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Cosmological Exact Solution of Petrov Type D of a Nonlinear Mixture of Fluids of Dark Energy, Dust, Zeldovich and a Non-Disrupted Primordial Magnetic Field

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Abstract

A cosmological exact solution is analyzed and obtained from Einstein's equations in an anisotropic and homogeneous symmetry of Petrov D. An ideal fluid is studied and which is the result of a mixture of fluids of dark-energy types, dust, Zeldovich and a nonlinear fluid that emerges due to the mixture of the three previous fluids and the presence of a primordial magnetic field, so it does not induce currents either electric fields. It is stated that the solution tends to be isotropic as time increases and equal to the dark energy solution for FRWL. It is determined that an initial singularity exists when $t = 0$. The temperature of the fluid mixture is studied and it is established the time dependence relation of this one with time and the roles that fluids play. Parameters of Hubble and deceleration, and their behaviors in time in that solution are also studied.

Keywords: cosmology, exact solution, Einstein, Magnetic field, non-lineal, Kretschmann, singularity, Mixture Fluids,Temperature

1 Introduction

In recent years, interest in cosmology has increased mainly because of the findings related to microwave background radiation obtained by COBE, WMAP,

and Planck satellites, universe acceleration and its possible scenarios (states) that it could have gone through. In this regard, other aspects in cosmology where its literature is based have been discussed in [1].
Another aspect of great interest in cosmology is the problem of the existence of primordial cosmic magnetic fields. Regarding this matter and from a theoretical point of view, some exact solutions of fluids with equations of state of fluid mixtures and nonlinear fluid elements between pressure and energy density have been discussed and analyzed some of them are [2, 3]. Recently, the possibility that the primordial magnetic field could relieve the Hubble tension has been studied [4] and simulations have occurred in order to determine its importance in regard to large-scale magnetic fields [5]. Models with fluid mixture and nonlinear fluids are the basis to explain the diversity of matter that exists in the universe, so their importance is essential in the study of the possible scenarios in the process of matter expansion and evolution. On the other hand, the possibility of existence of primordial magnetic fields and the discovery of a giant arc [6] in $z \sim 0.8$ that measures $\sim 1Gpc$, and that raises doubts about the validity of considering the space-time of the universe as isotropic and homogeneous in all its phases. The previous mentioned aspects are studied and analyzed in this work.

2 Symmetry, Einstein's Tensor, Electromagnetic Field and Dark Energy

2.1 Symmetry and Einstein's Tensor

The anisotropic symmetry of Petrov Type D has been considered in [1] as the form

$$ds^2 = F dt^2 - t^{2/3} K (dx^2 + dy^2) - \frac{t^{2/3}}{K^2} dz^2, \tag{1}$$

where F and K are functions of t.
Einstein's tensor components ($G_\alpha^\beta = R_\alpha^\beta - \frac{1}{2}\delta_\alpha^\beta R$) different from zero, of (1) are

$$G_0^0 = \frac{4\,K^2 - 9\,t^2 \dot{K}^2}{12 t^2 K^2 F}, \tag{2}$$

$$G_1^1 = -\frac{3Kt\dot{K}\left(2F - \dot{F}t\right) + 3Ft^2\left(2K\ddot{K} - 5\dot{K}^2\right) + 4K^2\left(\dot{F}t + F\right)}{12t^2K^2F^2}, \tag{3}$$

$$G_2^2 = G_1^1 = -\frac{G_3^3}{2} + \frac{9\,Ft^2\dot{K}^2 - 4\,K^2\dot{F}\,t - 4\,K^2F}{8t^2K^2F^2}, \tag{4}$$

where the points over these functions represent derivatives by time.

