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Fiat Numero: Trigintaduonion Emanation Theory and its Relation to the Fine-Structure Constant α, the Feigenbaum Constant c_∞, and π

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Abstract

In quantum physics unitary propagation is a standard part of the description. Efforts to move to algebras to describe such propagation leads to formulations based on the normed division algebras (real, complex, quaternion, and octonion). In prior work, the effort to achieve maximal information propagation led to relaxing the unitarity condition and showing that multiplication (right) on a unit norm base object by a unit norm chiral (10 subspace of trigintaduonions) emanator object, results in a new unit norm product [1]. A path is comprised of repeated (right) multiplications. Each step of the 'emanation' arrived at is a multiplication by a 10D chiral trigintaduonion. Use of methods from noise budget analysis, a constructive perturbation analysis, as well as analysis relating to maximal perturbation according to the Kato Rellich theorem, show that the chiral trigintaduonion with maximal perturbation (outside the 10D into surrounding 32D) has magnitude α, the fine structure constant. A relation between α and π results. Since repeated chiral emanation steps can be described as an iterative mapping, with unit-norm constraint resulting in a quadratic relation on components, we expect the Feigenbaum universal bifurcation parameter, c_∞, to appear according to the number of independent dimensions in a chiral trigintaduonion emanation step and the precise form of the "emanator" construction.

The number of effective dimensions is shown to be 29 plus a little more, and a relation between α, π and c_∞results that is in agreement with the choice of emanator examined in computational studies shown here. The computational studies with the emanator thus identified explore random walks in the Trigintaduonion space during emanation and explore non-classical noise additivity effects. A discussion is also included of the possibly fundamental role of analytic continuation in the emanator construct (and thus built-in, fundamental, dimensional regularization renormalizability and euclideanizability). Standard physics with path integral propagation, choice of time, and its assortment of fundamental constants, is then emergent from maximal information emanation via trigintaduonions.

Keywords: Trigintaduonion, alpha, fine-structure constant, Feigenbaum constant, Path Integral, Propagation; Cayley Algebra

Introduction

The chiral trigintaduonion emanation described here gives a precise derivation for the mysterious physics constant α (the fine-structure constant) from the mathematical physics formalism providing maximal information propagation, with α being the maximal perturbation amount (a fractal limit), and π being the maximum amount of overall imaginary component contributing to that maximal perturbation. The maximal imaginary component is hypothesized to be at antiphase, thus 'π' phase angle. Component sums becoming angle sums is an aspect of the analyticity hypothesized for the Emanator theory, and will be discussed further in the Results and Discussion. Thus, α can be determined by a (fractal) limit process, and separately, by a maximal information propagation argument, where a relation can be shown to exist with the maximum antiphase amount 'π', thereby providing an origin for the mathematical constant π. The ideal constructs of planar geometry, and related such via complex analysis, give methods for calculation of π to incredibly high precision (trillions of digits), thereby providing an indirect derivation of α to similar precision.

The trigintaduonion formulation provides that the structure of the space of initial 'propagation' (with initial propagation being referred to as 'emanation') has a precise derivation, with a unit-norm perturbative limit that leads to an iterative-map-like computed α (a limit that is precisely related to the Feigenbaum bifurcation constant and thus fractal). The familiar Mandelbroit set: $f(z) = z^2 + c$ (complex) has c_∞as a limit value on c in the iterative map for stability. Similarly, and directly relevant here, for the real one-parameter map $f(x) = a - x^2$, $c_\infty$is the (universal) limit value on a [2]. Since repeated chiral emanation steps can be described as an iterative mapping, with unit-norm constraint resulting in a quadratic relation on components, we expect the Feigenbaum universal bifurcation parameter, c_∞, to appear according to the number of independent dimension in a chiral trigintaduonion emanation step and the precise form of the "emanator" construction. The number of effective dimensions is shown to be 29 plus a little more, where a relation between α, π and c_∞results that is dependent on the choice of emanator. The emanator identified by the $\{\alpha, \pi\ c_\infty\}$ relation is used to explore random walks in the Trigintaduonion space during emanation and explore non-classical noise additivity effects (see Results). A discussion is also included of the possibly fundamental role of analytic continuation in the emanator construct (and thus built-in, fundamental, dimensional regularization renormalizability and euclideanizability). It is hypothesized that standard model physics with path integral propagation, choice of time, and its assortment of fundamental constants, is emergent from maximal information emanation via trigintaduonions. Thus, in Emanator Theory, the form of propagation is itself emergent, and within that construct, there is then emergent the functional optimization that describes how the system behaves, e.g., the Lagrangian and choice of time is part of that latter emergent step. Thus, Lagrangians originally introduced as a convenient mathematical constructs, and in later physics endowed with their own physicality, especially in conjunction with the path-integral description to properly capture topological features (the Aharanov-Bohm experiments), are here seen as direct mathematical encapsulations of the fundamental emergent nature of the physical system.

The mystery of alpha

The fine-structure constant, α, has been a mystery confounding physicists for over a century. In early work on spectral analysis where it first appeared, Sommerfeld noted the almost cabbalistic underpinnings of the mathematics (in his book Atombau and Spektrallinien [3], Sommerfeld referred to the Rydberg top square equation as a 'cabbalistic' formula). Wolfgang Pauli, a student of Sommerfeld's, shared his keen interest in the origins of α and turned it into a life-long obsession. So much so, that it practically drove him mad, to where he sought the help of famed psychoanalyst Carl Jung, with whom he eventually partnered to try to solve the mystery of α (the madness is contagious). From Pauli's Nobel Prize Lecture [4]:

> "From the view of logic my report on 'Exclusion principle and quantum mechanics' has no conclusion. I believe it will only be possible to write the conclusion if *a theory will be established which will determine the value of the fine structure constant* and will thus explain the atomistic of electric fields actually occurring in nature." (emphasis mine)

The obsession with α continued with the next generation of great Physicists as well, particularly Feynman, who said [5]:

> "There is a most profound and beautiful question associated with the observed coupling constant, *e* – the amplitude for a real electron to emit or absorb a real photon. It is a simple

> number that has been experimentally determined to be close to 0.08542455. (My physicist friends won't recognize this number, because they like to remember it as the inverse of its square: about 137.03597 with about an uncertainty of about 2 in the last decimal place. It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it.) Immediately you would like to know where this number for a coupling comes from: is it related to pi or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the "hand of God" wrote that number, and "we don't know how He pushed his pencil." We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the computer to make this number come out, without putting it in secretly!"

Fractal Reality

Consider maximum "unit-norm" propagation (via right multiplications), e.g., a projection (or 'emanation'), where a hypercomplex 'emanator' has maximum propagation dimensionality ten, a double-chiral 10dim subspace of the 32dim space of trigintaduonions. The maximum propagation perturbation allowed from the 10dim space into the embedded 32 dim space is given by the fraction α for the non-10dim part, where this is taken as the definition of α. Computational efforts to determine α recover the known α from QED, as in [6].

Exploration to high precision indicates a possible fractal limit (as noted in [6]), with possible pattern recurrences as in the Mandebroit Set on complex numbers. A further complication is that the 32 dim hypercomplex trigintaduonion numbers have also become non-associative (but still retain octonionic sub-space 'braid' rules, which are critical in what follows).

To see the fractal connection, consider the iterative mapping based on the function $z_n = (z_{n-1})^2 + c$. For choice c and initial $z_0=0$, if $z_\infty \rightarrow \infty$, then that c is outside the set, otherwise, if remains bounded, then it's in the (Mandelbroit) set. This is an example famous for its beautiful fractal images and mathematical properties. The largest c value (at the edge of chaos) is known as the bifurcation parameter and is $c_\infty = 1.401155189\ldots$ The maximum allowable 'perturbation' for z (not z^2) would then be $(c_\infty)^{(1/2)}$. In the trigintaduonion propagation we discover in what follows we have chiral propagation in the 32 dim trigintaduonion space, where the real dimension is fixed by the unit-norm property, leaving 29 'free' imaginary dimension/parameters, since two more are selected for a specific chirality. If we allow the same maximal bifurcation parameter as a factor for each of the 29 free dimensions (and for an imaginary part overall), a precise relation will be obtained according to the exact form of the trigintaduonion emanation (shown in Results).

