

and

$$W_{\pm} = \frac{\left(12\,\left(t^2 - 2\,a^2\right) B + 16\sqrt{a^2+t^2}\right)\Lambda}{9t^2\sqrt{a^2+t^2}} \pm \frac{32\sqrt{1 + 3\,B\sqrt{a^2+t^2} + 3\,\Lambda\,t^2}}{27t^4}$$

and the positive sign is taken when $C_1 = 2/3$ and the negative if $C_1 = -2/3$. From Kretschmann's invariant (20), it is determined that a singularity exists in $t = 0$ for any value of C_1 and of $B \neq 0$. When $C_1 = 2/3$, Krestchmann's invariant presents a singularity of $t = 0$ equivalent to Kasner's one E_{D_1} (with a depth of the order of t^{-4}) discussed in [1]. When $C_1 = -2/3$ and $B = 0$, Kretschmann's invariant is not singular in $t = 0$.

5 Temperature, Hubble Parameter H and Deceleration q

The temperature for the studied fluid type is obtained as in [5], so

$$\frac{dP}{\mu + P} = \frac{dT}{T}, \tag{21}$$

where T is the fluid temperature. From the solution (13), (12) and (21), it is known that the temperature for $a \neq 0$ depends on the time of the form

$$T = \frac{T_0\left(2a^2 + t^2\right)}{t\left(t^2 + a^2\right)}, \tag{22}$$

where $T_0 > 0$ is a constant of integration. From (22), when $t \to \infty$, the temperature tends to zero for any value of a, which is in agreement with the temperature of a fluid made up of a mixture of dark energy and dust and that tends to high values when $t \to 0$; in which case, it behaves similarly to a Zeldovich fluid which temperature is $T_Z = T_0/t$.
Hubble Parameter H and deceleration q are obtained as [5]

$$H = \frac{\left((g_{11}g_{22}g_{33})^{1/6}\right)^{\cdot}}{\sqrt{g_{00}}(g_{11}g_{22}g_{33})^{1/6}} = \frac{\sqrt{1 + 3\,B\sqrt{a^2+t^2} + 3\,\Lambda\,t^2}}{3t}, \tag{23}$$

where the components of the metric tensor have been considered $g_{\mu\nu}$ of (1) and

$$q = -(1 + \frac{\dot{H}}{\sqrt{g_{00}}H^2}) = 2 - \frac{9t^2\left(2\sqrt{a^2+t^2}\Lambda + B\right)}{2\left(1 + 3B\sqrt{a^2+t^2} + 3\Lambda t^2\right)\sqrt{a^2+t^2}}. \tag{24}$$

From (23) and (24), when $t \to 0$, $H \to \infty$, and when $t \to \infty$, $H \to \sqrt{\Lambda/3}$. The deceleration parameter q tends to $q \to 2$; when $t \to 0$, it is equal to zero in $t = t_1$ where t_1 is the positive real solution of the equation $2\sqrt{a^2+{t_1}^2}\,(3\Lambda {t_1}^2 - 2) - 12\,Ba^2 - 3\,{t_1}^2B = 0$ and when $t \to \infty$, $q \to -1$. Thus in this model, the universe expands initially decelerating and then, it transforms continuously in a universe in accelerating expansion.

6 Conclusions

Two cosmological exact solutions to Einstein's equations were obtained and found different from each other due to the initial expansion degree considering a mixture of two fluids, one of dark energy and other of nonlinear. The nonlinear fluid has the characteristic that for times $t \approx 0$ it is similar to a fluid of Zeldovich type and for large times $t \to \infty$, it is similar to a fluid of dust type. That fluid allows models to have a continuously transition through time by characteristic models of mixtures of linear fluids where the dark energy fluid is always present. It was established that both solutions present singularities in $t = 0$ and that as time increases, when $t \to \infty$, they become isotropic and equivalent to the solution of a dark energy fluid for the FRWL symmetry. The temperature of the mixture of fluids was studied and it was determined that $T \to \infty$, when $t \to 0$, and $T \to 0$, when $t \to \infty$. By studying the Hubble parameter, and the deceleration, it was found that for times $t \to 0$, the Hubble parameter tends to $H \to \infty$ and the deceleration parameter tends to $q \to 2$. As time increases, the Hubble parameter decreases its value in an asymptotic form to $H \to \sqrt{\Lambda/3}$, and the deceleration parameter decreases its value passing through a null value in time $t = t_1$ which is the real positive solution of the equation $2\sqrt{a^2+{t_1}^2}\,(3\Lambda {t_1}^2 - 2) - 12\,Ba^2 - 3\,{t_1}^2B = 0$ until a value of $q \to -1$ when $t \to \infty$; consequently, for these solutions, space-time initially decelerates, and as time increases, it starts to accelerate.

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