2.2 Magnetic Field

The magnetic field will be considered in a matter where the only tensor components of an electromagnetic field $F_{\mu\nu}$ different from zero are $F_{12} = -F_{21} = B_{0z} = const$, where it is obtained that the variant $F_{\mu\nu}F^{\mu\nu} = 2B(t)^2$ where $B(t) = B_{0z}\pi^{1/2}/(t^{2/3}K)$ is the effective-magnetic-field magnitude. The effective magnetic field does not generate currents either induced electric fields since the flow of the magnetic field Φ does not change in relation to time

$$d\Phi = B(t)dA(t) = B(t)\sqrt{g_{11}g_{22}}dxdy = B_{0z}\pi^{1/2}dxdy. \tag{5}$$

The choice of the field, in this given form, allows to meet the field equations $F^{\mu\nu}_{;\mu} = 0$ besides the equality to zero of the divergence of the energy-momentum tensor of the electromagnetic field ${}_{em}T^{\mu\nu}_{;\mu} = 0$ where the tensor ${}_{em}T^{\mu\nu}$ is the energy-momentum tensor of the electromagnetic field of which unique components different from zero for ${}_{em}T^{\mu}_{\nu}$ are

$${}_{em}T^0_0 = -{}_{em}T^1_1 = -{}_{em}T^2_2 =_{em} T^3_3 = \frac{{B_{0z}}^2}{8\,t^{4/3}K^2}. \tag{6}$$

2.3 Ideal Fluid Model

The ideal fluid model can be considered as a fluid of which energy-momentum tensor has the form

$${}_{fl}T_{\alpha\beta} = (\mu + P)\,u_\alpha u_\beta - g_{\alpha\beta}P \tag{7}$$

where ${}_{fl}T_{\alpha\beta}$ is the energy-momentum tensor of a perfect fluid, u_α the tetradimensional speed, $g_{\alpha\beta}$ the metric tensor, μ and P the energy density and the pressure of the fluid respectively.
A fluid with a tetradimensional speed will be considered as $u_\alpha = (u_0, 0, 0, 0)$. Hence, the energy-momentum-tensor components (7) different from zero are ${}_{fl}T^0_0 = \mu$, ${}_{fl}T^1_1 =_{fl} T^2_2 =_{fl} T^3_3 = -P$. In this work, it is assumed that $\mu = \mu_{DE} + \mu_{Dust} + \mu_{Zeld} + \mu_{nl} = \Lambda + C_0/t + C_1/t^2 + \mu_{nl}$ and $P = P_{DE} + P_{Dust} + P_{Zeld} + P_{nl} = -\Lambda + C_1/t^2 + P_{nl}$ ($P_{Dust} = 0$), and it complies with

$${}_{fl}T^{\mu\nu}_{;\mu} = 0,\ and\ {}_{fl}T^{\mu\nu} =_{DE} T^{\mu\nu} +_{Dust} T^{\mu\nu} +_{Zeld} T^{\mu\nu} +_{nl} T^{\mu\nu} \tag{8}$$

3 Einstein's Equations and the Solution of the Magnetic Field and the Fluid

Einstein's equations have the form $G^\beta_\alpha = \kappa T^\beta_\alpha$ where $T^\beta_\alpha = {}_{em}T^\beta_\alpha + {}_{fl}T^\beta_\alpha$. From (2-4, 6, 7), the next system of equations mutually independent is obtained

$$\frac{4\,K^2 - 9\,t^2\dot{K}^2}{12t^2K^2F} - \frac{{B_{0z}}^2 + 8\,\mu\ t^{4/3}K^2}{8t^{4/3}K^2} = 0, \tag{9}$$

$$-\frac{3Kt\dot{K}\left(2F-\dot{F}t\right)+3Ft^{2}\left(2K\ddot{K}-5\dot{K}^{2}\right)+4K^{2}\left(\dot{F}t+F\right)}{12t^{2}K^{2}F^{2}}+\frac{t^{2/3}{B_{0z}}^{2}+8\,P\;\;t^{2}K^{2}}{8K^{2}\,t^{2}}=0, \tag{10}$$

$$-\frac{-6Kt\dot{K}\left(2F-\dot{F}t\right)-3Ft^{2}\left(4K\dot{K}-\dot{K}^{2}\right)+4K^{2}\left(\dot{F}t+F\right)}{12t^{2}K^{2}F^{2}}+\frac{-t^{2/3}{B_{0z}}^{2}+8\,P\;\;t^{2}K^{2}}{8K^{2}\,t^{2}}=0, \tag{11}$$