Another impact of the analyticity of the 10D chiral emanation, is that a small complex component can effectively provide an 11th dimension of emanation. This appears to be a standard maneuver in a variety of 10D theories/constructions, occurring in going from 10D String Theory to 11D M-theory, or the appearance of the mysterious 11 Sefiroth, or in optimizing your amps to “go to 11” (This is Spinal Tap [7]).

In what follows we describe three relations, that parallel the construction of the emanator: (i) A relation {α} for α alone due to the fractal limit on chiral trigintaduonions with maximum perturbation; (ii) a relation {α, π} due to maximum perturbation occurring when noise has maximum antiphase; (iii) the {α, π, c_∞} relation due to maximum information flow occurring at the edge of chaos.

Background

The number system, or algebra, used to describe a physical system is typically the real numbers, sometimes the complex numbers (to describe wavelike phase information), and, rarely, the quaternionic numbers (to describe rotation and EM interactions). In recent theoretical efforts, attention has also been paid to octonionic numbers to describe Quantum Electrodynamics (QED) and Quantum chromodynamics (QCD) interactions [8-14]. The algebras given by real, complex, quaternionic, octonionic, sedenionic, trigintaduonionic, …., are known as the Cayley-Graves algebras, whose dimensions double at each step, one dimension for real, two for complex, four for quaternionic, etc. Maximal *unitary* propagation occurs with the octonion algebra and no higher (thus ‘maximal’ propagation, seemingly, only in 8 dimensions). What is actually needed in physics ‘propagation’ is right multiplication with a unit-norm ‘propagator’, for example, giving rise to a unit-norm result (then iterating to create a path from the infinitesimal propagator steps). If this is sought instead, then a chiral extension can be made from the octonions into the sedenions, and then again into the trigintaduonions, giving rise to a *maximal ‘propagation’, or projective emanation, in 10 dimensions* within the 32 dimensional trigintaduonions (as shown in [1,6,15,16]).

For Real numbers unit norm propagation is trivial, consisting of multiplying by +1 or -1. For Complex numbers unit norm propagation involves multiplication by complex numbers on the classic unit circle in the complex plane, which reduces to simple phase addition according to rotations about the center of that circle (motions on S^1). For quaternion numbers unit norm propagation is still straightforward since it’s still, in the end, a normed division algebra, where N(xy)=N(x)N(y). For the quaternions, instead of motion on S^1, we now have motion on S^3, the unit hypersphere in four dimensions. This still holds true for Octonions, with unit norm still directly maintained when multiplying unit norm objects in general. Now the motion is that of a point on a seven dimensional hypersphere S^7. Sedenions are not normed division algebras, lacking linear alternativity and the moufang loop identities [17], thus multiplication of unit norm objects for sedenions (points on S^{15}) will not, generally, remain unit norm, i.e., will leave the S^{15} space.

The question then arises is there is a sub-algebra or projection in the sedenions, that is not just trivially the octonions, that can still allow unit norm propagation? If this works for Sedenions, what about Bi-sedenions (trigintaduonions) and higher dimensional Cayley algebras? In [1] it is shown that there are two Sedenion subspaces where the unit norm property is retained. This is found again at the level of the Bi-Sedenions by a similar construction. The results were initially explored computationally [1], then later established in theoretical proofs [1,6,15,16]. In those proofs a key step fails when attempting to go to higher orders beyond the bi-sedenions and its sub-algebra propagation.

In the RCHO(ST) hypothesis Physics unification was thought to directly entail propagation in terms of hypercomplex numbers [18] (from Reals thru Trigintaduonions in Cayley sequence). This hypothesis was motivated by Maxwell, Feynman and Cayley, in hopes of being able to directly encode the standard model and statistical mechanics. In the end, this idea was not ambitious enough, with changes and clarifications as will be described in the Discussion. Part of the problem is that to get the 10D propagation formalism entails 'projections', not the more familiar mathematical objects directly giving rise to standard propagation (in a complex Hilbert space). Instead, the standard propagation is part of the emergent (with complex Hilbert space) description, as will described further in later sections.

The Feynman-Cayley Path Integral proposed in [1] involved use of chiral trigintaduonions in an effort to identify a mathematical framework within which to have a unified propagator theory (and maximal information propagation was sought for such a hypothesized propagator). At its root, this is a hypothesis for an algebraic reality, with algebraic elements describing 'reality' and algebraic multiplicative processes underlying propagation. All of the different 'paths' of propagation are then brought together in a sum – where stationary phase is selected out and the variational calculus basis for much of physics then takes over to offer all of the familiar elegant solutions of classical physics. This is still thought to be the process, but two stages of emergence are indicated: (1) emergence of the emanation (projective) process followed by the (2) emergence of standard propagation in a complex Hilbert space. So, even though we start with RCHO(ST) with the emergence of *emanation*, we end with a framework for emergence of standard propagation with complex propagator that requires a complex Hilbert space.

With the Feynman-Cayley construction there is a sum on all algebras, with selection for the highest order unit norm propagating algebra. It is shown that the highest order propagating structure is the ten dimensional (10D) unit-norm trigintaduonions elements, that are used here, that are (chirally) extended sedenions that are themselves made from chirally extended octonions. The nine dimensional space "free" dimensionality when paired with the implicit time dimension provides a 10 dim (1,9) spacetime theory, in agreement with string theory. If the time is augmented to be complex, we get an 11-dim theory, with a fundamental role for Euclideanization related thermodynamics properties.

Thus, for physical description a unit norm object can be used to represent a system, and by repeated transformation to other unit norm objects, it thereby evolves. Mathematical objects that can effect this 'transformation' simply by the rule of multiplication would be objects like division algebras, ideals, and what I'll simply call projections or emanations. In the universal propagator we have a unit norm trigintaduonion (32D) and perform a right multiplication with a chiral (10D) unit norm 'alpha-step' (defined by a max perturbation α into the 29 free dimensions given by 32 minus one for each chiral choice, and one for the unit normalization overall). Consider multiplication of a given (starting) trigintaduonion from the right with a chiral trigintaduonion as a 'projection' through the (chiral) step indicated. The repeated application and repeated 'chiral steps' thereby arriving at a path describing a chiral propagation. The resulting universal propagation consists of a 32D unit norm trigintaduonion with propagation via right multiplication using a unit-norm, chiral trigintaduonion, with max-α perturbation. Thus, we have selection on projections from an infinite space to an infinite-order Cayley-Graves algebraic space to a 32 dim trigintaduonion space to a 10 dim chiral 'propagation' space (where we will see that the parameter α arises in the limit of maximum information propagation, as does the familiar mathematical constant π).

Methods

We begin with constructing the theoretical expression for a general element of the trigintaduonion algebra after two chiral trigintaduonion multiplicative propagation steps. A simple analysis of the number of terms in this expression, when reduced to three-element algebraic 'braid-level', results in a count on algebraic braids of 137, plus a little extra (e.g. some lagniappe for the best 'cooking') of a contribution towards a 138th braid. (The extra involves a complex-dimensional extension outside the 10-dim propagation). This is used in the Results to show derivations for π and the Feigenbaum bifurcation constant.

Trigintaduonion Emanation and Emergence of the Critical Parameters 137 and α [1,6,15,16]

Consider a general Norm=1 (32D) Trigintaduonion (Bi-Sedenion): (A,B), where A and B are sedenions (16D). Then have (A,B) = ((a,b), (c,d)), where {a,b,c,d} are octonions. Slightly different than a propagator, we have an 'emanator' with the following notation and properties, where the emanator describes a 10D multiplicative step. The emanator is a chiral bi-sedenion: a trigintaduonion whose first sedenion half is itself a chiral bi-octonion, and the second sedenion half is a pure real (as is the second octonion half): $(\tilde{A},\beta)$, $\tilde{A} = (\tilde{a},\alpha)$, where the norm is 1, α is a real octonion, and β is a real sedenion. Thus:

Emanator: $(\tilde{A},\beta) = ((\tilde{a},\alpha), \beta)$.