From the equation (9), it is determined that

$$F=\frac{2\left(4\,K^{2}-9\,t^{2}\dot{K}^{2}\right)}{3t^{2/3}\left({B_{0z}}^{2}+8\,t^{4/3}\mu\;\;K^{2}\right)}. \tag{12}$$

Bearing in mind (8) and (12) in (10) and (11), both equations are satisfied if

$$\begin{aligned}&\left(-27t^{3}\dot{K}^{3}+48K^{2}t\dot{K}+36\ddot{K}K^{2}t^{2}-16K^{3}\right){B_{0z}}^{2}+\\&+72t^{4/3}K^{2}\mu\left(-9\,t^{3}\dot{K}^{3}-4t^{2}K\dot{K}^{2}+4\ddot{K}\,K^{2}t^{2}+8\,K^{2}t\dot{K}\right)+\\&+36K^{2}t^{10/3}\left(4K^{2}\dot{K}-9\dot{K}^{3}t^{2}\right)\dot{\mu}=0.\end{aligned} \tag{13}$$

As stated above, the fluid being considered is a mixture of fluids of dark energy ($\mu_{DE}=\Lambda$), dust ($\mu_D=C_D/t$), Zeldovich ($\mu_Z=C_Z/t^2$) and a nonlinear element μ_{nl}, so $\mu=\Lambda+C_D/t+C_Z/t^2+\mu_{nl}$ (for more information about μ_{DE}, μ_D and μ_Z see [1]).

The solution of the equation (13) can be obtained following the method discussed in [3], so the equation (13) is satisfied if

$$\mu\left(t\right)=\Lambda+\frac{2\Lambda\,a}{t}+\frac{15\Lambda\,a^{2}}{16t^{2}}+\frac{{B_{1}}^{2}\left(a+t\right)^{2/3}\left(15\,a+16\,t\right)}{8at^{2}}\;\;\textit{and}\;\;K=\frac{\sqrt[6]{a+t}}{\sqrt[6]{t}}. \tag{14}$$

From (14) and (12), F takes the form

$$F=\frac{a}{3\left(\Lambda\,\sqrt[3]{a+t}a+2\,{B_{1}}^{2}\right)\left(a+t\right)^{5/3}}, \tag{15}$$

and pressure P is

$$P=-\Lambda+\frac{15\Lambda\,a^{2}}{16t^{2}}+\frac{B_{1}^{2}\left(15\,ta-32\,t^{2}+45\,a^{2}\right)}{24\sqrt[3]{a+t}t^{2}a}. \tag{16}$$

The metric interval can be established as

$$ds^2 = \frac{a\,(a+t)^{-5/3}\,dt^2}{3\left(\Lambda\,\sqrt[3]{a+t}a + 2\,{B_1}^2\right)} - \sqrt[6]{a+t}\sqrt{t}\left(dx^2 + dy^2\right) - \frac{t\,dz^2}{\sqrt[3]{a+t}}, \tag{17}$$

or making a change of coordinate of the type

$$t = 8\,\sinh\left(\frac{1}{6}\sqrt{3}\sqrt{\Lambda}\eta + \frac{1}{4}\ln\left(\frac{\Lambda\,a}{{B_1}^4}\right)\right)^6 {B_1}^6\Lambda^{-3}a^{-3} - a,$$

of the form

$$ds^2 = d\eta^2 - T\,(\eta)\,\sqrt{T\,(\eta)^6 - a}\left(dx^2 + dy^2\right) - \frac{\left(T\,(\eta)^6 - a\right)\,dz^2}{T\,(\eta)^2}, \tag{18}$$

where $T\,(\eta) = \sqrt{2}\sinh\left(\frac{1}{6}\sqrt{3}\sqrt{\Lambda}\eta + \frac{1}{4}\ln\left(\frac{\Lambda\,a}{{B_1}^4}\right)\right) B_1\frac{1}{\sqrt{\Lambda}}\frac{1}{\sqrt{a}}$ and for values $t \geq \frac{\left(\sqrt{\Lambda\,a} - {B_1}^2\right)^6}{8\Lambda^4 a^4\sqrt{\Lambda\,a}} - a$.