Note: $\tilde{A}^* = (\tilde{a}^*, -\alpha)$.

Let's set up a description of the Universal 'Emanation' resulting from a few emanation steps. To begin, suppose we have already arrived at, or received, a unit norm trigintaduonion (32D) state 'T', and suppose our emanations are the result of right multiplication with a chiral trigintaduonion (bi-sedenion) 'step', and suppose we consider one such path after just a few steps. Here's the notation to begin:

T = (A,B), a unit norm trigintaduonion.

$\boldsymbol{\tau} = (\tilde{A},\beta) = ((\tilde{a},\alpha), \beta)$, the 'emanator' above (so named to distinguish from a 'propagator').

Universal Emanation from T on single path with three steps: $((\mathbf{T} \bullet \boldsymbol{\tau}_1) \bullet \boldsymbol{\tau}_2) \bullet \boldsymbol{\tau}_3)$ …

Consider the first emanation step:
$\mathbf{T} \bullet \boldsymbol{\tau}_1 = (A,B) \bullet (\tilde{A},\beta) = ([A\bullet\tilde{A}-\beta^*\bullet B] , [B\bullet\tilde{A}^*+\beta\bullet A])$. (Standard Cayley algebra multiplication rules.)
$A\bullet\tilde{A} = (a,b) \bullet (\tilde{a},\alpha) = ([a\bullet\tilde{a}-\alpha^*\bullet b] , [b\bullet\tilde{a}^*+\alpha\bullet a])$
$B\bullet\tilde{A}^* = (c,d) \bullet (\tilde{a}^*,-\alpha) = ([c\bullet\tilde{a}^*+\alpha^*\bullet d] , [d\bullet\tilde{a}-\alpha\bullet c])$
Thus,
$\mathbf{T} \bullet \boldsymbol{\tau}_1 = (A,B) \bullet (\tilde{A},\beta) = ([(a\bullet\tilde{a}-\alpha^*\bullet b-\beta c) , (b\bullet\tilde{a}^*+\alpha\bullet a-\beta d)] , [(c\bullet\tilde{a}^*+\alpha^*\bullet d+\beta a) , (d\bullet\tilde{a}-\alpha\bullet c+\beta b)])$.

At the lowest octonion level, that covers the pure real trigintaduonion, we have:

$(a\bullet\tilde{a}-\alpha^*\bullet b-\beta c) \rightarrow 8\text{x}8 + 8 + 8 - 2 = 64+14 = 78$ independent octonion terms (78 independent generators of motion). The −2 comes from the unit norm constraints on T and τ.

Now consider the second propagation step:
$(\mathbf{T} \bullet \boldsymbol{\tau}_1) \bullet \boldsymbol{\tau}_2) = ([(a\bullet\tilde{a}-\alpha^*\bullet b-\beta c) , (b\bullet\tilde{a}^*+\alpha\bullet a-\beta d)] , [(c\bullet\tilde{a}^*+\alpha^*\bullet d+\beta a) , (d\bullet\tilde{a}-\alpha\bullet c+\beta b)]) \bullet (\tilde{A},\beta)$,
where $\boldsymbol{\tau}_2 = (\tilde{A}',\beta') = ((\tilde{a}',\alpha'), \beta')$.

Let $(\mathbf{T} \bullet \boldsymbol{\tau}_1) \bullet \boldsymbol{\tau}_2) = ([Z_{11},Z_{12}] , [Z_{21}, Z_{22}])$.
$Z_{11} = (a\bullet\tilde{a}-b\alpha-c\beta)\bullet\tilde{a}' - (b\bullet\tilde{a}^*+\alpha a-\beta d)\, \alpha' - (c\bullet\tilde{a}^*+d\alpha+a\beta)\beta'$.

In Z_{11} we can replace the octonions with their unit component forms:
$a = a_1e_1 + a_2e_2 + \ldots + a_8e_8$,
where $\{e_1, e_2, \ldots, e_8\}$ are the unit octonions (one real, seven imaginary), while 'α'=αe_9 and 'β'=βe_{17}, originally, but in expressions, are reduced to just their real part. All expressions, thus, involve 10 components: $\{e_1, e_2, \ldots, e_8, e_9, e_{17}\}$, and as

the equations for Z_{11} shows, grouped in factors of three (three-element octonionic 'braids'). We don't have associativity but we do have alternativity and the braid rules on three-element octonionic products that allows their regrouping. Applying these rules to have only ordered $e_i \bullet e_j \bullet e_k$ products in a simplified expression, we will then have 10x9x8/3! = 120 independent terms when the products involve different components. We have 8 independent terms when the first product are on the same component (equals 1), have 8 independent terms when the second product involves the same component, and have 1 independent term when the three-way product equals 1. There are, thus, 137 independent terms in Z_{11}, where each term has norm less than unity (since each octonionic component has norm less than one and the norm of a product of octonions is the product of their norms). The terms involving products with the same component, or with the components three-way product equal unity, correspond to the 'telescoping terms' in what follows.

When **T=((a,b),(c,d)) → ((T • $\boldsymbol{\tau_1}$) • $\boldsymbol{\tau_2}$**)=((Z_{11},Z_{12}),(Z_{21}, Z_{22})). we have a→Z_{11} and the terms involving 'a' in Z_{11} are referred to as 'telescoping' due to their simple math properties with further emanation steps. In particular, the terms involving 'a' are:

Z_{11}[a terms]= $a \bullet \tilde{\alpha} \bullet \tilde{\alpha}' - a\alpha\alpha' - a\beta\beta'$.

We can see that the original 'a' information is passed along three (telescoping) channels, one involving repeated full octonionic factors $\tilde{\alpha}$, one involving repeated real-octonion α factors, and one involving repeated real-octonion β factors:

(1) a → $(a \bullet \tilde{\alpha}) \bullet \tilde{\alpha}'$, if this product is continued indefinitely, then we have the random product of a collection of octonions, all of which have norm less than one (although their norms can be quite close to one). If their norms were perfectly equal to one, then the addition of their random 'phases' would tend to cancel to zero, giving only a real octonionic component (same argument for phase cancelation on S1 as on S7 or S15). What results is a 'mostly' real octonion, having some imaginary part.

(2) a → $a\alpha\alpha'$, if this product is continued indefinitely, 'telescoped' with repeated α products, we see that the original 8 independent terms arising from 'a' are passed forward with an overall real octonion product, giving rise to 8 independent terms.

(3) a → $a\beta\beta'$, as with (2), we have 8 independent terms.

From the above, we see an alternative accounting of the extra 17 independent terms to go with the 120 for a total of 137 independent terms in the propagation of the octonionic sectors of the universal emanation. A benefit of the telescoping analysis is it clarifies how in (1) an imaginary component may arise, and in perturbation expansions it will then be natural to refer to an overall imaginary component.

There are 137 terms in the dually chiral 'emanation', each with norm bounded by unity, with total bi-sedenion norm equal to unity. In the analysis that led to the computational discovery of α [1], an imaginary (non 10D) component was added of growing magnitude until unit-norm propagation failed. In essence, a maximum perturbation, from propagation strictly in the 10D subspace of the 32D trigintaduonions, was sought.

In the Results we identify maximal perturbation by doing an independent term analysis, and by adding a maximum perturbation term that implicitly identifies a definition of maximum antiphase. From this definition of maximum antiphase, there results the parameter π.

The construction of an achiral emanator

Consider the emanator described in the previous section: $(\tilde{A},\beta) = ((\tilde{a},\alpha), \beta)$. Let's shift to representing the full octonion part by O: ((O,α), β). There are four types of chiral emanation:

$$T^{(k)}_{chiral} = \begin{cases} ((\mathrm{O},\alpha),\beta) \\ ((\alpha,\mathrm{O}),\beta) \\ (\beta,(\mathrm{O},\alpha)) \\ (\beta,(\alpha,\mathrm{O})) \end{cases}, where\ T^{(k)}_{chiral} = \mathbf{1} + i\boldsymbol{\delta}.$$

From unit norm we have $\alpha^2 = 1 - \mathrm{O}^2 - \beta^2$, with ± sign choice on α, similarly for β.

Suppose we have a unit norm base trigintaduonion as before, but let's now attempt to construct an achiral form of emanation from the set of four types of chiral emanations (and summing over the four sign conventions for ±α and ±β).

$$\mathrm{Emanation}(\mathbf{T}) = \frac{1}{N}\sum_{\{k\}} \mathbf{T} \bullet \boldsymbol{T}^{(\boldsymbol{k})}_{\boldsymbol{chiral}},$$

where (1/N) is a normalization (to recover unit norm) and the Results show analysis of this emanator as well as the chiral emanators $T^{(k)}_{chiral}$ individually.

A straightforward perturbative analysis, or noise budget analysis, can be used on the above emanator to determine the effective number of dimensions in the iterative mapping corresponding to the emanation step. This is a construction for dimensional regularization (another analyticity argument, used in QFT renormalization [19]), and it shows that the achiral emanation definition above is on the right track, but has not differentiated within the four chiralities properly. Consider the first chiral emanation family with the template $((\mathrm{O},\alpha),\beta)$. In the achiral emanation by simply summing over each of the four chiralities, the emanator for a given chirality is generated randomly and according to the indicated template, where all seven imaginary components of the octonion O have a small perturbative contribution. Let's now consider 14 possible perturbations within a given "suit" (chirality) from the pure imaginary octonion modulations (positive or negative). For the $((\mathrm{O},\alpha),\beta)$ chirality this corresponds to generating emanators with the form: $\mathrm{O} = (\sim 1,0,0,\ldots\delta,\ldots 0)$, with δ perturbation in each of the seven positions and then further divided according to whether it is positive or negative. Let's also consider 4 additional perturbations according to the template when $\hat{\alpha}^2 = 1 - \mathrm{O}^2 - \beta^2$ and the template has perturbation (positive or negative) at the $\hat{\alpha}$ component:$((\mathrm{O},\hat{\alpha}+\delta),\beta)$. Similarly for $((\mathrm{O},\alpha),\hat{\beta}+\delta)$. Thus, each chirality is split into 18 subtypes, and for the four chiralities this results in 4x18=72 terms in the emanator sum, with 4 separate sums on the 72 according to the $\pm\alpha$ and $\pm\beta$ conventions on the template. Thus

$$\mathrm{Emanation}(\mathbf{T}) = \frac{1}{N}\sum_{k\in\{4x72\}} \mathbf{T} \bullet \boldsymbol{T}_{\boldsymbol{chiral}}^{(\boldsymbol{k})},$$

This is referred to as the Emanator with the 72-card deck and in the Results it will be shown to provide a relation between the fine structure constant, π, and the Feigenbaum Universal bifurcation parameter c_∞, that is correct to the highest level of experimental and theoretical precision known on the fine structure constant.

If we only consider the 14 subtypes from pure imaginary octonion contributions, there are 4x14=56 card types. Respective to a particular chiral template, there are 22 zero-positions from the imaginary octonion sector with α (7 components) and the imaginary sedenion sector associated with β (15 components), giving rise to 22 chiral propagators of the form $\boldsymbol{T} = (\sim 1,0,0,\ldots\delta,\ldots 0)$. If we combine the 56 minor subtypes or 'cards' and the 22 major cards, we arrive a similar complete system of perturbations, whose sum would again be achiral. This latter case, with a 78 card deck, is referred to as "Tarot Emanation" due to the similarity to the Tarot deck with 56 minor arcana and 22 major arcana, and it may be equivalent to the 72 card deck. For the derivation to follow in the Results, however, the 72 deck is most accessible to analysis. Further understanding of the multiple-multiplication 'paths' in an achiral path sum of chiral emanations is left to the discussion (and may eventually relate trigintaduonion zero-divisor density to Planck's constant).

Kato-Rellich Theorem and Noise Budget Analysis

Definition of "B is A-bounded": Let A: $D(A) \to H$ be a self-adjoint operator. Let B: $D(B) \to H$ be symmetric. If $D(A) \subset D(B)$ and $\exists$ b s.t. $\|Bf\| \leq a\|Af\| + b\|f\|$ $\forall$ $f \in D(A)$, then B is A-bounded with bound a.

Kato-Rellich Theorem: Let A: $D(A) \to H$ be a self-adjoint operator. Let B be A-bounded with bound $a < 1$. Then $A + B: D(A) \to H$ is self-adjoint.

Corollary to Kato-Rellich Theorem: If K-R theorem applies and A is bounded below, then so is $A + B$.

The above corollary is significant in the analysis that follows since the forms of the emanator (sums over 4-suits, or 72deck, or 78deck, especially), before normalization, are here seen to be bounded from below if within the perturbation-limit of the sum-type emanation considered. This means that the normalization step won't fail (divide by zero), furthermore, it indicates no zero divisors when operating within the perturbation limit when considering a single chiral path. Interestingly, the reverse is also indicated, a loss of boundedness just beyond the perturbation limit, including the existence of zero-divisor "land mines".

To apply this to our trigintaduonion analysis, let's "lift" the trigintaduonion T into a formal operator setting as a T-position operator, whose distance operator (from the origin) is the norm(T). Let's denote the Emanation of T by one step by $Em_{\delta}^{(n)}(T)$, where $\delta = 0$ is the case for no perturbation. We arrive at the form necessary for a self-consistent emanation rule (with well-defined sum from above) when:

$$\left\| Em_{\delta}^{(n)}(T) \right\| \leq n^{*}\delta \left\| Em_{0}^{(n)}(T) \right\| + b\|f\|,$$

where choosing b=1 for simplicity, and noting that $\left\| Em_{0}^{(n)}(T) \right\| = 1$, leads to:

$$\left\| Em_{\delta}^{(n)}(T) \right\| \leq n^{*}\delta + 1,$$

where n^{*} is related to n according to the noise-budget analysis appropriate to the form of Emanator implementation, as discussed in the Results. When n refers to the 137 independent terms, each of max norm 1, comprising each Trigintaduonion emanation, n^{*} is the familiar $1/\alpha$. When n refers to the 29 independent dimensions of emanation along a particular chiral emanation (with perturbations), then $n^{*}$is the effective number of dimensions in the iterative mapping that results. This is needed in the Results that follow to get the $\{\alpha, \pi\ c_{\infty}\}$ relation.

From the form above, application of the Kato-Rellich theorem is equivalent to a noise budget analysis to arrive at the same inequality. We inject an amount of noise δ in the form of the emanation chosen and determine the amount of noise, worst-case, that might present after the chosen emanation operation is performed. Since the terms are all max norm 1, this decomposes into a simple counting on the number of independent terms, free dimensions, etc. This allows for a straightforward counting process to arrive at a number of solutions as will be shown in the Results that follow.

Results

The {α, π} relation

In the methods we saw that there are 137 independent tri-octonionic braid propagations contributing to the overall chiral trigintaduonion propagation (further details in [6]), in each of its octonion subparts, along with an independent imaginary component (in those sub-parts with 137 terms). At the component level of the base trigintaduonion, we similarly have 137 independent (real) terms, each with maximum one, thus an evaluation of the maximum at the component level involves a simple counting on the (unit max) independent terms. Aside from an overall scale factor, the maximum magnitude at component level involves a real part of magnitude 137 and an imaginary part. We hypothesize that the imaginary part has magnitude π in relation to the 137, for maximum antiphase when viewed as a phase angle (to be justified in the next paragraph), and we thereby arrive at component-level having an overall maximum perturbation given by $137+i\pi$, i.e. an overall perturbation magnitude of the injected perturbation amount δ and the multiplier $137/\cos(\pi/137)$.

At trigintaduonion level, we see that the overall maximum perturbation is given by the individual component-level perturbation amounts in the chiral emanation and their possible convergence into a maximum magnitude factor of $137/\cos(\pi/137)$, for maximum perturbation amount δ x $137/\cos(\pi/137)$ (see Fig. 1). Thus, at the level of the independent terms (137) in each of the chiral trigintaduonion independent components (29), each such term has a maximum perturbation contribution with magnitude δ x $137/\cos(\pi/137)$, each with phase angle $\theta=(\pi/29\text{x}137)$ to have equipartitioning of phase among the 29x137 independent terms (see Fig. 2).