4 Analysis of the Solution

For large times, the metric (18) tends toward the isotropic form

$$\begin{aligned} ds^2 \approx d\eta^2 - \frac{4{B_1}^4}{\Lambda^2 a^2}\sinh\left(\frac{\sqrt{3}\sqrt{\Lambda}\eta}{6} + \frac{1}{4}\ln\left(\frac{\Lambda\,a}{{B_1}^4}\right)\right)^4 \left(dx^2 + dy^2 + dz^2\right) \approx \\ \approx d\eta^2 - e^{2\sqrt{\frac{\Lambda}{3}}\eta}\left(dX^2 + dY^2 + dZ^2\right) \end{aligned} \tag{19}$$

where $X = \frac{x}{2\sqrt{\Lambda\,a}}$, $Y = \frac{y}{2\sqrt{\Lambda\,a}}$ y $Z = \frac{z}{2\sqrt{\Lambda\,a}}$, and it is equal to analogous solutions in FRWL space-time for the case of a fluid of dark energy.

For the analysis of the solutions in proximity to $t = 0$ or $t \longrightarrow \infty$, for instance, the Kretschmann's invariant will be used. The condition $Krets < \infty$ is required and sufficient for the finitude of all invariants of algebraic curvatures (see [1] and its respective literature). It is defined as $Krets = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ and has the form for the metric (1),

$$\begin{aligned} Krets = \frac{\left(2520\,a^3t + 1161\,a^4 + 1704\,a^2t^2 + 640\,at^3 + 320\,t^4\right){B_1}^4}{48\,(a+t)^{2/3}\,a^2t^4} + \\ + \frac{\Lambda\,\left(1161\,a^4 + 1248\,a^2t^2 + 2268\,a^3t + 512\,at^3 + 384\,t^4\right){B_1}^2}{48a\sqrt[3]{a+t}t^4} + \\ + \frac{\Lambda^2\left(512\,t^4 + 512\,at^3 + 864\,a^2t^2 + 2016\,a^3t + 1161\,a^4\right)}{192t^4}, \end{aligned} \tag{20}$$

from (20), it is stated that when $t \to 0$,

$$Krets \to \frac{387}{16}\,\frac{\left(a^{2/3}{B_1}^2 + \Lambda\, a^2/2\right)^2}{t^4} \to \infty \tag{21}$$

therefore, the solution is singular in $t = 0$. From (21), the singularity (its depth $\sim t^{-4}$) is essentially due to the presence of the fluid of Zeldovich type and the nonlinear element in (14). For the case where the dark energy fluid and the magnetic field are studied and as it is also mentioned in [2], the singularity is not present in the solution.

5 Temperature, Hubble Parameter H and Deceleration q

The temperature for the studied fluid type is defined as [2], so

$$\frac{dP}{\mu + P} = \frac{dT}{T}, \tag{22}$$

where T is the fluid temperature. From the solution (14), (16) and (22), it is known that

$$T = T_0 \left(2\,\frac{(45\,a^2 + 54\,ta + 8\,t^2)\,{B_1}^2}{t\sqrt[3]{a+t}} + 48\,\Lambda\,a^2 + 45\,\frac{a^3\Lambda}{t} \right), \tag{23}$$