According to the Kato-Rellich Theorem, the maximum perturbation is such that the magnitude of the total perturbation is ≤ 1 (as described in the Methods). Before we can do this step, however, we must rescale such that component-level imaginary component equals component level phase (thereby introducing a factor $\theta/\sin\theta$, see Fig. 3). This is a result of the normalization step in the achiral emanator forms described in the Methods, the existence of which is related to the hypothesis that component sums are made interchangeable with angle sums. Here the result is we arrive back at a component-level sum of all of the imaginary parts totaling π, which was the initial hypothesis, and we have for maximum perturbation $\alpha_{max} = (1/137)(\cos\beta/\cos\theta)(\sin\theta/\theta)$, where $\beta = (\pi/137)$ and $\theta = \pi/(137 \times 29)$.

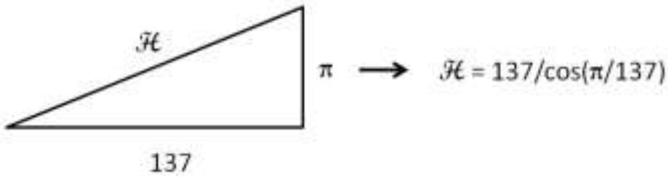

Fig. 1 The magnitude relation at Trigintaduonion-level.

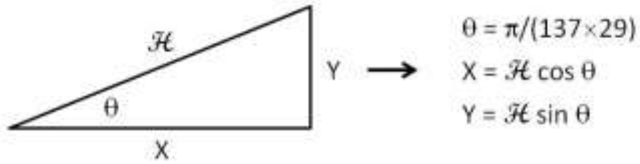

Fig. 2. The magnitude-angle relation at independent terms level, given 29 independent dimensions and 137 independent terms in each.

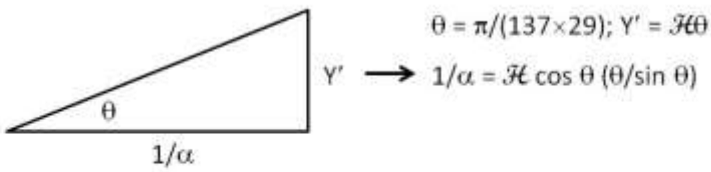

Fig. 3. Emanator definition gives imaginary component sums are made interchangeable with angle sums (given sum with normalization in definition). This can be described in terms of analyticity in general or Euclideanizability.

To recap, First, consider a trigintaduonion element of propagation that results from multiple achiral emanation steps, for which it's octonion subsectors will have 137 independent terms (resulting from tri-octonion products) with perturbation (or noise) magnitude having a factor of $\mathcal{H}=|137+i\pi|$ (see Fig. 1), where the unit norm upper bound on the tri-octonion products gives the 137 and the "maximal antiphase" π phase amount is justified and made self-consistent, at the next step. Second, now consider the maximal noise element at the level of each 137 independent term in each of the 29 (free) dimensions in each of the chiral product terms in $\mathbf{T} \bullet \boldsymbol{T}^{(\boldsymbol{k})}_{\boldsymbol{chiral}}$ in the Emanator (in essence, interpret the multiplication as projecting the other way, **T** onto the chiral basis specified by $\boldsymbol{T}^{(\boldsymbol{k})}_{\boldsymbol{chiral}}$). Again, we postulate that the total imaginary amount will be at maximal antiphase, or such that the amount of phase for each of the 137x29 independent terms is $\theta=(\pi/29\text{x}137)$, indicating the general relation shown in Fig. 3. Now consider the magnitude rescaled in Fig. 3 such that the hypotenuse is 1 (unit norm), it is then clear that the maximum allowed perturbation, $1/\alpha$, satisfies $((1/\alpha)/\mathcal{H})=(\theta/\tan\theta)$ (see Fig. 4). Note the distinctive arrangement that the maximal noise, or perturbation, hypothesis reveals in Fig. 4, where the phase angle and imaginary component value are equal (already suggested by both component-level sum in the Emanator, and phase-angle sums from the chiral product terms, must total maximum antiphase π).

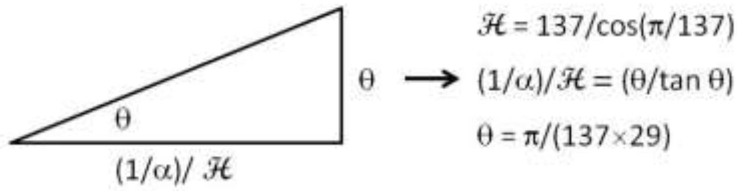

Fig. 4 Unit norm case. $((1/\alpha)/\mathcal{H})=(\theta/\tan\theta)$. Shows phase angle = imaginary component magnitude. .

Thus, maximal noise, or perturbation transmission occurs when the noise phase angle equals the noise imaginary component, when noise scaled to total magnitude 1. This resulted in part by assumptions going into the construction of an achiral emanator from summing over chiral emanations (multiplications), and would generally result for a variety of such emanators. Further specifics of the emanator construction are required for the next section, however, so this emanator dependency will be developed further soon. Before moving on, however, it appears that the only constraint on emanators would be that they generate, through inclusion of unbiasing sums on chiral multiplications, that the noise phase angle equals the noise imaginary component relation. I'll refer to this relation as the proto-Euclideanizability, or proto-analyticity, property of the Emanator. If we start with the hypothesis that the Emanator will induce a proto-Euclideanizability relation, this allows us to start directly at Fig. 4 in evaluating the maximum perturbation allowed, and we get fundamental Euclideanizability as a side effect. Regardless of starting hypotheses, the end result for the maximum perturbation magnitude is

$\alpha = (1/137)(\cos\beta/\cos\theta)(\sin\theta/\theta)$, where $\beta = (\pi/137)$ and $\theta = \pi/(137 \times 29)$.

Thus, $1/\alpha \cong 137.0359998$, where the last digit is uncertain given the precision used (this relation originally appeared in [20] but without explanation in terms of trigintaduonions).

Since α is a fundamental parameter that emerges for a maximal propagation, and we find here another relation on α that ties it to the maximal antiphase amount 'π', we find that this is the origin of the fundamental parameter π from mathematics. Although the idealizations of planar geometry can be used to derive

π (or modern variants from complex analysis involving the complex plane) it is interesting that we have here an origin of π via what leads to maximal anti-phase when computing α_{max} , where $\alpha=\alpha_{max}$ is selected for maximal information propagation.

The relation of c_∞ to α (and thus π)

Recall from the Methods that we have emanation in the form:

$$\text{Emanation}(\mathbf{T}) = \frac{1}{N} \sum_{k \in \{4x72\}} \mathbf{T} \bullet \boldsymbol{T}^{(\boldsymbol{k})}_{\boldsymbol{chiral}},$$

Each of the chiral trigintaduonions has a template of fixed parameters, involving 3 of its 32 dimensions, leaving 29 dimensions 'free'. The effective dimension will be 29 plus a correction due to imaginary contributions to the noise transmission with each chiral multiplication. Consider a noise, or perturbation, contribution δ, in generating the chiral emanators of the various types, as described in the Methods. From a base trigintaduonion with chiral multiplication in the Emanator sum, for the 29 'free' dimensions respective to that chiral multiplication path, we have noise transmission for each of the independent 72 elements from the 72-deck sum, assume worst-case noise transmission into each of the independent emanator sum terms (72) in each of the 29 dimensions, whose imaginary component is again maximal antiphase at maximum noise transmission, thus (π/29x72) for each of these terms. Thus, as a conditioning step, consider a trigintaduonion resulting from Emanation with a 72-deck as described, with noise in each free dimension going as $\delta(1+i(\pi/72)/29)$. Now let's consider this noise transmitted through a general emanation step:

The real part, δ, will transmit to δ in the new trigintaduonion, but since the emanation process uses a 'deck' of 72 valid chiral emanation types, a correction is needed since 3 of these emanation types are not valid for the real emanation path (the 3 chiral emanation that have α or β at the T[0] position are locked into α positive (~1) or β positive (~1), respectively, thus exclude 3 of the 4 $\{\pm\alpha,\pm\beta\}$ cases). This amounts to the real part $\delta \rightarrow \delta(1 - \frac{1}{29}\left(\frac{3}{72}\right)4(\frac{\pi}{72})(\frac{\pi}{137x29})\hat{\boldsymbol{\imath}}$), where the correction on the real part is (3/72) of the $\delta(\pi/72)/29)$ imaginary part transmitting as a new noise factor δ', where there are four transmission chiralities (each with its own resulting imaginary, so sum to 4 after renormalization, unlike the real part where "1" is the same in the four chiral sums, thus normalization, divide by four at this stage, reduces to "1" for the real part shown). For convenience, the four different imaginary values are summed as the 4i shown. When considering effective dimensions later this will be valid when linear additivity is assumed (not adding in quadrature). The modified noise factor $\delta''=\frac{1}{29}\left(\frac{3}{72}\right)4(\frac{\pi}{72})$ described thus far, is then multiplied, in the emanation product, by the maximal noise imaginary component allowed in the 137 independent terms in the 29 independent dimensions, θ=(π/29x137), thus the form shown.

The imaginary part,

$$\delta(i(\pi/72)/29) \rightarrow 4\delta((\pi/72)/29)\hat{\boldsymbol{j}} + 4\delta((\pi/72)/29)(\frac{\pi}{72})(\frac{\pi}{137x29})\hat{\boldsymbol{k}},$$

where the first term simply results from the δ(i(π/72)/29) noise injection hitting the (~1) real component in the chiral triginataduonion multiplication, again with a factor of 4 from the 4 separate chiral sums. The second term has the δ((π/72)/29)i noise injection factor, the 4-factor, as before, and a (π/72)j factor for the 72-deck chiral emanations and within that a (π/29x137)k factor respective to a particular chiral emanation. Again, the ijk imaginary products for different i, j, k's, is all grouped as $\hat{\boldsymbol{k}}$**.**

Now to multiply the noise for one of 29 free dimensions by 29 and sum the magnitudes of the real and imaginary components. Dividing out the noise injected δ, we thereby arrive at an expression for the effective dimensionality as seen by noise transmission:

Dim effective = 29 + (4π/72)[(1+θ{(π/72)+(3/72)}]

Thus, we expect the maximum perturbation amount α, when inverted, to be related to the Feigenbaum bifurcation constant according to the number of effective dimensions:

$\alpha^{-1} = (c_\infty)^\gamma$ = 137.035999206…,
where **γ = (1/2)(29 + (4π/72)[(1+θ{(π/72)+(3/72)}]),** and **θ=π/(137x29).**

Random Emanation-Walk Results

$T^{(1)}$chiral emanation: $((0,\alpha),\beta)$form, with noise δ at the indicated template positions aside from T[0] component, which is ~1 with unit-norm normalization (where all other components, if non-zero, involve a max δ/2 noise, noise uniformly distributed $\pm|\delta/2|$). Emanation is then simply multiplication: $\left(\mathbf{T} \bullet \boldsymbol{T}^{(\mathbf{1})}_{chiral}\right) \bullet \boldsymbol{T}^{(\mathbf{1})}_{chiral} \bullet \bullet \bullet$, where here we see how many emanation steps it take to go from T[0]=1 in the initial base trigintaduonion to T[0]=0 (the number of steps to the first zero-crossing). These are effectively random walk simulations on the unit-norm trigintaduonion subspace S^{15}, where the emanation step is chiral (see Table 1).

δ	N_{avg} (5 samples)	$\left(\frac{\delta}{2}\right)\sqrt{N_{avg}}\sqrt{2/\pi}$
0.7	9.0	0.8376
0.6	17.2	0.9932
0.5	32.4	1.1368
0.4	34.2	0.9334
0.3	75.2	1.0371
0.2	147.8	0.9693
0.1	706.0	1.061
0.05	2819.4	1.057
0.01	43,136.0	0.8297
0.005	206,454.4	0.9055
0.002	1,613,224.8	1.0131
0.0016	3,532,666.8	1.1997

Table 1. $T^{(1)}$ chiral emanation random-walk simulation.

Let's now consider off-template $T^{(1)}$chiral emanation: $((0,\alpha),\beta)$form with T[7] and T[24] swapped (thus have δ noise off template, which breaks the unit-norm preserving property in the emanation multiplication $\left(\mathbf{T} \bullet \boldsymbol{T}^{(\mathbf{1})*}_{\boldsymbol{chiral}}\right) \bullet \boldsymbol{T}^{(\mathbf{1})*}_{\boldsymbol{chiral}} \bullet \bullet \bullet$, where the * denotes the off-template form (see Table 2).

δ	N_{avg} (5 samples)	$\left(\frac{\delta}{2}\right)\sqrt{N_{avg}}\sqrt{2/\pi}$	norm(**T**) at zero-crossing
0.7	12.0	0.9674	~1
0.6	14.6	0.9146	~1
0.5	19.4	0.8786	~1
0.4	19.0	0.6956	~1
0.3	66.4	0.9752	~1
0.2	254.2	1.2721	~1
0.1	721.8	1.0718	1.0179
0.05	3103.2	1.1112	1.0011
0.01	46883.6	0.8638	1.0001
0.005	213,397.0	0.9214	1.0009
0.002	975,838.4	0.7882	1.0001
0.0016	2,248,293.0	0.9571	1.0001

Table 2. $T^{(1)*}$ chiral emanation random-walk simulation.

Let's now consider an Emanator definition that involves a 4-suit (chirality) generation process that is summed and renormalized to 1 at each step (to be achiral). The 'deck' of four cards (general chiral class members) that is summed leads to a modification of the ranwalk equation:

$$ranwalk(\delta, N_{avg}, |deck|) = \delta\sqrt{N_{avg}\frac{\sqrt{2/\pi}}{\sqrt{|deck|}}} = \delta\sqrt{\frac{2N_{avg}}{\pi|deck|}}$$

The results for 4-suit Emanation are shown in Table 3. If the perturbation is generated in a range (uniformly in$[-\delta..\delta]$) it has half the step-size (on average) and has possible mixing of chiral emanations that are within the perturbative limit and not within the limit.

δ	4-suit with[-1..1] N_{avg} (5 samples)	$ranwalk$ ($\frac{\delta}{2}$ on avg.)	4-suit with[-1,1] N_{avg} (5 samples)	$ranwalk$
0.1	2583.0	1.013775	772.0	1.108457
0.05	11584.0	1.073445	2926.6	1.079100
0.01	246437.8	0.990220	144019	1.513979
0.005	792124.4	0.887660	347164.2	1.175297
0.001	15049973.4	0.773834	6079663.0	0.983671
0.0005	-----	-----	22,486,524 (one)	0.945891
0.00025	-----	-----		
0.000125	-----	-----	>100M	

Table 3. 4-suit Emanation Steps. Random walk transitions from noise additivity in quadrature to linear. Also, at transition to analyticity have $\sqrt{2}$.

In Table 3, the [-1,1] case shows possible transition at 0.01: into "analytic" domain, where random walks explore more terrain, have more movement, get possible $\sqrt{2}$ effect, then transition to linear perturbative domain. A transition from quadratic to linear noise dependency is becoming apparent.

In Table 4 is shown the results for when the Emanator Deck is 72, with 4 sums to get the different ±α and ±β chiral templates, and a different four sums associated with the 4 'suits' or chiralities. Consider linear noise additivity within the noise components of a given chirality during the chiral emanation:

$$ranrun(\delta, N_{avg}, deck) \propto \delta N_{avg}$$

Shown in table results from runs with 72-deck with noise drawn from δ x [-1,1]. A clear linear relationship exists. The same strong linear relations exists for δ x [-1..1] based emanation, but proceeds more slowly, so dataruns not as complete and not shown. The result "with Major 1" has noise injection δ at the position of the 1st Major perturbation (as in major arcana, since similar card subgroups as in the construct of the tarot deck). The result with "Tarot" Emanation uses the 72-deck with random noise injection according to the probability of a "card" from a 78-style emanation "card" tarot deck (this appears to be the most complete case for achiral emanation with the full range of non-chiral perturbations allowed).