wherein T_0 is a constant. In (23), the terms in parentheses from left to right represent the contribution of the nonlinear element of the fluid, of dust type, and of Zeldovich type respectively. Then, it is concluded that temperature increases when time does it, in the form $T \to 16T_0{B_1}^2t^{2/3}$ mainly because of the nonlinear element of the fluid. In close proximity to the singularity, temperature $T \to T_0\left(90\,a^{5/3}{B_1}^2 + 45\,a^3\Lambda\right)t^{-1}$ being singular in $t = 0$ due to the nonlinear element of the fluid and the Zeldovich fluid, the contribution in the fluid temperature of dust type gives at all times a constant value equal to $48T_0\Lambda a^2$ meanwhile the contribution of the dark energy fluid is null.
Hubble parameter H and deceleration q are described as [5]

$$H = \frac{\left((g_{11}g_{22}g_{33})^{1/6}\right)^{\cdot}}{\sqrt{g_{00}}(g_{11}g_{22}g_{33})^{1/6}} = \frac{\sqrt{\left(\Lambda\,\sqrt[3]{a+t}a + 2\,{B_1}^2\right)(a+t)^{5/3}}}{t\sqrt{3a}} \tag{24}$$

where the being-considered components of the metric tensor $g_{\mu\nu}$ of (1) and

$$q = -(1 + \frac{\dot{H}}{\sqrt{g_{00}}H^2}) = 2 - \frac{3t}{a+t} + \frac{t{B_1}^2}{\left(\Lambda\,\sqrt[3]{a+t}a + 2\,{B_1}^2\right)(a+t)}. \tag{25}$$

From (24) and (25), when $t \to 0$, $H \to \infty$, and when $t \to \infty$, $H \to \sqrt{\Lambda/3}$ and the deceleration parameter q tends to $q \to 2$, when $t \to 0$, it is equal to zero in $t = a - \chi^3$, where χ satisfies the equation $\chi^4 + \frac{{B_1}^2\chi^3}{\Lambda\, a} - 3\,a\chi - 5\,\frac{{B_1}^2}{\Lambda} = 0$ and a $q \to 1$, when $t \to \infty$, so in this model, the universe expands initially decelerated and then, it transforms continuously into a universe in accelerated expansion where the Hubble parameter tends to be constant.

6 Conclusions

In this work, a cosmological exact solution was determined using the symmetry of Petrov D with a fluid that represents a mixture of dark energy, dust, Zeldovich, a nonlinear element, and a primordial magnetic field that does not induce currents either electric fields. It was stated as well that the solution of the simple mixture with a primordial magnetic field of dark energy fluids, dust and Zeldovich require a special connection between some constants of fluids and of a nonlinear-fluid element that emerges from the mixture of fluids and magnetic field. This nonlinear fluid for not small values t is close in behavior (see [1]) to the mixture of a quintessence fluid $P_q = -2\mu_q/3 = -4B_{0z}^2/(3at^{1/3})$ and a radiation fluid $P_r = \mu_r/3 = (77B_{0z}^2/(72t^{4/3}))$, so $\mu_{nl} \to \mu_q + \mu_r$ and $P_{nl} \to P_q + P_r$. The term μ_{nl} could be related to the filaments in between the clusters of galaxies and could be fundamental to explain them. If what abovementioned is valid, their temperature should increase in the form $T_q \sim t^{2/3}$, and the radiation temperature should decrease with time in the form $T_r \sim t^{-1/3}$.
It was also established that the solution is singular in $t = 0$ and that tends to the model solution of dark energy for FRWL for very huge values of t. The dependence in relation to the time of the fluid temperature was determined, so for large times $T \approx T_q \sim t^{2/3}$, that dependence is present due to the nonlinear element of the fluid. Established Hubble parameter H and deceleration q determined that when $t \to 0$, parameters $H \to \infty$ and $q \to 2$, hence, the space is affected initially by a deceleration process, but when $t \to \infty$, parameters $H \to \sqrt{\Lambda/3} = const$ and $q \to -1$, the space suffers an acceleration process and goes through a value $q = 0$ in a finite time equal to $t = a - \chi^3$ where χ satisfies the equation $\chi^4 + \frac{{B_1}^2\chi^3}{\Lambda\, a} - 3\,a\chi - 5\,\frac{{B_1}^2}{\Lambda} = 0$.

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