δ	72-deck [-1,1] N	72-deck [-1,1] N With Major 1	Effective 78-deck “Tarot” Emanation
0.1	212	214	422
0.05	427	428	851
0.01	2137	2133	4298
0.005	4274	4264	8531
0.001	21372	21353	42740
0.0005	42745	42751	85476
0.00025	85493	85572	170961
0.000125	170986	171139	---------

Table 4. 72-deck Emanation Steps. Random walk in linear noise additivity regime.

The repeated experiments show remarkably small difference in the 72 deck counts even with non-pathology cases [-1..1] and outside mixing domain (if we are even seeing one) – i.e, all 72 deck runs appear to be in the perturbative regime, with linear growth seen for the entirety of the Results in Table 4.

The computational results shown above confirms that noise, or random walk steps, add in quadrature, thus $\propto\sqrt{N}$ distance, until an analytic perturbation regime reached, where noise then adds linearly, thus random walk goes $\propto N$ distance with N steps.

Discussion

Consider numerogenesis from an infinite-order hypercomplex unit-norm ‘Number’ and ‘Emanation’ process (algebraic multiplication) giving rise to a propagating structure, with time and chirality self-selected, with QED and QCD gauge bundles emergent, for example, with their associated parameters fixed (including α). With the proposed chiral trigintaduonion emergence have 10dim propagation with α-perturbation into the full 32 dimensions. Thus, have a hypercomplex Big Bang with emergence of unit-norm base elements and the unit-norm resulting emanation step (or path sum). The receiving of the universal emanation results in emergent spacetime and chirality, perhaps akin to the emergence of Amman bars and orientation with a Penrose tiling once seeded [21].

The emanation construct is self-selected to have maximum information flow beginning with selection of the maximum dimensionality algebraic subspace for states (Step I below singles out chiral trigintaduonions and with perturbation limit $\{\alpha\}$ that is fractal.). Next is allowing the 10D chiral subspace emanation element to have maximum perturbation into the surrounding 32D trigintaduonion algebra. As mentioned in the Kato-Rellich and noise-budget analysis in the Methods, and related Results, maximum noise transmission is from unit-norm trigintaduonion base with right multiplication by emanation involving a unit-normal, maximal perturbation, chiral trigintaduonion. This results in the {α, π} relation (Step II), where maximum perturbation occurs when noise has maximum antiphase (thus introduction of 'π'). Next is allowing for emanation processes that are achiral, but composed of the high dimensional flow chiral elements, that are then summed (Step III). In other words, a fundamental sum on emanation paths is posited in the emanation process even if the paths are only a single step long (possible path conventions are discussed later). Here are the three steps:

> **Step I:** Selection, or projective emergence, of maximum-dimensionality subspace emanation process (10D) operating within its Cayley algebra (32D). By allowing the maximum allowed perturbation into the surrounding 32D algebra of elements, a fractal limit can be probed, as done with the Mandelbrot set images, this limit reveals {α} purely computationally (theoretically this is related to the universal c_∞ derivation, as will be quantified at Step III).
>
> **Step II:** Selection, or emergence, of emanation from Step I with maximum allowed perturbation into the surrounding 32D algebra of elements, where analyticity is assumed (it is part of the optimal selection process in the emergence from the higher dimensional hypercomplex numbers). The property of analyticity allows application of the Kato-Rellich theorem in related domains, and lays the foundation for Euclideanization and dimensional regularization (QFT renormalization) methods later. At this step, the noise-budget analysis is only based on the structure of the chiral trigintaduonion elements T and the structure of their right multiplication on a norm=1 base trigintaduonion (that is being emanated to a new base trigintaduonion), e.g. the structure T•T, from which the {α, π} relation is obtained. At this stage in the Emanator construction described in the Methods/Results we see a fundamental hypothesis of maximal noise when phase angle and imaginary component magnitude are equal (a Euclideanization, or analyticity, type relation).
>
> **Step III:** Selection, or emergence, of emanation from Step II with maximum allowed perturbation into the surrounding 32D algebra of elements, at the boundary where analyticity is not assumed, but where an iterative mapping is induced with resulting universal limit properties from [2], giving rise to an effective dimension analysis for the iterative mapping, and thus a new relation "from the edge of chaos": {α, π, c_∞}. This analysis can be developed even further, since Kato-Rellich is used in the Methods to argue that there are no zero-divisors for perturbations up to the α limit. If we attempt to probe a little further, we start to encounter zero-divisors. Consider the limit density on zero-divisors at the α limit (taken from greater than α) how might this relate to quantum properties and the distinctive quantum constant h?

Once a 10dim propagation is emergent, there is likely an emergent semiclassical string theory. The emergence process with analyticity also helps explain the validity of the various renormalization methods (dimensional regularization, in particular). In the latter regard, the dimensional regularization trick whereby a higher complex dimensional extension is invoked is here seen to actually be true. Similarly, string theory is an emergent construct, along with the manifold and the standard model, and Lagrangian encapsulations, etc. Thus, invoking a higher dimensional space, often through complexification of real variables, is natural in this emergent from a higher hypercomplex algebraic space context, since a higher dimensional complex embedding is already posited to exist in the emanation emergence process. The complex-extension method is critical in QED, Euclideanized path integral formulations, and thermal quantum field theory in general, where complex time relates to introducing a thermal background temperature for the system (thus the complex extension allows unification with thermal physics and emergent, Law of Large Numbers based, statistical mechanics constructs).

Consistency with the semiclassical first quantized string theory, allowing an alternative renormalization, also indicates the flat-space oddity of the seemingly general formalism of string theory (in other regards) having an odd flat spacetime reference. This is here understood as simple consistency with the maximum information propagation in the universal algebra formalism, where the 10 dimensions are resulting from the ‘free’ algebra parameters in the 32 D trigintaduonions, and as such have no other structure between them other than the implied ‘flat metric’ of the trigintaduonion algebra. This also demotes the string to being an artifact of the emergence, albeit on a higher level than the quantum field theory based on point particle descriptions.

In [9], with split octonions alone it is possible to describe spacetime, EM-fields, and uncertainty relations… This is very promising as regards extracting the familiar standard model from the much larger, already chiral, 10D propagation with maximal perturbation α (and 22 parameters from the non-propagating dimensionalities [15]). From this we get complete propagation with 78 generators (consistent with string theory, as is the 10dim). Also, we shall see that we have 137 tri-octonionic ‘braids’ of information flowing in the 10dim chiral propagation, this was critical in the derivation of π from α.

Just from the propagation structure on one path we have already seen core emergent structure that results in a universal emanation with structural parameters 10,22,78,137 and perturbation maximum α=~1/137. The central notion in the universal emanation hypothesis is that there should be maximal information flow, where this is accomplished by finding the highest theoretical dimensionality of unit-norm ‘propagation’, here called an emanation, which turns out to be 10, then add the maximal perturbation that still allows unit-norm propagation, where that perturbation is into the space the 10D motion is embedded in, here a 32 dimensional (trigintaduonion algebra) space.

Given maximum information flow, the universal emergence will arrive at the 10D propagation splitting (compaction) into spacetime geometry and matter gauge fields. The parameters and structure described are consistent with string theory and quantum field theory, where we fundamentally arrive at emergence of ‘propagation’ as conventionally known, with a complex Hilbert Space. A complex Hilbert Space description is the only one with propagation [22] (details below), thus it is necessarily the emergent construct that must encapsulate the geometry/matter split/compaction, into the familiar Standard Model formulations. This ties into emergence of the standard formalisms of QED and QCD. Likewise for the emergence of elegant geometrically optimal solutions relating to General Relativity (GR). Where there was conflict between QED/QCD and GR, e.g. the question of Quantum Gravity (QG), it will be solved by considering the universal emanation of not just one path but all paths, summed with the usual phase cancelations down to a ‘classical path’ with stationary phase. The latter, in this context, is the emergence of standard propagator theory with standard model. So proposing here an earlier phase of universal evolution described by a theory of emanations, where mathematically invariant emergent structures appear. From this early phase, one of the emergent constructs is the familiar path integral based on standard (unitary) propagators in a complex Hilbert space.

The implication of an emergent phase of universal evolution with standard propagators, etc., is not only a framework within which to answer the questions of quantum gravity, but also a framework where the emergent trajectory has emergent ‘time’ (and parameter h, and euclideanization/thermality). In the end, the Black Hole (BH) conundrum in quantum gravity might reduce to a scattering calculation, where semiclassical string theory (at 1^{st} quantization as known) may suffice, once ‘boundary terms’ are understood. With reference to the originating ‘emanator’ construct, we have a higher level second quantization but not based on standard propagators, but emanators. This new type of second quantization might shift to a notation where the stringiness is no longer discernable, and the trigintaduonion (bi-sedenion) structure dominates.

To recap: α, 10,22,78,137, are parameters resulting from analysis on a single path construct, where the number 22 corresponds to the number of emergent parameters in the description of the propagating construct. In addition, the time choice is emergent via a multi-path construct, along with the propagator construct, and is coupled in both time step (by h) and imaginary time increment (with Euclideanization regularization ‘built in’). The formulation is inherently embedded in a higher dimensional complex space, thus all of the QFT complex analysis analyticity tricks are valid as the assumptions made are now part of the maximal information flow emergent construct.

Maximal Information Propagation requires a complex Hilbert Space [22]
As mentioned previously, according to [22], a complex Hilbert space is selected by the quantum deFinetti theorem, since it is required for information propagation (and thereby consistent with the maximum information propagation concept in its selection). Because it's a complex Hilbert space, this explains why the path integral operates in a complex space, even though the underlying universal algebraic construct from which it is emergent is hypercomplex to the level of the trigintaduonions.

From Caves [22], a simple derivation shows why the quantum deFinetti Theorem requires amplitudes to be complex. Suppose f(n) is the number of real parameters to specify an n-dimensional mixed state. For real amplitudes f(n)=n(n+1)/2, for complex amplitudes f(n)=n^2, and for quaternionic f(n) = n(2n-1). For propagation, etc., need f(n1n2)=f(n1)f(n2), which only works for complex amplitudes.

Stringiness
It has been shown in numerous papers that the (1,9) dimensional superstring has a natural parameterization in terms of octonions [23-25]. In [8,9] the Dirac and Maxwell equations (in vacuum) are derived using octonionic algebras. In [10] a quaternionic equation is described for electromagnetic fields in inhomogeneous media. In [11], the D4-D5-E6 model that includes the Standard Model plus Gravity is constructed using octonionic fermion creators and annihilators. In [12] octonionic constructions are shown to be consistent with the $SU(3)_C$ gauge symmetry of QCD. It would appear that there are a number of implementations involving hypercomplex numbers that are consistent with the Standard Model. But there is still the question of why bother? What is shown here is why the bother might be worth it as a critical new link to string theory is provided, that may explain what dimensional compactification will relate to what experiments involving the standard model, and the formalism also allows for an explanation for Dark matter, all in a mathematics that can be absorbed into a Lagrangian formulation that could be consistent with a theory of Gravity.

To be more specific as regards the different strings. Type I superstring theory is an "open" string theory with critical dimension 10, with strings unoriented, and gauge SO(32). For closed string theories the left and right moving modes no longer have to be of the same type. If they are the same and don't obey supersymmetry, then D=26 and there are tachyons. If they are the same and obey supersymmetry (Type II), then the critical dimension is 10 with no gauge but two supersymmetries. If they are a mix with D10 'right-movers' and D26 'left-movers' (with 36 degrees of freedom), they are known as 'heterotic'. For heterotic strings with the two critical dimensionalities (D=10 and D=26), the 26 must compactify 16 as gauge degrees of freedom to reduce to 10. If compactification done with gauge E8xE8, spin(32)/Z2, or SO(16)xSO(16), then anomaly and tachyon free. Note, from [26]: "E6 is a subgroup of E8. E6 has 78 generators that form a sub-algebra of E8. E6 has a maximal subgroup SU(3)xSU(3)xSU(3)."

Often overlooked, but critical to agreement with the emanator hypothesis, is that a relation between spinors and vectors is required in classical superstring theory, and this can only happen when the space of direction perpendicular to the string worldsheet forms a normed division algebra [27,28]. So, classical superstring theories must exist in 8+2=10 dimensions as well.

Emergent Time

A variety of efforts have been made to find a definition of time that is somehow implicit to the main QFT and GR formalisms, whether it be a choice of vacuum for QFT in curved spacetime (and even if the spacetime is not curved [29]) which is indirectly a choice of time. Or seeking an internal time-reference in a full-GR quantum minisuperspace analysis of dust-shell collapse [30]. Or in seeking a notion of time in full general relativistic (GR) models, in the equilibrium sense, with an assumption of euclideanizability [31,32]. For the latter, the self-consistent stable solutions that were indicated showed the general utility of the euclideanizability hypothesis on emanation/propagation solutions in general (that is especially relevant, or interpretable, when the system is in equilibrium). In none of these efforts, however, was there success in identifying some internal notion of time, time, it seems, is an added construction, and this is consistent with the results shown in this paper, where we find that time is likely an emergent construct.

Emergent Evolution and Emergent Universal Learning

We see that the definition of the emanator process is not known, but that consistency arguments (such as achirality constructed from a collection of chiral emanations) lead to a certain set of forms. And that consistency with the {α, π c_∞} relation imposes further constraints on the form of the emanator. What is hypothesized is that the emanator is selected for maximal information transmission, thus emergent itself under that criterion. Let's now consider the maximal information transmission idea from the receiving end, e.g., maximal information receiving, or learning, in this context. If we turn to the information geometry analysis of learning in neural nets [33-35] (which uses differential geometry) we obtain a fundamental origin for statistical entropy (Shannon Entropy), and we identify optimal learning processes, based on expectation/maximization, that involves two-steps (as the name suggests) that may be done according to two fundamentally different conventions, e.g. the optimal learning involves four types of step, or is doubly chiral, consistent with the emanator 4-chiral processes described [36]. The potential applications of these results to Trigintaduonion encoded neuromanifolds is beyond the scope of this paper.

Objective Reduction, Zero-Divisors, and possible origins of Planck 's constant

A new mechanism for objective reduction [37,38] is also indicated by the way π enters the theory as a maximum anti-phase amount comprising part of the maximal perturbation propagation. Consider in the context where there is a 'classical' trigintaduonion path in a congruence of paths (a flow-line description). On the classical path in the congruences, we have α calculated using a +π maximal anti-phase, but this could also occur with –π maximal anti-phase as well, thus we could have a ±π phase toggle when a zero divisor is encountered in the 32D propagation (given the perturbations extending outside the 10D somewhat into the entire 32D). The zero-divisor discontinuity requires the field to reformulate a new 'consistency' with the 32D algebraic propagation (and 64D and higher, as well), which could have the result that since the prior π phase had the discontinuity, then it must toggle to the other, negative, phase, e.g., objective reduction may occur as a zero-divisor phase-toggle event.

Conclusion

Maximal information propagation as an emergent construct appears to require two from of propagation, an early hypercomplex 'emanation' that reduces to a chiral 10D propagation in a 32D trigintaduonion space, and standard propagation with complex propagators (consistent with the quantum deFinetti relation) operating inside that 10D propagation of geometry and gauge field. From the 'emanation' stage we see the maximum dimensionality and fractal limits provide the fundamental constants that then imprints upon the emergent geometry and gauge field, including giving rise to the constants α and π. The origin of α has been a long-standing mystery. So much so that the central role of α in modern physics is literally engraved in stone, the tombstones of Sommerfeld (which displays $\frac{e^2}{\hbar c}$, which is α) and Schwinger (which displays $\frac{\alpha}{2\pi}$) for example. Its origin has eluded physics for over a century, and appears to reside in the algebra of trigintaduonions.

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